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A SHORT NOTE

A note to our paper "Automorphisms of homogeneous symmetric groups and hierarchomorphisms of rooted trees"

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Abstract. The results on automorphisms of homogeneous alternating groups are corrected and improved.

This note is a supplement to the recent paper [LS]. We assume that the paper is available to the reader, and freely use notations and terminology introduced there.

It was falsely asserted in [LS] that every automorphism of $A(\partial T_\Theta)$ is locally inner. So, some parts of assertions of Proposition 10, Corollary 2, Theorem 13, namely the assertions in parenthesis, are false.

We give below slightly modified and corrected Proposition 10 with corrected and more clear proof and state result on the automorphisms of the group $A(\partial T_\Theta)$.

Proposition 1. *Let X denote either H or AH. Let $\alpha \in \operatorname{Aut} X_\Omega$ be such that $\alpha(X_n) \leq X_k$ where $1 < n \leq k$. Then $\alpha|_{X_n} \in \operatorname{Inn} H_k$.*

Proof. Let $g \in X_\Omega$. We have $g \in X_n$ for some $n \in \mathbb{N}$. Since X_Ω is union of its subgroups X_n ($n \in \mathbb{N}$), there exists $k \in \mathbb{N}$ such that

$$\alpha(X_n) \leq X_k.$$

Let us show that $\alpha|_{X_n}$ is induced by an inner automorphism of H_k. By Corollary 3.13c of [Rub] the automorphism α is induced by a homeomorphism γ of ∂T_Ω. Note that every homeomorphism belonging to H_k is determined by its action on $V_k(T_\Omega)$. Suppose that for some $1 \leq i, j \leq f_\Omega(n)$, $i \neq j$ and $1 \leq l \leq f_\Omega(k)$ we have

$$\gamma^{-1}(P_{kl}) \cap P_{ni} \neq \emptyset, \tag{1}$$

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$$\gamma^{-1}(P_{kl}) \cap P_{nj} \neq \emptyset. \tag{2}$$

Let $g \in X_n$ be such that

$$g(P_{ni}) = P_{ni}, \tag{3}$$

$$g(P_{nj}) \neq P_{nj}. \tag{4}$$

Since $g^\gamma \in X_k$, we get $g^\gamma(P_{kl}) = P_{km}$, where $1 \leq m \leq f_\Omega(k)$. Taking into account (1) and (3) we get $g^\gamma(P_{kl}) = P_{kl}$ and by $g^\gamma \in X_k$ we have $g^\gamma(x) = x$ for all $x \in P_{kl}$.

Also taking into account (2) and (4) we get that there exists $x_0 \in P_{kl} \cap \gamma(P_{nj})$ such that $g^\gamma(x_0) \neq x_0$. This is a contradiction. Hence, $\gamma^{-1}(P_{kl}) \subset P_{ni}$ for some i such that $1 \leq i \leq f_\Omega(n)$.

Let

$$\gamma(P_{ni}) = P_{kl_{i1}} \sqcup \ldots \sqcup P_{kl_{i,r(i)}}$$

for some $r(i)$, where $1 \leq l_{i1}, \ldots, l_{i,r(i)} \leq f_\Omega(k)$.

Since X_n is transitive, we can choose $g_i \in X_n$, $1 \leq i \leq f_\Omega(n)$ such that $g_i(P_{ni}) = P_{n1}$. Since $g_i^\gamma \in X_k$ and the sets $\gamma(P_{ni})$ and $\gamma(P_{nj})$ do not intersect, there exists a mapping

$$t : \{1, \ldots, r(i)\} \times \{1, \ldots, f_\Omega(n)\} \to \{1, \ldots, r(1)\}$$

such that for every m and i, $1 \leq m \leq r(i)$, $1 \leq i \leq f_\Omega(n)$, the following equality holds

$$g_i\left(P_{kl_{im}}^{\gamma^{-1}}\right) = P_{kl_{1,t(m,i)}}^{\gamma^{-1}}. \tag{5}$$

Note that $t(m,i)$ does not depend on the choice of g_i. Obviously, $t(m,i)$ is a bijection for every fixed i. Thus $r(i)$ does not depend on i for $1 \leq i \leq f_\Omega(n)$. It is easy to see that $r = r(i) = f_\Omega(n)/f_\Omega(n-1)$ for all $1 \leq i \leq f_\Omega(n)$.

Let us define a homeomorphism $\pi \in H_k$ as follows: the homeomorphism π maps the vertex $v_{kl_{im}}$ corresponding to the ball $P_{kl_{im}}$ to the vertex $g_i^{-1}(v_{k,t(m,i)})$ corresponding to the ball $g_i^{-1}(P_{k,t(m,i)})$, for every i, $1 \leq i \leq f_\Omega(n)$, and for every m, $1 \leq m \leq r$. Here $t(m,i)$ is as in (5). Since $g_i \in X_n$, $g_i^{-1}(P_{n1}) = P_{ni}$ and the vertex $v_{k,t(m,i)}$ lies under the vertex v_{n1}, the homeomorphism π is well-defined and does not depend on the choice of g_i. We remind that g_i is an element of X_n such that $g_i(P_{ni}) = P_{n1}$.

Let $g \in X_n$ such that $g(P_{ni}) = P_{nj}$. According to (5), we get

$$g\left(P_{kl_{i,t^{-1}(m,i)}}^{\gamma^{-1}}\right) = P_{kl_{j,t^{-1}(m,j)}}^{\gamma^{-1}}. \tag{6}$$

Obviously $g' = \pi\gamma g\gamma^{-1}\pi^{-1}$ belongs to H_k. Let $P_{ks} \subset P_{ni}$. Using (6), we get

$$g'(P_{ks}) = \pi\gamma g\gamma^{-1}\left(P_{kl_{i,t^{-1}(s+r-ir,i)}}\right) = \\ = \pi\left(P_{kl_{j,t^{-1}(s+r-ir,j)}}\right) = P_{k,s+(j-i)r} = g(P_{ks}).$$

Therefore $\pi\gamma$ centralizes X_n. Hence, $g^\gamma = g^{\pi^{-1}}$ for all $g \in X_n$.

So we have proved that $\alpha|_{X_n}$ is induced by an inner automorphism of H_k. $\square$

The main result of this note is then stated as follows.

Theorem 2. *The automorphism group of the subgroup AH_Ω coincides with the automorphism group of the group H_Ω.*

Proof. Let α be an automorphism of AH_Ω. The relation $\alpha|_{AH_n} \in \operatorname{Inn} H_k$ follows from Proposition 1. The last inclusion implies that $\alpha \in N_{\mathrm{Homeo}(\partial T_\Omega)}(H_\Omega)$. Hence $\operatorname{Aut} AH_\Omega \leq \operatorname{Aut} H_\Omega$. On the other hand, AH_Ω is the commutant of H_Ω. Thus $\operatorname{Aut} H_\Omega$ is a subgroup of $\operatorname{Aut} AH_\Omega$. So $\operatorname{Aut} AH_\Omega = \operatorname{Aut} H_\Omega$. $\square$

We also correct a misprint in [LS]. On p. 38, line 7 of [LS], the phrase "we have $P_{ni} \cap P_{mj} \neq \emptyset$ if and only if $P_{ni} = P_{mj}$ that is $n = m$ and $i = j$" should be replaced by "we have $P_{ni} \cap P_{nj} \neq \emptyset$ if and only if $P_{ni} = P_{nj}$, that is $i = j$".

References

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