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On equivalence of some subcategories of modules in Morita contexts

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ABSTRACT. A Morita context $(R,\ {}_RV_S,\ {}_SW_R, S)$ defines the isomorphism $\mathcal{L}_0(R) \cong \mathcal{L}_0(S)$ of lattices of torsions $r \geq r_I$ of R-Mod and torsions $s \geq r_J$ of S-Mod, where I and J are the trace ideals of the given context. For every pair (r, s) of corresponding torsions the modifications of functors $T^W = W\otimes_R$- and $T^V = V\otimes_S$- are considered:

$$R\text{-}Mod \supseteq \mathcal{P}(r) \quad \underset{\bar{T}^V = (1/r)\cdot T^V}{\overset{\bar{T}^W = (1/s)\cdot T^W}{\longleftrightarrow}} \quad \mathcal{P}(s) \subseteq S\text{-}Mod,$$

where $\mathcal{P}(r)$ and $\mathcal{P}(s)$ are the classes of torsion free modules. It is proved that these functors define the equivalence

$$\mathcal{P}(r) \cap \mathcal{J}_I \approx \mathcal{P}(s) \cap \mathcal{J}_J,$$

where $\mathcal{P}(r) = \{_RM \mid r(M) = 0\}$ and $\mathcal{J}_I = \{_RM \mid IM = M\}$.

Let $(R,\ {}_RV_S,\ {}_SW_R, S)$ be an arbitrary Morita context with the bimodule morphisms

$$(,):\ V \otimes_S W \longrightarrow R, \qquad\qquad [,]:\ W \otimes_R V \longrightarrow S,$$

satisfying the conditions of associativity:

$$(v, w)v_1 = v[w, v_1], \qquad\qquad [w, v]w_1 = w(v, w_1) \tag{1}$$

for $v, v_1 \in V$ and $w, w_1 \in W$. We denote by $I = (V, W)$ and $J = [W, V]$ the *trace ideals* of this context, where I is ideal of R and J is ideal of S.

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They define the torsions r_I in R-Mod and r_J in S-Mod such that the classes of torsion free modules are:

$$\begin{aligned}\mathcal{P}(r_I) &= \{{}_RM \mid I\,m = 0,\, m \in M \implies m = 0\},\\ \mathcal{P}(r_J) &= \{{}_SN \mid J\,n = 0,\, n \in N \implies n = 0\},\end{aligned}$$

i.e. r_I and r_J are determined by the smallest Gabriel filters, containing I and J, respectively [7].

In the lattices $\mathcal{L}(R)$ and $\mathcal{L}(S)$ of all torsions of R-Mod and S-Mod, respectively, we distinguish the following sublattices:

$$\begin{aligned}\mathcal{L}_0(R) &= \{r \in \mathcal{L}(R) \,|\, r \geq r_I\},\\ \mathcal{L}_0(S) &= \{s \in \mathcal{L}(S) \,|\, s \geq r_J\}.\end{aligned} \tag{2}$$

The following result is well known ([1], [4], [5], [7]).

Theorem 1. *There exists a preserving order bijection between the torsions of R-Mod containing r_I and torsions of S-Mod containing r_J, i.e. $\mathcal{L}_0(R) \cong \mathcal{L}_0(S)$.* □

This bijection is obtained with the help of the functors:

$$R\text{-}Mod \;\underset{H^W = Hom_S(W,-)}{\overset{H^V = Hom_R(V,-)}{\rightleftarrows}}\; S\text{-}Mod, \tag{3}$$

acting by H^V and H^W to the injective cogeneratots of torsions [4]. From the definitions it follows

Lemma 2. *([4], Lemma 4). If (r, s) is a pair of corresponding torsions in the sense of Theorem 1 (i.e. $H^V(r) = s$ and $H^W(s) = r$), then $H^V(\mathcal{P}(r)) \subseteq \mathcal{P}(s)$ and $H^W(\mathcal{P}(s)) \subseteq \mathcal{P}(r)$, where $\mathcal{P}(r)$ and $P(s)$ are the classes of torsion free modules.* ($\mathcal{P}$) □

