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Neighborhood-prime labeling of some product graphs

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Abstract. We have introduced the concept of neighborhood-prime labeling and investigated it for paths, cycles, wheels and union of such graphs earlier. In this paper, we construct neighborhood-prime labelings for graphs that are cartesian or tensor product of two paths.

1. Introduction

All graphs considered in this paper are simple, finite and undirected. We begin with the definition of prime labeling and prime graphs.

Definition 1. Let G be a graph with n vertices. A bijective function $f : V(G) \to \{1, 2, 3, \ldots, n\}$ is said to be a *prime labeling*, if for every pair of adjacent vertices u and v, $\gcd(f(u), f(v)) = 1$. A graph that admits a prime labeling is called a *prime graph*.

The notion of prime labeling for graphs originated with Roger Entringer and was introduced in a paper by Tout et al. [6] . After the introduction of prime labeling, varieties of graphs or families of graphs have been tested for primality and it has been an active area of research for more than thirty years now. See [1] for a summary on prime labelings and prime graphs. Our definition of neighborhood-prime labeling is motivated by the definition of prime labeling.

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Definition 2. Let G be a graph with n vertices and for $v \in V(G)$, let $N(v)$ denote the open neighborhood of v. A bijective function $f : V(G) \rightarrow \{1, 2, 3, \ldots, n\}$ is said to be a *neighborhood-prime labeling* on G, if for every vertex $v \in V(G)$ with $\deg(v) \geqslant 2$, $\gcd\{f(u) : u \in N(v)\} = 1$. A graph that admits neighborhood-prime labeling is called a *neighborhood-prime graph.*

Remark 1. If in a graph G, every vertex is of degree at most 1, then such a graph is neighborhood-prime vacuously.

The notion of neighborhood-prime labeling was introduced in [3] where we showed that the path P_n is a neighborhood-prime graph for all n and the cycle C_n is a neighborhood-prime graph iff $n \not\equiv 2 \pmod 4$. We also showed that certain path and cycle related graphs are neighborhood-prime graphs. In [4], we characterized neighborhood-prime graphs amongst the class of union of two cycles. Further in the same paper, we proved that the union of two wheels and the union of a finite number of paths are neighborhood-prime graphs. Our present work is motivated by some of the results on prime labelings of product of paths, but the strategy is completely different over here. We consider two types of graph products.

Definition 3. The cartesian product $G \times H$ of graphs G and H is a graph such that its vertex set is $V(G \times H) = \{(u, v) : u \in V(G), v \in V(H)\}$; and any two vertices (u, v) and (u', v') are adjacent in $G \times H$ if and only if either $u = u'$ and v is adjacent to v' in H, or $v = v'$ and u is adjacent to u' in G.

Definition 4. The tensor product $G \otimes H$ of two graphs G and H is a graph such that its vertex set is $V(G \otimes H) = \{(u, v) : u \in V(G), v \in V(H)\}$; and any two vertices (u, v) and (u', v') are adjacent in $G \otimes H$ if and only if u is adjacent to u' in G and v is adjacent to v' in H.

Sundaram et al. [5] proved that the grid $P_m \times P_n$ is prime when n is prime and $n > m$. In the same paper they proved that $P_n \times P_n$ is prime when n is prime. Later, Kanetkar [2] proved that few other grids are also prime. In this paper, we show that the grid $P_m \times P_n$ and the graph $P_n \otimes P_n$ are neighborhood-prime graphs. We also show that the tensor product of a path and a wheel $P_m \otimes W_n$ is neighborhood-prime. But so far we have not been able to get a neighborhood-prime labeling for the tensor product of two arbitrary paths of different order.

