AN IDENTITY ON AUTOMORPHISMS OF LIE IDEALS IN PRIME RINGS

NADEEM UR REHMAN

Abstract

In the present paper it is shown that a prime ring R with center Z satisfies s_4, the standard identity in four variables if R admits a non-identity automorphism σ such that $[u,v]-u^m[u^\sigma,u]^n u^\sigma \in Z$ for all u in some noncentral ideal L of R, whenever $char(R) > n+m$ or $char(R)=0$, where n and m are fixed positive integer.

Key words: Prime ring; Automorphisms; Maximal right ring of quotients; Generalized polynomial identity(GPI).

2010 Mathematics Subject Classification: 16N60; 16W20; 16R50.

1. Introduction

Throughout this article, R is a prime ring with center Z. For given $x,y \in R$, the Lie commutator of x,y is denoted by $[x,y]$ and defied by $[x,y]=xy-yx$. Recall that a ring R is prime if for any $a,b \in R$, $aRb=(0)$ implies $a=0$ or $b=0$. The standard identity s_4 in four variables is defined as follows:

$$s_4 = \sum (-1)^\tau X_{\tau(1)} X_{\tau(2)} X_{\tau(3)} X_{\tau(4)}$$

where $(-1)^\tau$ is the sign of a permutation τ of the symmetric group of degree 4.

The theory of commuting and centralizing maps on (semi-)prime rings was motivated by the result of Posner [20] and was developed by Brešar [4–6]. Posner's second theorem sates that if there exists a nonzero centralizing derivation on a prime ring R, then R is commutative. Mayne [18] obtained an analogous result for automorphisms of prime rings. Many people have extended Posner's result in various ways and obtained many powerful results. In [16], Lee and Lee generalized Posner's result by showing that if $char(R) \neq 2$ and $[d(x),x] \in Z$ for all x in a noncentral Lie ideal of R, then R is commutative. In [15], Lanski proved that if $[d(x),x]_n = 0$ for all x in a noncommutative Lie ideal of R, then $char(R)=2$ and $R \subseteq M_2(\mathbb{F})$ for $\mathbb{F}$ a field. A similar extension for Lie ideals in automorphism case was obtained by Mayne [19].

In [7], Carini and De Filippis studied the power-centralizing derivations on noncentral Lie ideals of prime rings. They proved that, if $char(R) \neq 2$ and $[d(x),x]^n \in Z$ for all x in a noncentral Lie ideal of R, then R satisfies s_4, the standard identity in four variables.

Recently, Wang [22], obtained similar result for automorphisms of prime rings. To be more specific, Wang discussed the following: Let R be a prime ring with center Z, L a noncentral Lie ideal of R and σ a nontrivial automorphism of R such that $[u^\sigma, u]^n \in Z$ for all $u \in L$. If either $Char(R) > n$ or $char(R) = 0$, then R satisfies s_4.

On other hand, the representative work of Herstein should be mention at least. Herstein [12], proved that if there exists a nonzero derivation d on a prime ring R such that the map $x \mapsto d(x)$ is commuting on R, then R may be noncommutative. That is, the following relation $[d(x), x]d(x) + d(x)[d(x), x] = 0$ for all $x \in R$ does not imply that $d = 0$. Motivated by the above result Cheng [10] proved the following, which can be considered as an extension of Posner's second theorem: if R is a 2-torsion free noncommutative prime ring and d be a derivation of R such that $[d(x), x]d(x) = 0$ for all $x \in R$, then $d = 0$.

The property $x^n = x$ has been among the favourites of many ring theorists over the last many decades since Jacobson [14]first studied the commutativity of rings satisfying this condition in order to generalize the classical Wedderburn theorem [23]. This result was further generalized by Sercoid and MacHale [21] who proved that commutativity of an arbitrary ring R (not necessarily prime) follows even if the above condition is weakened as $(xy)^n = xy$ for all $x, y \in R$ and integer $n = n(x, y) > 1$. Further, Bell and Ligh [3] obtained direct sum decomposition of ring satisfying the property $xy = (xy)^2 f(x, y)$, where $f(X, Y) \in \mathbb{Z} < X, Y >$, the ring of polynomial in two non-commuting indeterminates. Later, Ashraf [1] established a decomposition theorem for ring satisfying $yx = x^m f(xy)x^n$ or $xy = x^m f(xy)x^n$ where m, n are non-negative integers and $f(X) \in X^2\mathbb{Z}[X]$, which in turn allows us to determine the commutativity of R. Now in this perspective and inspired by Wang [22] and Cheng [10] works, one can consider the following related ring property:

Let $m \geq 0$, $n \geq 0$ be fixed integers and L a Lie ideal of prime ring R which admits an automorphism σ such that $[u, v] - u^m[u^\sigma, u]^n u^\sigma \in Z$.