Now we consider the following functors accompanying the given Morita context:

$$R\text{-}Mod \;\underset{T^V = V \otimes_S -}{\overset{T^W = W \otimes_R -}{\rightleftarrows}}\; S\text{-}Mod \tag{4}$$

with the natural transformations

$$\eta : T^V T^W \longrightarrow 1_{R-Mod}, \qquad \rho : T^W T^V \longrightarrow 1_{S-Mod},$$

defined by the rules:

$$\eta_M(v \otimes w \otimes m) = (v, w)m, \qquad \rho_N(w \otimes v \otimes n) = [w, v]n, \tag{5}$$

for $v \otimes w \otimes m \in T^V T^W(M)$, $M \in R\text{-}Mod$ and $w \otimes v \otimes n \in T^W T^V(N)$, $N \in S\text{-}Mod$. By definitions it follows:

$$Im\,\eta_M = IM, \qquad Im\,\rho_N = JN.$$

It is easy to verify the following relations:

$$T^W(\eta_M) = \rho_{T^W(M)}, \tag{6}$$

$$T^V(\rho_N) = \eta_{T^V(N)}, \tag{7}$$

for every $M \in R\text{-}Mod$ and $N \in S\text{-}Mod$ (i.e. (T^W, T^V) and (η, ρ) define a wide Morita context in the sense of [3]).

For an arbitrary class of modules $\mathcal{K} \subseteq R\text{-}Mod$ we denote:

$$\begin{aligned} \mathcal{K}^{\uparrow} &= \{X \in R\text{-}Mod \mid Hom_R(X,Y) = 0 \ \forall Y \in \mathcal{K}\}, \\ \mathcal{K}^{\downarrow} &= \{Y \in R\text{-}Mod \mid Hom_R(X,Y) = 0 \ \forall X \in \mathcal{K}\}. \end{aligned}$$

If r is a torsion of $R\text{-}Mod$, $\mathcal{R}(r) = \{M \in R\text{-}Mod \mid r(M) = M\}$ and $\mathcal{P}(r) = \{M \in R\text{-}Mod \mid r(M) = 0\}$, then $\mathcal{R}(r) = \mathcal{P}(r)^{\uparrow}$ and $\mathcal{P}(r) = \mathcal{R}(r)^{\downarrow}$ ([5], [7], [8]).

The following statement is known ([6], lemma 3), but for convenience we give the proof.

Lemma 3. *If (r, s) is a pair of corresponding torsions in the sense of Theorem 1, then $T^W(\mathcal{R}(r)) \subseteq \mathcal{R}(s)$ and $T^V(\mathcal{R}(s)) \subseteq \mathcal{R}(r)$.*

Proof. Let ${}_SN \in \mathcal{R}(s) = \mathcal{P}(s)^{\uparrow}$, i.e. $Hom_S(N, Y) = 0$ for every $Y \in \mathcal{P}(s)$. If $M \in \mathcal{P}(r)$, then by Lemma 2 ${}_SH^V(M) = Hom_R(V, M) \in \mathcal{P}(s)$. Now from $N \in \mathcal{R}(s)$ it follows that $Hom_S(N, Hom_R(V, M)) = 0$. By adjunction

$$Hom_R(V \otimes_S N, M) \cong Hom_S(N, Hom_R(V, M)) = 0$$

for every $M \in \mathcal{P}(r)$, therefore $V \otimes_S N \in \mathcal{P}(r)^{\uparrow} = \mathcal{R}(r)$, i.e. $T^V(\mathcal{R}(s)) \subseteq \mathcal{R}(r)$. By symmetry the relation $T^W(R(r)) \subseteq R(s)(\mathcal{R})$ is true. □