2. Main results

Theorem 1. *The cartesian product of two paths is a neighborhood-prime graph.*

Proof. Let $G = P_m \times P_n$. Without loss of generality we may assume that $n \leqslant m$. Let $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ denote the (consecutive) vertices of the paths P_m and P_n respectively. Thus

$$V(G) = \{(u_k, v_j) : k = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n\}.$$

Note that the vertices $(u_1, v_1), (u_1, v_n), (u_m, v_1)$ and (u_m, v_n) are of degree 2 and the other vertices in G are of degree 3 or 4. For $l = 2, 3, \ldots, m+n$, we introduce the sets

$$D_l = \{(u_k, v_j) \in V(G) : k + j = l\},$$

which form a partition of the vertex set $V(G)$. The sets D_l are called diagonal sets and can also be expressed as

$$D_l = \{(u_k, v_{l-k}) \in V(G) : k \in S_l\}$$

where

$$S_l = \begin{cases} \{1, 2, \ldots, l-1\} & \text{if } 2 \leqslant l \leqslant n+1 \\ \{l-n, l-n+1, \ldots, l-1\} & \text{if } n+2 \leqslant l \leqslant m+1 \text{ (collapses if } m=n) \\ \{l-n, l-n+1, \ldots, m\} & \text{if } m+2 \leqslant l \leqslant m+n. \end{cases}$$

Two vertices (u_k, v_{l-k}) and $(u_{k'}, v_{l-k'})$ of the set D_l are called consecutive vertices if $|k - k'| = 1$. Further, the vertex (u_k, v_{l-k}) is called the first (last) vertex of D_l if k is the minimum (maximum) value of the corresponding set S_l.

Now we define a bijective function $f : V(G) \to \{1, 2, \ldots, mn\}$ by defining it on the diagonal sets D_l. For $(u_k, v_{l-k}) \in D_l$,

$$f(u_k, v_{l-k}) = f_l(u_k, v_{l-k})$$

where

$$f_l(u_k, v_{l-k}) = \begin{cases} \frac{(l-2)(l-1)}{2} + k & \text{if } 2 \leqslant l \leqslant n+1 \\ (n-1)l - \frac{(n-1)(n+2)}{2} + k & \text{if } n+2 \leqslant l \leqslant m+1 \\ \frac{(l-m)(m-l+3)+2l(n-1)-n(n+1)}{2} + k & \text{if } m+2 \leqslant l \leqslant m+n. \end{cases}$$

One can observe that any two consecutive vertices of a diagonal set D_l are mapped to consecutive integers under f. More precisely, if (u_k, v_{l-k}) and $(u_{k'}, v_{l-k'})$ are the vertices on the same diagonal set D_l and say $k' > k$, then

$$f_l(u_{k'}, v_{l-k'}) = f_l(u_k, v_{l-k}) + (k' - k). \tag{2.1}$$

Further, it may be easily verified that if (u_k, v_{l-k}) is the last vertex of D_l and $(u_{k'}, v_{(l+1)-k'})$ is the first vertex of D_{l+1}, then

$$f_{l+1}(u_{k'}, v_{(l+1)-k'}) = f_l(u_k, v_{l-k}) + 1. \tag{2.2}$$

From (2.1) and (2.2), it follows that f_l may also be defined recursively. For better understanding of these formulas, we illustrate the labeling of f on $P_9 \times P_6$ in Figure 1a below. Now in view of (2.1), if $w = (u_k, v_{l-k})$ is an arbitrary vertex of G different from (u_1, v_n) and (u_m, v_1), then

$$\gcd\{f(p) : p \in N(w)\} = 1 \tag{2.3}$$

because such $N(w)$ contains at least one pair of consecutive vertices. But (2.3) may not hold for the vertices (u_1, v_n) and (u_m, v_1). For instance, in case of the graph $P_9 \times P_6$,

$$N(u_1, v_n) = N(u_1, v_6) = \{(u_1, v_5), (u_2, v_6)\}.$$

But $f(u_1, v_5) = 11$ and $f(u_2, v_6) = 22$ and so f does not define a neighborhood-prime labeling on G. Similar problem occurs for the vertex (u_m, v_1) of this graph. To eliminate this problem, we modify f and show that the modified function is a neighborhood-prime labeling on G. The strategy is to bring consecutive labels on the two neighbors of (u_m, v_1) whereas to overcome the problem of (u_1, v_n) we exchange the label of (u_1, v_1) (which is 1) with the neighbor (u_1, v_{n-1}) of (u_1, v_n). This is formally explained by the definition of g below but for the better understanding of the effect of g we refer the reader to Figure 1. Now define $g : V(G) \to \{1, 2, \ldots, mn\}$ as follows. For $(u_k, v_{l-k}) \in D_l$,

$$g(u_k, v_{l-k}) = \begin{cases} f(u_k, v_{l-k}) & \text{if } l \neq 2, n, m+1, m+2 \\ f(u_1, v_{n-1}) & \text{if } l = 2 \\ f(u_1, v_1) & \text{if } l = n \text{ and } k = 1 \\ f(u_k, v_{l-k}) & \text{if } l = n \text{ and } k > 1 \\ \frac{(n-1)(2m+2-n)}{2} + k & \text{if } l = m+1 \\ \frac{(1-n)(n+2)+2m(n+1)}{2} - k & \text{if } l = m+2. \end{cases}$$