In the present paper, it is shown that if R admits an automorphismσ satisfy the above condition, if $char(R) > n + m$ or $char(R) = 0$, then R satisfies s_4, the standard identity in four variables.

2. Preliminaries

For the sake of completeness we shall touch upon a few preliminary notions required for the exposition of the main theorem. Some of these notions are classical and we present them briefly, R will be prime ring with center Z and maximal right ring of quotients $Q = Q_{mr}(R)$. Note that Q is also a prime ring and the center C of Q, which is called the extended centroid of R, is a field. Moreover, $Z \subseteq C$ (for more explanation we refer to [2]). It is well known that any automorphism of R can be uniquely extended to an automorphism of Q. An automorphism σ of R is called Q-inner if there exists an invertible element $g \in Q$ such that $x^{\sigma} = gxg^{-1}$ for all $x \in R$. Otherwise, σ is called Q-outer. We denote by G the group of all automorphisms of R and by A_i the group consisting of all Q-inner automorphisms of R. Recall that a subset $\mathfrak{A}$ of G is said to be independent (modulo A_i) if for any $a_1, a_2 \in \mathfrak{A}$, $a_1 a_2^{-1} \in A_i$ implies $a_1 = a_2$. For instance, if a is an outer automorphism of R, then 1 and a are independent (modulo A_i). We present some well-known facts that will be used in the sequel.

Fact 2.1. *It is well known that any automorphisms of R can be extended to Q.*

Fact 2.2. *Let R be a prime ring and I a two-sided ideal of R. Then I, R, and Q satisfy the same generalized polynomial identities with coefficients in Q (see [8]).*

Fact 2.3. *Suppose that R is a prime ring and $\mathfrak{A}$ an independent subset of G modulo A_i. Let $\phi = \chi(x_i^{a_j}) = 0$ be a generalized identity with automorphisms of R reduced with respect to $\mathfrak{A}$. If for all $x_i \in X$, $a_j \in \mathfrak{A}$, the $x_i^{a_j}$-word degree of $\phi = \chi(x_i^{a_j})$ is strictly less than $char(R)$ when $char(R) \neq 0$, then $\chi(z_{ij}) = 0$ is also a generalized polynomial identity of R (see [9, Theorem 3]).*

Fact 2.4. Recall that, in case $char(R) = 0$, an automorphism σ of Q is called *Frobenius* if $(x)^{\sigma} = x$ for all $x \in C$. Moreover, in case $char(R) = p \geq 2$, an automorphism σ is *Frobenius* if there exists a fixed integer t such that $(x)^{\sigma} = x^{p^t}$ for all $x \in C$. In [9, Theorem 2] Chuang proves that if $\Phi(x_i, \alpha(x_i))$ is a generalized polynomial identity for R, where R is a prime ring and $\sigma \in Aut(R)$ an automorphism of R which is not Frobenius, then R also satisfies the non-trivial generalized polynomial identity $\Phi(x_i, y_i)$, where x_i and y_i are distinct indeterminates.

Fact 2.5. *Let R be a prime ring and L a noncentral Lie ideal of R. If $char(R) \neq 2$, then there exists a nonzero ideal I of R such that $0 \neq [I, R] \subseteq L$. If $char(R) = 2$ and $dim_c RC > 4$, then there exists a nonzero ideal I of R such that $0 \neq [I, R] \subseteq L$. Thus if either $char(R) \neq 2$ or $dim_C RC > 4$, then we may conclude that there exists a nonzero ideal I of R such that $[I, I] \subseteq L$.*

Fact 2.6. *Let R be a prime ring with extended centroid C. Then the following conditions are equivalent:*

(i) *$dim_C RC \leq 4$.*
(ii) *R satisfies S_4, the standard identity in four variables.*
(iii) *R is commutative or R embeds in $M_2(\mathbb{F})$, where $\mathbb{F}$ is a field.*
(iv) *R is algebraic of bounded degree 2 over C.*
(v) *R satisfies $[[x^2, y], [x, y]] = 0$.*

3. The results in Prime Rings

We begin with the following results which are imperative to establish of our main theorem.