In continuation we mention some facts about the classes of modules determined by trace ideals $I \lhd R$ and $J \lhd S$ in the categories $R\text{-}Mod$ and $S\text{-}Mod$, respectively. The ideal $I \lhd R$ defines in $R\text{-}Mod$ the following classes of modules:

$$\begin{aligned} \mathcal{A}(I) &= \{M \in R\text{-}Mod \mid IM = 0\}, \\ \mathcal{J}_I &= \{M \in R\text{-}Mod \mid IM = M\}, \\ \mathcal{F}_I &= \{M \in R\text{-}Mod \mid I\,m = 0,\, m \in M \Longrightarrow m = 0\} = \mathcal{P}(r_I). \end{aligned}$$

The modules of $\mathcal{J}_I$ are called *I-accessible* and

$$\mathcal{J}_I = \{M \in R\text{-}Mod \mid Im\, \eta_M = M\}.$$

The following relations are known ([7], [8]):

$$\mathcal{J}_I = \mathcal{A}(I)^{\uparrow}, \qquad \mathcal{F}_I = \mathcal{A}(I)^{\downarrow}. \tag{8}$$

Similarly we define the classes $\mathcal{A}(J)$, $\mathcal{J}_J$ and $\mathcal{F}_J$ in $S\text{-}Mod$ with the relations $\mathcal{J}_J = \mathcal{A}(J)^{\uparrow}$ and $\mathcal{F}_J = \mathcal{A}(J)^{\downarrow}$, where $\mathcal{F}_J = \mathcal{P}(r_J)$.

Lemma 4. *Let (r, s) be a pair of corresponding torsions (Theorem 1). Then $\mathcal{A}(I) \subseteq \mathcal{R}(r)$ and $\mathcal{A}(J) \subseteq \mathcal{R}(s)$.*

Proof. From $r \geq r_I$ it follows $\mathcal{P}(r) \subseteq \mathcal{P}(r_I) = \mathcal{F}_I$ and by (8) we obtain

$$\mathcal{R}(r) = \mathcal{P}(r)^{\uparrow} \supseteq \mathcal{P}(r_I)^{\uparrow} = \mathcal{F}_I^{\uparrow} = \mathcal{A}(I)^{\downarrow\uparrow} \supseteq \mathcal{A}(I).$$

Similarly, $\mathcal{R}(s) \supseteq \mathcal{A}(J)$. □

From now on we fix an arbitrary pair (r, s) of corresponding torsions, i.e. $r \geq r_I$, $s \geq r_J$, $s = H^V(r)$ and $r = H^W(s)$ (Theorem 1). We consider the following modifications of the functors T^W and T^V:

$$\begin{array}{ccc}
R\text{-}Mod & \underset{T^V}{\overset{T^W}{\rightleftarrows}} & S\text{-}Mod \\
\Big\downarrow {\scriptstyle 1/r} & & \Big\downarrow {\scriptstyle 1/s} \\
R\text{-}Mod & \underset{\bar{T}^V}{\overset{\bar{T}^W}{\rightleftarrows}} & S\text{-}Mod,
\end{array}$$

where $(1/r)(M) = M/r(M)$, $(1/s)(N) = N/s(N)$, $\bar{T}^W = (1/s) \cdot T^W$ and $\bar{T}^V = (1/r) \cdot T^V$. So, by definition:

$$\bar{T}^W({}_R M) = (W \otimes_R M)/s(W \otimes_R M), \quad \bar{T}^V({}_S N) = (V \otimes_S N)/r(V \otimes_S N) \tag{9}$$

for $M \in R\text{-}Mod$ and $N \in S\text{-}Mod$. Denote by α and β the natural transformations:

$$\alpha : T^W \longrightarrow \bar{T}^W, \qquad \beta : T^V \longrightarrow \bar{T}^V,$$

where

$$\alpha_M : T^W(M) \longrightarrow T^W(M)/s(T^W(M))$$

and

$$\beta_N : T^V(N) \longrightarrow T^V(N)/r(T^V(N))$$

are the natural epimorphisms. Since the functors $T^W$and T^V are right exact, it is clear that the functors $\bar{T}^W$ and $\bar{T}^V$ preserve epimorphisms. By definitions of $\bar{T}^W$ and $\bar{T}^V$ it follows that $\bar{T}^W(M) \in \mathcal{P}(s)$ and $\bar{T}^V(N) \in \mathcal{P}(r)$ for every $M \in R\text{-}Mod$ and $N \in S\text{-}Mod$, therefore we can consider the restrictions of these functors on the subcategories $\mathcal{P}(r)$ and $\mathcal{P}(s)$:

$$\mathcal{P}(r) \quad \underset{\bar{T}^V}{\overset{\bar{T}^W}{\rightleftarrows}} \quad \mathcal{P}(s). \tag{10}$$