It may be verified that any two consecutive vertices of the set D_l, none of which is (u_1, v_{n-1}), are mapped to consecutive integers under g. We now verify that g is a neighborhood-prime labeling on G by considering the following cases.

Case 1. $w \neq (u_m, v_1), (u_1, v_n), (u_1, v_{n-2})$.

In this case $N(w)$ contains at least one pair of consecutive vertices none of which is (u_1, v_{n-1}) and so (2.3) holds.

Case 2. $w = (u_1, v_n), (u_1, v_{n-2})$.

This case may be dispensed by observing that $N(w)$ contains (u_1, v_{n-1}) and $g(u_1, v_{n-1}) = f(u_1, v_1) = 1$.

Case 3. $w = (u_m, v_1)$.

Here $N(w) = \{(u_{m-1}, v_1), (u_m, v_2)\}$. Observe that $(u_m, v_2) \in D_l$ with $l = m + 2$ (and $k = m$) and therefore

$$\begin{aligned} g(u_m, v_2) &= \frac{(1-n)(n+2) + 2m(n+1)}{2} - k \\ &= \frac{(1-n)(n+2) + 2m(n+1)}{2} - m \\ &= mn - \frac{(n-1)(n+2)}{2}. \end{aligned}$$

Now $(u_{m-1}, v_1) \in D_l$ with $l = m$ (and $k = m - 1$) and so to determine $g(u_{m-1}, v_1)$ we have to consider the following two sub-cases.

Sub-case 1. $n \leqslant l = m \leqslant n + 1$.

Here

$$\begin{aligned} g(u_{m-1}, v_1) &= \frac{(l-2)(l-1)}{2} + k \\ &= \frac{(m-2)(m-1)}{2} + (m-1) \\ &= \frac{m(m-1)}{2}. \end{aligned}$$

Now if $m = n$ or $n + 1$, then it can be easily verified that

$$g(u_m, v_2) - g(u_{m-1}, v_1) = 1 \tag{2.4}$$

and so we are through.

Sub-case 2. $l = m \geqslant n + 2$.

In this sub-case

$$\begin{aligned} g(u_{m-1}, v_1) &= (n-1)l - \frac{(n-1)(n+2)}{2} + k \\ &= (n-1)m - \frac{(n-1)(n+2)}{2} + (m-1) \\ &= mn - \frac{(n-1)(n+2)}{2} - 1 \end{aligned}$$

and thus (2.4) holds. □

Example 1. The labelings f and g of $P_9 \times P_6$ are given in Figure 1a and Figure 1b respectively.

Our next result is about the tensor product of two paths of same order.

Theorem 2. *Let P_n be a path graph. Then the graph $G = P_n \otimes P_n$ is neighborhood-prime.*

Proof. We note that the graph $P_2 \otimes P_2$ is neighborhood-prime vacuously because all its vertices are of degree 1. Thus we may assume that $n > 2$. Let $\{u_1, u_2, \ldots, u_n\}$ and $\{v_1, v_2, \ldots, v_n\}$ denote the vertex sets of the two paths involved in the product graph G so that

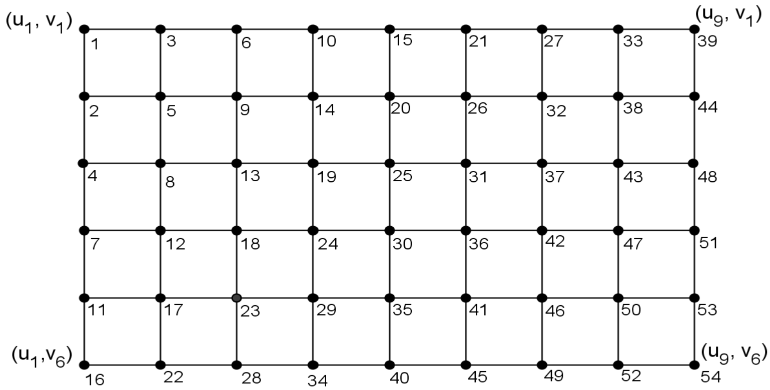
$$V(G) = \{(u_i, v_j) : 1 \leqslant i, j \leqslant n\}.$$