Theorem 3.1. *Let R be a prime ring and σ a non-identity automorphism of R such that $[u,v] - u^m[u^\sigma, u]^n u^\sigma = 0$ for all u, v in a noncentral Lie ideal L of R, where n, m are fixed positive integer. If either $char(R) > n+m$ or $char(R) = 0$, then R satisfies s_4, the standard identity in four variables.*

Proof. We assume that $dim_C RC > 4$. In view of Fact 2.5, there exists a nonzero ideal I of R such that $[I, I] \subseteq L$. Using our hypothesis, we find that

(3.1) $$[[x,y],[z,w]] - [x,y]^m[[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma = 0 \text{ for all } x,y \in I.$$

Firstly, if σ is Q-inner, then there exists an invertible element $q \in Q$ such that $x^\sigma = qxq^{-1}$ for all $x \in R$. By [8, Theorem 2],

$$[[x,y],[z,w]] - [x,y]^m[q[x,y]q^{-1},[x,y]]^n q[x,y]q^{-1} = 0$$

is also an identity for RC. By Martindale's theorem in [17], RC is a primitive ring with nonzero socle. Since RC is primitive, there exist a vector space $\mathcal{V}$ over a division ring $\mathcal{D}$ such that RC is a dense ring of $\mathcal{D}$-linear transformations over $\mathcal{V}$. We divide the proof into two steps:

Step 1. Our aim is to show that for any $v \in \mathcal{V}$, v and vq are linearly $\mathcal{D}$-dependent. If v and vq are linearly $\mathcal{D}$-independent for some $v \in \mathcal{V}$, then we consider the following cases:

If $vq^{-1} \notin Span_{\mathcal{D}}\{v, vq\}$, then the set $\{v, vq, vq^{-1}\}$ is linearly $\mathcal{D}$-independent. By the density of RC there exist $x_0, y_0 \in RC$ such that

$$\begin{array}{lllll} vx_0 = v, & vqx_0 = 0, & vz_0 = vq & vqz = v & vq^{-1}x_0 = -vq \\ vy_0 = v, & vqy_0 = -v, & vw_0 = v & vqw = 0 & vq^{-1}y_0 = 0. \end{array}$$

We can easily see that

$$0 = v([[x,y],[z,w]] - [x,y]^m[q[x_0,y_0]q^{-1},[x_0,y_0]]^n q[x_0,y_0]q^{-1}) = v \neq 0, \text{ a contradiction.}$$

On the other hand if $vq^{-1} \in Span_{\mathcal{D}}\{v, vq\}$, then $vq^{-1} = v\alpha + vq\beta$ for some $\alpha, \beta \in \mathcal{D}$. In view of the density of RC, there exist $x_0, y_0, z_0, w_0 \in RC$ such that

$$\begin{array}{llll} vx_0 = v, & vqx_0 = 0 & vz_0 = qv & vqz_0 = v \\ vy_0 = v, & vqy_0 = v & vw_0 = v & vqw_0 = 0. \end{array}$$

Hence we find that

$$0 = v([q[x_0,y_0]q^{-1},[x_0,y_0]]^n q[x_0,y_0]q^{-1}) = \gamma v \neq 0$$

for some $\gamma \in \mathcal{D}$, again a contradiction.