In the situation (10) there exist the modifications of natural transformations η and ρ:

$$\bar{\eta} : \bar{T}^V\bar{T}^W \longrightarrow 1_{\mathcal{P}(r)}, \qquad \bar{\rho} : \bar{T}^W\bar{T}^V \longrightarrow 1_{\mathcal{P}(s)},$$

which are defined (see [3]) as follows. For every $M \in \mathcal{P}(r)$ applying T^V to the exacte sequence

$$0 \to s(T^W(M)) \ \underset{\subseteq}{\xrightarrow{i_M}} \ T^W(M) \ \underset{nat}{\xrightarrow{\alpha_M}} \ T^W(M)/s(T^W(M)) \to 0, \tag{11}$$

we obtain the diagram:

$$\begin{array}{c} r(T^V\bar{T}^W(M)) \\ \Big\downarrow i \cap\! \mid \\ T^V(s(T^W(M))) \xrightarrow{T^V(i_M)} T^VT^W(M) \xrightarrow{T^V(\alpha_M)} T^V\bar{T}^W(M) \xrightarrow{\beta_{\bar{T}^W(M)}} \bar{T}^V\bar{T}^W(M) \to 0 \\ \eta_M \searrow \qquad \Big\downarrow \eta'_M \qquad \swarrow \overline{\eta}_M \\ M \end{array} \tag{12}$$

Since $s(T^W(M)) \in \mathcal{R}(s)$, by Lemma 3 $T^V(s(T^W(M))) \in \mathcal{R}(r)$, so from $M \in \mathcal{P}(r)$ it follows $Hom_R(T^V(s(T^W(M))), M) = 0$, therefore $\eta_M \cdot T^V(i_M) = 0$. Since $Im\, T^V(i_M) = Ker\, T^V(\alpha_M) \subseteq Ker\, \eta_M$ and $T^V(\alpha_M)$ is an epimorphism, there exists an unique morphism η'_M such that $\eta'_M \cdot T^V(\alpha_M) = \eta_M$. The following step: from $M \in \mathcal{P}(r)$ and $r(T^V\bar{T}^W(M)) \in \mathcal{R}(r)$ it follows $\eta'_M \cdot i = 0$ and there exists an unique morphism $\bar{\eta}_M$ such that $\bar{\eta}_M \cdot \beta_{\bar{T}^W(M)} = \eta'_M$. So, by definitions we have:

$$\eta_M = \overline{\eta}_M \cdot \beta_{\bar{T}^W(M)} \cdot T^V(\alpha_M). \tag{13}$$

In such a way it is obtained a natural transformations $\overline{\eta}$ ([3]) and symmetrically $\bar{\rho}$ is defined. From these definitions follows immediately

Lemma 5. *a) If the module $M \in \mathcal{P}(r)$ is I-accessible (i.e. η_M is epi), then $\bar{\eta}_M$ is an epimorphism.*
b) If the module $N \in \mathcal{P}(s)$ is J-accessible, then $\bar{\rho}_N$ is an epimorphism. □

Now we consider in $\mathcal{P}(r)$ and $\mathcal{P}(s)$ the following subcategories of torsion free and accessible modules:

$$\mathcal{A} = \mathcal{P}(r) \cap \mathcal{J}_I \subseteq R\text{-}Mod, \qquad \mathcal{B} = \mathcal{P}(s) \cap \mathcal{J}_J \subseteq S\text{-}Mod.$$