Note that the vertices $(u_1, v_1), (u_1, v_n), (u_n, v_1)$ and (u_n, v_n) are of degree 1 and the other vertices in G are of degree 2 or 4. Our method of obtaining the neighborhood-prime labeling for G is different for odd and even n . So first we prove the theorem assuming that n is odd.

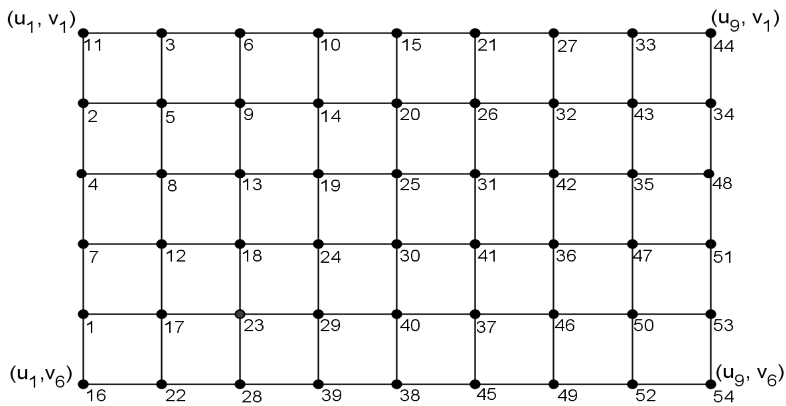
We define a bijective function $f : V(G) \to \left\{1, 2, \ldots, n^2\right\}$ as

$$\begin{aligned} f(u_{2i-1}, v_{2j}) &= (i-1)n + j, & 1 \leqslant i \leqslant \tfrac{n+1}{2}, \quad 1 \leqslant j \leqslant \tfrac{n-1}{2}, \\ f(u_{2i}, v_{2j-1}) &= \frac{n-1}{2} + (i-1)n + j, & 1 \leqslant i \leqslant \tfrac{n-1}{2}, \quad 1 \leqslant j \leqslant \tfrac{n+1}{2}, \\ f(u_{2i-1}, v_{2j-1}) &= \frac{n^2-1}{2} + (i-1)n + j, & 1 \leqslant i \leqslant \tfrac{n+1}{2}, \quad 1 \leqslant j \leqslant \tfrac{n+1}{2}, \\ f(u_{2i}, v_{2j}) &= \frac{n(n+1)}{2} + (i-1)n + j, & 1 \leqslant i \leqslant \tfrac{n-1}{2}, \quad 1 \leqslant j \leqslant \tfrac{n-1}{2}. \end{aligned}$$

In order to show that f is a neighborhood-prime labeling on G, we need to verify that if w is an arbitrary vertex of G with degree at least 2 and



(a) The labeling f of $P_9 \times P_6$.



(b) The labeling g of $P_9 \times P_6$.

FIGURE 1. Labeling of $P_9 \times P_6$ under f and g.

$L = \{f(p) : p \in N(w)\}$, then the gcd of the integers appearing in the set L is 1. This is justified by considering the following cases.

Case 1. $w = (u_i, v_j)$, $1 \leqslant i < n$, $1 < j < n$.

In this case L contains $f(u_{i+1}, v_{j-1})$ and $f(u_{i+1}, v_{j+1})$ which are consecutive integers by the definition of f, and so we are through.

Case 2. $w = (u_n, v_j)$, $1 < j < n$.

Here L contains $f(u_{n-1}, v_{j-1})$ and $f(u_{n-1}, v_{j+1})$ which are again consecutive integers.

Case 3. $w = (u_i, v_1)$, $1 < i < n$.