Step 2. We have that v and qv are $\mathcal{D}$-dependent for every $v \in \mathcal{V}$. For each $v \in \mathcal{V}$, we write $vq = v\lambda_v$ where $\lambda_v \in \mathcal{D}$. Fix $0 \neq u \in \mathcal{V}$. Let $0 \neq v \in \mathcal{V}$ and write $vq = v\lambda_v$. Suppose first that v and u are $\mathcal{D}$-independent. Then $(u+v)\lambda_{u+v} = (u+v)q = uq + vq = u\lambda_u + v\lambda_v$. So $u(\lambda_{u+v} - \lambda_u) = v(\lambda_v - \lambda_{u+v})$, and hence $\lambda_{u+v} = \lambda_u = \lambda_v$. Suppose next that u and v are $\mathcal{D}$-dependent. Indeed, for any $w \in \mathcal{V}$, w and u are $\mathcal{D}$-independent, and then, by the proof above, we have $\lambda_w = \lambda_v$. Clearly, w and v are $\mathcal{D}$-independent. So $\lambda_w = \lambda_v$, implying that $\lambda_u = \lambda_v$. Thus λ_v is the independent choice of $v \in \mathcal{V}$. Consequently, $vq = v\lambda$ for all $v \in \mathcal{V}$, where $\lambda = \lambda_v$. By standard argument we see that $q \in C$, a contradiction. Thus $dim_C RC \leq 4$, and by Fact 2.6, R satisfies s_4, the standard identity in four variables.

Next we assume that σ is not Q-inner, then by Chuang [10, Main Theorem], R satisfies $[[x,y],[z,w]] - [x,y]^m[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma = 0$. Since either $char(R) > n$ or $char(R) = 0$, it follows from Fact 2.3 that $[[x,y],[z,w]] - [x,y]^m[[w_1,z_1],[x,y]]^n[w_1,z_1] = 0$ for all $x,y,z,w \in R$. Note that this is a polynomial identity and thus there exists a field $\mathbb{F}$ such that $R \subseteq M_k(\mathbb{F})$, the ring of $k \times k$ matrices over a field $\mathbb{F}$, where $k \geq 1$. Moreover, R and $M_k(\mathbb{F})$ satisfy the same polynomial identity [15, Lemma 1], that is $[[x,y],[z,w]] - [x,y]^m[[w_1,z_1],[x,y]]^n[w_1,z_1] = 0$ for all $x,y,w,z,z_1,w_1 \in M_k(\mathbb{F})$. But by choosing $x = e_{11}$, $y = e_{21}$, $w = e_{12}$, $z = e_{12}$, $w_1 = e_{11}$, $z_1 = e_{12}$ we get

$$0 = [[e_{11},e_{21}],[e_{12},e_{12}]] - [e_{11},e_{21}]^m[[e_{11},e_{12}],[e_{11},e_{21}]]^n[e_{11},e_{12}] = (-1)^n e_{12},$$

a contradiction. This completes the proof. □

Let $\mathcal{V}_\mathcal{D}$ be a right vector space over a division ring $\mathcal{D}$. We denote $End(\mathcal{V}_\mathcal{D})$ the ring of $\mathcal{D}$-linear transformations on $\mathcal{V}_\mathcal{D}$. A map $T : \mathcal{V} \to \mathcal{V}$ is called a semilinear transformation if T is additive and there is an automorphism ζ of $\mathcal{D}$ such that $T(v\gamma) = (Tv)\zeta(\gamma)$ for all $v \in \mathcal{V}$ and $\gamma \in \mathcal{D}$. Moreover, by a theorem of Jacobson [13, Isomorphism Theorem, p.79], there exists an invertible semilinear transformation $T : \mathcal{V} \to \mathcal{V}$ such that $\sigma(x) = TxT^{-1}$ for all $x \in End(\mathcal{V}_\mathcal{D})$, where σ is an automorphism of $End(\mathcal{V}_\mathcal{D})$.

Lemma 3.1. *Let σ be an automorphism of $End(\mathcal{V}_\mathcal{D})$ such that for every $x,y,z,w,z_1 \in End(\mathcal{V}_\mathcal{D})$, $[[x,y],[z,w]] - [x,y]^m[[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma,z_1] = 0$, where n,m are fixed positive integer. If $dim(\mathcal{V}_\mathcal{D}) \geq 2$, then σ is identity map of $End(\mathcal{V}_\mathcal{D})$.*

Proof. By a theorem of Jacobson [13, Isomorphism Theorem, p.79], there exists an invertible semilinear transformation $T : \mathcal{V} \to \mathcal{V}$ such that $\sigma(x) = TxT^{-1}$ for all $x \in End(\mathcal{V}_\mathcal{D})$, where σ is an automorphism of $End(\mathcal{V}_\mathcal{D})$. In particular, there exists an automorphism ζ of $\mathcal{D}$ such that $T(v\gamma) = (Tv)\zeta(\gamma)$ for all $v \in \mathcal{V}$ and $\gamma \in \mathcal{D}$. Using our hypothesis, we find that $0 = [[x,y],[z,w]] - [x,y]^m[[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma,z_1] =$