Lemma 6. *The functors $\bar{T}^W$ and $\bar{T}^V$ transfer subcategories $\mathcal{A}$ and $\mathcal{B}$ each one in another, i.e. $\bar{T}^W(\mathcal{A}) \subseteq \mathcal{B}$ and $\bar{T}^V(\mathcal{B}) \subseteq \mathcal{A}$.*

Proof. Let $M \in \mathcal{A}$. Since $\bar{T}^W(M) \in \mathcal{P}(s)$, it is sufficient to check that $\bar{T}^W(M) \in \mathcal{J}_J$. For that we consider the following commutative diagram:

$$\begin{array}{ccc} T^W T^V T^W(M) & \xrightarrow{\quad \rho_{T^W(M)} \quad} & T^W(M) \\ \Big\downarrow {\scriptstyle T^W T^V(\alpha_M)} & & \Big\downarrow {\scriptstyle \alpha_M} \\ T^W T^V \bar{T}^W(M) & \xrightarrow{\quad \rho_{\bar{T}^W(M)} \quad} & \bar{T}^W(M) \end{array} \qquad (14)$$

Since $M \in \mathcal{J}_I$, η_M is epi, therefore $T^W(\eta_M)$ is epi. From (6) $\rho_{T^W(M)} = T^W(\eta_M)$, so $\rho_{T^W(M)}$ is epi, therefore $\alpha_M \cdot \rho_{T^W(M)}$ also is epi. Now diagram (14) shows that $\rho_{\bar{T}^W(M)}$ is epimorphism, i.e. $\bar{T}^W(M) \in \mathcal{J}_J$. This proves that $\bar{T}^W(\mathcal{A}) \subseteq \mathcal{B}$. By symmetry $\bar{T}^V(\mathcal{B}) \subseteq \mathcal{A}$. □

Another proof of Lemma 6 follows from the remark that

$$T^W(\mathcal{J}_I) \subseteq \mathcal{J}_J, \qquad T^V(\mathcal{J}_J) \subseteq \mathcal{J}_I. \tag{15}$$

Indeed, if $M \in \mathcal{J}_I$ then:

$$J(W \otimes_R M) = [W, V] W \otimes_R M = W(V, W) \otimes_R M = \\ = W \otimes_R (V, W) M = W \otimes_R IM = W \otimes_R M,$$

i.e. $T^W(M) \in \mathcal{J}_J$, and similarly for the second relation.

Now from (15) for every $M \in \mathcal{J}_I$ we obtain:

$$J \cdot \bar{T}^W(M) = J \cdot [(W \otimes_R M)/s(W \otimes_R M)] = \\ = [J(W \otimes_R M) + s(W \otimes_R M)]/s(W \otimes_R M) \overset{(15)}{=} \\ = [W \otimes_R M + s(W \otimes_R M)]/s(W \otimes_R M) = \\ = (W \otimes_R M)/s(W \otimes_R M) = \bar{T}^W(M),$$

therefore $\bar{T}^W(M) \in \mathcal{J}_J$.

Lemma 6 permits to obtain by restriction the functors:

$$\mathcal{A} \underset{\bar{T}^V}{\overset{\bar{T}^W}{\rightleftarrows}} \mathcal{B} \tag{16}$$

with the natural transformations $\bar{\eta}$ and $\bar{\rho}$.

Lemma 7. *a) For every $M \in \mathcal{P}(r)$, $I \cdot Ker\,\bar{\eta}_M = 0$, i.e. $Ker\,\bar{\eta}_M \in \mathcal{A}(I) \subseteq \mathcal{R}(r)$.*

b) For every $N \in \mathcal{P}(s)$, $J \cdot Ker\,\bar{\rho}_N = 0$, i.e. $Ker\,\bar{\rho}_N \in \mathcal{A}(J) \subseteq \mathcal{R}(s)$.

Proof. From definition of $\bar{\eta}_M$ (see (12), (13)) it is clear that $\bar{\eta}_M$ acts as follows:

$$\bar{\eta}_M\overline{(v \otimes (w \otimes m + s(W \otimes_R M))} = \eta_M(v \otimes w \otimes m) = (v, w)m,$$

where $\overline{(v \otimes (w \otimes m + s(W \otimes_R M))} = \beta_{\bar{T}^W(M)} T^V(\alpha_M)(v \otimes w \otimes m)$.