Here we observe that $L = \{f(u_{i-1}, v_2), f(u_{i+1}, v_2)\}$. But these two numbers of the set L are relatively prime since

$$f(u_{i+1}, v_2) - f(u_{i-1}, v_2) = n \text{ and also } f(u_{i+1}, v_2) \equiv 1 (\bmod n).$$

Case 4. $w = (u_i, v_n)$, $1 < i < n$.

In this case $L = \{f(u_{i-1}, v_{n-1}), f(u_{i+1}, v_{n-1})\}$. But

$$f(u_{i+1}, v_{n-1}) - f(u_{i-1}, v_{n-1}) = n \text{ and also } f(u_{i+1}, v_{n-1}) \equiv \frac{n-1}{2} (\bmod n)$$

and so the two numbers in L are relatively prime.

Now we assume that n is even. Consider a bijective function $f : V(G) \to \left\{1, 2, \ldots, n^2\right\}$ which is defined as

$$\begin{aligned} f(u_i, v_{2j}) &= (i-1)n + j, & 1 \leqslant i \leqslant n, \quad 1 \leqslant j \leqslant \frac{n}{2}, \\ f(u_i, v_{2j-1}) &= (2i-1)\frac{n}{2} + j, & 1 \leqslant i \leqslant n, \quad 1 \leqslant j \leqslant \frac{n}{2}. \end{aligned}$$

Note that f does not work as a neighborhood-prime labeling on G since for the vertices (u_i, v_n), $1 < i < n$, $N(u_i, v_n) = \{(u_{i-1}, v_{n-1}), (u_{i+1}, v_{n-1})\}$; but $f(u_{i-1}, v_{n-1}) = (2i-3)n/2 + n/2$ and $f(u_{i+1}, v_{n-1}) = (2i+1)n/2 + n/2$ are both even . So we modify f by exchanging the labels of the vertices in the set $\{(u_i, v_{n-1}) : 1 \leqslant i \leqslant n\}$ with the set $\{(u_1, v_j) : 1 \leqslant j \leqslant n\}$. More precisely, consider a bijective function $g : V(G) \to \left\{1, 2, \ldots, n^2\right\}$ which is defined as

$$g(u_i, v_r) = \begin{cases} f(u_i, v_r) & \text{if } i \neq 1, r \neq n-1 \\ f(u_1, v_i) & \text{if } 1 \leqslant i \leqslant n, \ r = n-1 \\ f(u_r, v_{n-1}) & \text{if } i = 1, r \neq 1, n-1, n \\ f(u_n, v_{n-1}) & \text{if } i = 1, r = 1 \\ f(u_{n-1}, v_{n-1}) & \text{if } i = 1, r = n. \end{cases}$$

We illustrate the effect of f and g on $P_8 \otimes P_8$ in Figure 2a and Figure 2b respectively. Now we show that g works as a neighborhood-prime labeling on G. For this we consider the following cases and justify that the gcd of the integers in the set $L = \{g(p) : p \in N(w)\}$ is 1 for all w with degree at least 2.

Case 1. $w = (u_i, v_r), 1 \leqslant i < n, \ 1 < r < n, \ r \neq n-2$.

Here L contains $g(u_{i+1}, v_{r-1})$ and $g(u_{i+1}, v_{r+1})$ which are consecutive integers by the definition of f since $g(u_{i+1}, v_{r-1}) = f(u_{i+1}, v_{r-1})$ and $g(u_{i+1}, v_{r+1}) = f(u_{i+1}, v_{r+1})$.

Case 2. $w = (u_n, v_r), 1 < r < n$.

Here $L = \{g(u_{n-1}, v_{r-1}), g(u_{n-1}, v_{r+1})\}$. If $r \neq n-2$ then once again the two integers in the set L are consecutive by the definition of f. If $r = n-2$, then we observe that

$$g(u_{n-1}, v_{n-3}) = f(u_{n-1}, v_{n-3}) = n(n-1) - 1$$

and

$$g(u_{n-1}, v_{n-1}) = f(u_1, v_{n-1}) = n$$

which are relatively prime numbers.

Case 3. $w = (u_i, v_1),\ 1 < i < n$.

Here $L = \{g(u_{i-1}, v_2), g(u_{i+1}, v_2)\}$. Now if $i = 2$, $g(u_1, v_2) = f(u_2, v_{n-1}) = 2n$ and $g(u_3, v_2) = f(u_3, v_2) = 2n+1$ which are consecutive integers.