$[[x,y],[z,w]] - [x,y]^m[[T[x,y]T^{-1},[x,y]]^nT[x,y]^{-1},z_1]$ for all $x,y,z,w,z_1 \in End(\mathcal{V}_{\mathcal{D}})$. We divide our proof into the following cases:

There exists $v \in \mathcal{V}$ such that v and $T^{-1}v$ are $\mathcal{D}$-independent. Suppose first that $\{v, vT, vT^{-1}\}$ is $\mathcal{D}$-independent. Let $x,y,z \in End(\mathcal{V}_{\mathcal{D}})$ such that

$$\begin{array}{lll} xv = Tv, & xT^{-1}v = -v, & xTv = 0 \\ yv = Tv, & yT^{-1}v = 0, & yTv = v \\ wv = Tv & z_1T^{-1}v = 0 & zTv = Tv \\ zv = 0 & w_1T^{-1} = 0 & wTv = 0 \\ z_1v = 0, & z_1T^{-1}v = v, & z_1Tv = -v. \end{array}$$

Then $[x,y]v = 0$, $[x,y]T^{-1}v = v$, $[x,y]Tv = Tv$, $[z,w]v = Tv$ and hence

$$0 = ([[[x,y],[z,w]] - [x,y]^m[T[x,y]T^{-1},[x,y]]^nT[x,y]T^{-1},z])v = v, \text{ a contradiction}$$

Suppose next that $\{v, Tv, T^{-1}v\}$ is $\mathcal{D}$-dependent. Then there exist $\mu, \chi \in \mathcal{D}$ such that $Tv = v\mu + T^{-1}v\chi$. Moreover, we claim that $\chi \neq 0$. Indeed, if $\chi = 0$, then $Tv = v\mu$ and $v = T^{-1}v\mu$, a contradiction. Let $x,y,z,w,z_1 \in End(\mathcal{V}_{\mathcal{D}})$ such that

$$\begin{array}{lll} xv = Tv, & xT^{-1}v = -v, & z_1v = 0 \\ yv = Tv, & yT^{-1}v = 0, & z_1T^{-1}v = -v \\ zv = 0 & wv = Tv & zT^{-1}v = v. \end{array}$$

We can easily see that $0 = ([[[[x,y],[z,w]] - [x,y]^m[T[x,y]T^{-1},[x,y]]^nT[x,y]T^{-1},z])v = \eta v$, for some $\eta \in \mathcal{D}$, a contradiction.

We have that v and $T^{-1}v$ are $\mathcal{D}$-dependent for every $v \in \mathcal{V}$. For each $v \in \mathcal{V}$, we write $T^{-1}v = v\alpha_v$ where $\alpha_v \in \mathcal{D}$. Fix $0 \neq u \in \mathcal{V}$. Let $0 \neq v \in \mathcal{V}$ and write $T^{-1}v = v\alpha_v$. Suppose first that v and u are $\mathcal{D}$-independent. Then $(u+v)\alpha_{u+v} = (u+v)q = uq + vq = u\alpha_u + v\alpha_v$. So $u(\alpha_{u+v} - \alpha_u) = v(\alpha_v - \alpha_{u+v})$, and hence $\alpha_{u+v} = \alpha_u = \alpha_v$. Suppose next that u and v are $\mathcal{D}$-dependent. Since $dim(\mathcal{V}_{\mathcal{D}}) \geq 2$, there exists $w \in \mathcal{V}$ such that w and u are $\mathcal{D}$-independent, and then, by the proof above, we have $\alpha_w = \alpha_v$. Clearly, w and v are $\mathcal{D}$-independent. So $\alpha_w = \alpha_v$, implying that $\alpha_u = \alpha_v$. Thus α_v is independent of the choice of $v \in \mathcal{V}$. Consequently, $T^{-1}v = v\alpha$ for all $v \in \mathcal{V}$, where $\alpha = \alpha_v$. Now we have $\sigma(x)v = T(x(v\alpha)) = T((xv)\alpha) = xv$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$ and $v \in \mathcal{V}$. In particular, $(\sigma(x) - x)V = 0$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$. Thus $\sigma(x) = x$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$. This implies σ is the identity map of $End(\mathcal{V}_{\mathcal{D}})$, proving the lemma. □

Using both of these lemmas, we are ready to prove our main theorem.