If $\overline{(v \otimes (w \otimes m + s(W \otimes_R M))} \in Ker\,\bar{\eta}_M$,
then $\eta_M(v \otimes w \otimes m) = (v, m)m = 0$ and for every $(v', w') \in I$ we obtain:

$$\begin{aligned}
(v', w')&\overline{(v \otimes (w \otimes m + s(W \otimes_R M))} = \\
&= \overline{(v', w')v \otimes (w \otimes m + s(W \otimes_R M))} = \\
&= \overline{v'[w', v] \otimes (w \otimes m + s(W \otimes_R M))} = \\
&= \overline{v' \otimes ([w', v]w \otimes m + s(W \otimes_R M))} = \\
&= \overline{v' \otimes (w'(v, w) \otimes m + s(W \otimes_R M))} = \\
&= \overline{v' \otimes (w' \otimes (v, w)m + s(W \otimes_R M))} = 0,
\end{aligned}$$

because $(v, w)m = 0$. From this we can conclude that $I \cdot Ker\,\bar{\eta}_M = 0$ and by Lemma 4 $Ker\,\bar{\eta}_M \in \mathcal{A}(I) \subseteq \mathcal{R}(r)$. The statement (b) follows from symmetry. □

Lemma 8. *a) $Ker\,\bar{\eta}_M = 0$ for every $M \in \mathcal{P}(r)$. b) $Ker\,\bar{\rho}_N = 0$ for every $N \in \mathcal{P}(s)$.*

Proof. Since $Ker\,\bar{\eta}_M \subseteq \bar{T}^V\bar{T}^W(M) \in \mathcal{P}(s)$, we have $Ker\,\bar{\eta}_M \in \mathcal{P}(r)$. By Lemma 7 $Ker\,\bar{\eta}_M \in \mathcal{R}(r)$, therefore $Ker\,\bar{\eta}_M \in \mathcal{R}(r) \cap \mathcal{P}(r) = \{0\}$. Similarly $Ker\,\bar{\rho}_N = 0$ for $N \in \mathcal{P}(s)$. □

Theorem 9. *For every pair (r, s) of corresponding torsions (in the sense of Theorem 1) the functors $\bar{T}^W$ and $\bar{T}^V$ (see (10)) with natural transformations $\bar{\eta}$ and $\bar{\rho}$ define an equivalence between the subcategories of torsion free and accessible modules $\mathcal{A} = \mathcal{P}(r) \cap \mathcal{J}_I \subseteq R$-Mod and $\mathcal{B} = \mathcal{P}(s) \cap \mathcal{J}_J \subseteq S$-Mod.*

Proof. If $M \in \mathcal{A}$, then by Lemma 5 a) $\bar{\eta}_M$ is epi. Moreover, from $M \in \mathcal{P}(r)$ by Lemma 8 a) we conclude that $\bar{\eta}_M$ is mono, so $\bar{\eta}_M$ is an ismorphism. Symmetrically, for every $N \in \mathcal{B}$ we obtain that $\bar{\rho}_N$ is an isomorphism. Therefore the functors $\bar{T}^W$ and $\bar{T}^V$ with the natural transformations $\bar{\eta}$ and $\bar{\rho}$ establish the equivalence $\mathcal{A} \approx \mathcal{B}$. □

The more general situation of wide Morita contexts is studied in [3]. The equivalence of Theorem 9 can be proved by [3, Theorem 2.6], using the preceding lemmas. We exposed the direct proof of this result.

For the particular case of the smallest pair (r_I, r_J) of corresponding torsions we have

Corollary 10. *([2], [3]). The subcategories of torsion free and accessible modules* $\mathcal{P}(r_I) \cap \mathcal{J}_I$ *and* $\mathcal{P}(r_J) \cap \mathcal{J}_J$ *are equivalent.* □

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