If $2 < i < n$, then the gcd of the two integers in L is 1 because their difference is $2n$ and they are congruent to 1 modulo n and as n is even.

Case 4. $w = (u_i, v_{n-2}), 1 \leqslant i < n$.

Note that if $i = 1$, then L contains $g(u_2, v_{n-1}) = f(u_1, v_2) = 1$ and so we are through. If $1 < i < n$, L contains $g(u_{i-1}, v_{n-1}) = f(u_1, v_{i-1})$ and $g(u_{i+1}, v_{n-1}) = f(u_1, v_{i+1})$ which are consecutive integers.

Case 5. $w = (u_i, v_n), 1 < i < n$.

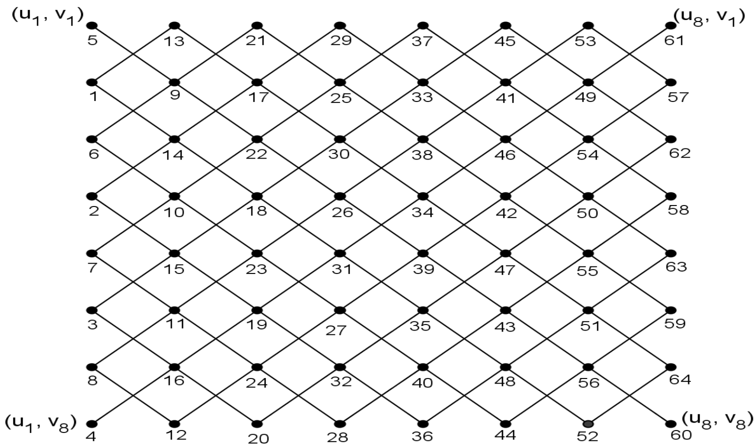
In this case $L = \{g(u_{i-1}, v_{n-1}), g(u_{i+1}, v_{n-1})\}$ which are consecutive integers as observed in Case 4. □

Example 2. The labelings f and g of $P_8 \otimes P_8$ are given in Figure 2a and Figure 2b respectively.

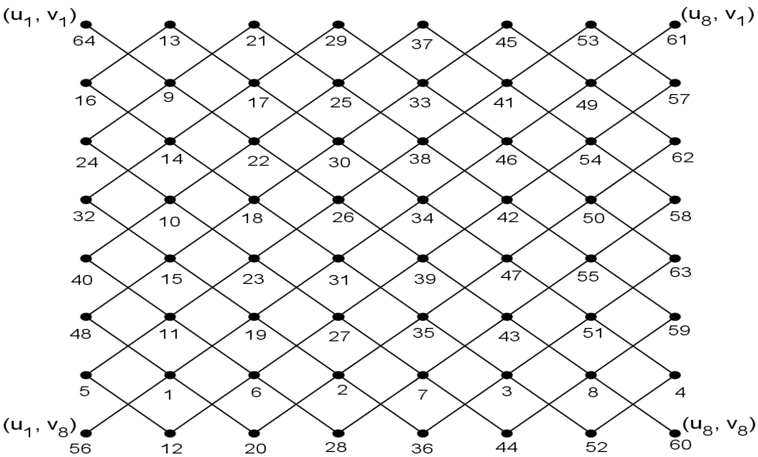
So far we have seen that the cartesian product of two paths (of same or different orders) is a neighborhood-prime graph and the tensor product of two paths of same order is a neighborhood-prime graph. Although, we believe that the tensor product of two paths of different orders is neighborhood-prime, we do not have a general proof for this. However, we do have a neighborhood-prime labeling for the tensor product of a path and a wheel.

Theorem 3. *If W_n is a wheel graph, then the graph $G = P_m \otimes W_n$ is neighborhood-prime.*

Proof. It is quite easy to show that the graphs $P_2 \otimes W_n$ and $P_3 \otimes W_n$ are neighborhood-prime and so we assume that $m \geqslant 4$. Let $u_1, u_2, \ldots, u_m$



(a) The labeling f of $P_8 \otimes P_8$.



(b) The labeling g of $P_8 \otimes P_8$.