Theorem 3.2. *Let R be a prime ring with center Z which admits a non-identity automorphism σ such that $[u,v] - u^m[u^\sigma,u]^nu^\sigma \in Z$ for all u in a noncentral ideal L of R, where n,m are* *fixed positive integer. If $char(R) > n+m$ or $char(R) = 0$, then R satisfies s_4, the standard identity in four variables.*

Proof. We assume that $dim_C RC > 4$. Then by Fact 2.5, there exists a nonzero ideal I of R such that $0 \neq [I, I] \subseteq L$. By assumption, we get

$$[[x,y],[z,w]] - [x,y]^m[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma \in Z \text{ for all } x, y \in I. \tag{3.2}$$

Suppose σ is Q-inner automorphism, there exists an invertible element $g \in Q$ such that $x^\sigma = gxg^{-1}$ for all $x \in R$. Then I satisfies

$$[[x,y],[z,w]] - [x,y]^m[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma \in Z. \tag{3.3}$$

By a Theorem of Chuang [8], I and Q satisfy the same generalized polynomial identities. Thus Q satisfied

$$[[x,y],[z,w]] - [x,y]^m[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma \in C. \tag{3.4}$$

Since $g \notin C$, therefore $\phi(t) = [[[x,y],[z,w]] - [x,y]^m[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma, z_1]$ for all $x, y, z, w, z_1 \in Q$ is a nontrivial generalized polynomial identity on Q. Denote by F the algebraic closure of C if C is infinite and set $F = C$ for C finite. Then $Q \otimes_C F$ is a prime ring with extended centroid F [11, Theorem 3.5]. Clearly $Q \cong Q \otimes_C C \subseteq Q \otimes_C F$. So we may regards Q as a subring $Q \otimes_C F$ and hence $\phi(t)$ is also a nontrivial generalized polynomial identity of $Q \otimes_C F$. Let $\mathcal{Q} = Q_{mr}(Q \otimes_C F)$, the maximal right ring of quotients of $Q \otimes_C F$. By [2, Theorem 6.4.4], $\phi(t)$ is also a nontrivial generalized polynomial identity on $\mathcal{Q}$. By Martindale's theorem [17], $\mathcal{Q} \cong End(\mathcal{V}_\mathcal{D})$, where $\mathcal{V}$ is a vector space over a division ring $\mathcal{D}$ and $\mathcal{D}$ is finite dimension over its center F. Recall that F is either algebraically closed or finite. From the finite dimensionality of D over F, it follows that $\mathcal{D} = F$. Hence $\mathcal{Q} \cong End(\mathcal{V}_F)$. By Lemma 3.1, we get a contradiction.

We now assume that σ is Q-outer automorphism, due to Chuang [8, Main Theorem], I and Q satisfies the same polynomial identity and hence R as well. Therefore R satisfies $[[[x,y],[z,w]] - [x,y]^m[[x,y]^\sigma,[x,y]]^n[x,y]^\sigma, z_1] = 0$. Since either $char(R) > n+m$ or $char(R) = 0$, it follows from Lemma 3.1 that $[[[x,y],[z,w]] - [x,y]^m[[s,t],[x,y]]^n[s,t], z] = 0$ for all $x, y, s, t, z, w, z_1 \in R$. Note that this is a polynomial identity and thus there exists a field $\mathbb{F}$ such that $R \subseteq M_k(\mathbb{F})$, the ring of $k \times k$ matrices over a field $\mathbb{F}$, where $k > 1$. Moreover, R and $M_k(\mathbb{F})$ satisfy the same polynomial identity [15, Lemma 1], that is $[[[x,y],[z,w]] - [x,y]^m[[s,t],[x,y]]^n[s,t], z] = 0$ for all $x, y, s, t, z \in M_k(\mathbb{F})$. Let e_{ij} be a matrix unit with 1 in the (i,j)-entry and zero elsewhere. Since $dim_C RC > 4$, we see that $k > 2$. By choosing $x = e_{11}$, $y = e_{21}$, $z = e_{12}$, $w = e_{12}$, $s = e_{11}$, $t = e_{12}$, $z_1 = e_{31}$ we get $0 = [[[x,y],[z,w]] - [x,y]^m[[s,t],[x,y]]^n[s,t], z] =$