FIGURE 2. Labeling of $P_8 \otimes P_8$ under f and g.

denote the vertices of the path P_m and $v_0, v_1, v_2, \ldots, v_{n-1}$ denote the vertices of the wheel W_n where v_0 is its apex vertex. Then

$$V(G) = \{(u_i, v_j) : 1 \leqslant i \leqslant m \text{ and } 0 \leqslant j \leqslant n-1\}.$$

We shall see that if we define the labels of the vertices (u_i, v_0), (where $1 \leqslant i \leqslant m$) carefully then it generates a neighborhood-prime labeling on

G since the neighborhood of every vertex (u_i, v_j), (where $j > 0$), contains at least one vertex from the set $\{(u_i, v_0) : 1 \leqslant i \leqslant m\}$.
Let p_0 be a randomly chosen prime number lying strictly between $mn/2$ and mn; which exists due to Bertrand's postulate which states that there is always a prime number between any number and its double. Obviously p_0 is relatively prime to every integer (different from p_0) of the set $\{1, 2, \ldots, mn\}$. Define a bijective function $f : V(G) \to \{1, 2, \ldots, mn\}$ by first defining it on the set $\{(u_i, v_0) : 1 \leqslant i \leqslant m\}$ as follows.

If m is even

$$\begin{aligned} f(u_{2i}, v_0) &= i, & 1 \leqslant i \leqslant \frac{m}{2} \\ f(u_{2i-1}, v_0) &= \frac{m}{2} + i, & 1 \leqslant i < \frac{m}{2} \\ f(u_{m-1}, v_0) &= p_0; \end{aligned}$$

and if m is odd

$$\begin{aligned} f(u_{2i}, v_0) &= i, & 1 \leqslant i < \frac{m-1}{2} \\ f(u_{2i-1}, v_0) &= \frac{m-3}{2} + i, & 1 \leqslant i \leqslant \frac{m+1}{2} \\ f(u_{m-1}, v_0) &= p_0. \end{aligned}$$

The remaining vertices may be labeled from the set

$$\{m, m+1, \ldots, p_0 - 1, p_0 + 1, \ldots, mn\}$$

such that $f(u_i, v_1)$ and $f(u_i, v_2)$ are two consecutive integers for each i. (Note that this can be done in many ways).

Now we consider the following four cases in order to verify that the gcd of the integers in the set $L = \{f(p) : p \in N(w)\}$ is 1 for every $w \in V(G)$.

Case 1. $w = (u_i, v_j); 1 < i < m$ and $1 \leqslant j \leqslant n - 1$.

Here L contains $f(u_{i-1}, v_0)$ and $f(u_{i+1}, v_0)$ which are either consecutive integers (if $i \neq m-2$) or one of them is a (Bertrand) prime p_0 (if $i = m-2$).

Case 2. $w = (u_1, v_j), 1 \leqslant j \leqslant n - 1$.

In this case L contains $f(u_2, v_0) = 1$.

Case 3. $w = (u_m, v_j),\ 1 \leqslant j \leqslant n - 1$.

Here L contains $f(u_{m-1}, v_0)$ which is (Bertrand) prime p_0.

Case 4. $w = (u_i, v_0), 1 \leqslant i \leqslant m$.

For $1 \leqslant i < m$, L contains $f(u_{i+1}, v_1)$ and $f(u_{i+1}, v_2)$ which are consecutive integers by the definition of f. For $i = m$, L contains$f(u_{m-1}, v_1)$ and $f(u_{m-1}, v_2)$ which are also consecutive integers. □

Concluding remarks. Let H be a subgraph of G such that $V(H) = V(G)$ and suppose $\deg_H(v) \geqslant 2$ for every vertex v. Then from the definition of neighborhood-prime labeling it follows that every neighborhood-prime labeling on H is also a neighborhood-prime labeling on G. Therefore in view of Theorem 1, the graph obtained by taking the cartesian product of any two cycles is also neighborhood-prime. But a similar conclusion cannot be made for the tensor product of two cycles (of same order) via Theorem 2 because $P_n \otimes P_n$ contains vertices of degree 1. For instance, it may be verified that the neighborhood-prime labeling of $P_{14} \otimes P_{14}$ as defined by Theorem 2 does not work for $C_{14} \otimes C_{14}$. Thus right now it is an open question to obtain a neighborhood-prime labeling for the tensor product of two cycles of same or different orders.

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