$[[[e_{11}, e_{21}, [e_{12}, e_{12}]] - [e_{11}, e_{21}]^m[[e_{11}, e_{12}], [e_{11}, e_{21}]]^n[e_{11}, e_{12}], e_{31}] = (-1)^{n+1}e_{31}$, a contradiction. Thus $dim_C RC \leq 4$. In View of Fact 2.6, we get required result. With this the proof is complete. □

References

[1] Ashraf, M., *Structure of certain periodic rings and near-rings*, Rend. Sem. Mat. Univ. Pol. Torino 53 (1995), 61-67.

[2] Beidar, K. I., Martindale III, W. S., Mikhalev, A. V., Rings with Generalized Identities, *Pure and Applied Mathematics, Marcel Dekker 196, New York*, 1996.

[3] Bell, H. E. and Ligh, S., *Some decomposition theorems for periodic rings and near-rings*, Math. J. Okayama Univ. 31(1989), 93-99.

[4] Brešar, M., *Centralizing mappings and derivations in prime ring*, J. Algebra 156 (1993), 385-394.

[5] Brešar, M., *Commuting traces of biadditive mappings, commutativity preserving mappings and Lie mappings*, Trans. Amer Math. Soc. 335 (1993), 525-546.

[6] Brešar, M., *On a generalization of the notion of centralizing mappings*, Proc. Amer. Math. Soc. 114 (1992), 641-649.

[7] Carini, L. and De Filliippis, V., *Commutators with power central values on a Lie ideals*, Pacfic J. Math. 193 (2000), 269-278.

[8] Chuang, C. L., *GPIs having coefficients in Utumi quotient rings*, Proc. Amer. Math. Soc. 103 (1988), 723-728.

[9] Chuang, C. L., *Differential identities with automorphism and anti-automorphism-II*, J. Algebra 160(1993), 291-335.

[10] Cheng, H., *Some results about derivations of prime rings*, J. Math. Reser. Expos. 25(4) (2005), 625-633.

[11] Erickson, T. S., Martindale III, W., Osborn J. M., *Prime nonassociative algebras*, Pacfic. J. Math. 60 (1975), 49-63.

[12] Herstein, I. N., *Derivations of prime rings having poer central values*, Contemp. Math. 13 (1982), 163-171.

[13] Jacobson, N., Structure of rings, *Amer. Math. Soc. Colloq. Pub. 37 Rhode Island* (1964).

[14] Jacobson, N., *Structure theory of algebraic algebras of bounded degree*, Ann. of Math. 46 (1945), 695-707.

[15] Lanski, C., *An Engel condition with derivation*, Proc. Amer. Math. Soc. 118 (1993), 731-734.

[16] Lee, P. H. and Lee, T. K., *Lie ideals of prime rings with derivations*, Bull. Inst. Math. Acad. Sin. 11 (1983), 75-80.

[17] Martindale III, W. S., *Prime rings satisfying a generalized polynomial identity*, J. Algebra 12 (1969), 576-584.

[18] Mayne, J. H., *Centralizing automorphisms of prime rings*, Canad. Math. Bull. 19 (1976), 113-115.

[19] Mayne, J. H., *Centralizing automorphisms of Lie ideals in prime rings*, Canad. Math. Bull. 35 (1992), 510-514.

[20] Posner, E. C., *Derivations in prime rings*, Proc. Amer. Math. Soc., 8 (1957), 1093-1100.

[21] Searcoid, M.O. and MacHale, D., *Two elementary generalizations for Boolean rings*, Amer. Math. Monthly 93 (1986), 121-122.

[22] Wang, Y., *Power-centralizing automorphisma of Lie ideals in prime rings*, Comm. Algebra 34 (2006), 609-615.

[23] Wedderburn, J. H. M., *A theorem on finite algebras*, Trans. Amer. Math. Soc. 6(1905), 349-352.

Nadeem ur Rehman, Department of Mathematics, Aligarh Muslim University, Aligarh-202002 India

Email address: nu.rehman.mm@amu.ac.in