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Criterions of supersolubility of some finite factorizable groups

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Abstract. Let A, B be subgroups of a group G and $\emptyset \neq X \subseteq G$. A subgroup A is said to be X-permutable with B if for some $x \in X$ we have $AB^x = B^xA$ [1]. We obtain some new criterions for supersolubility of a finite group $G = AB$, where A and B are supersoluble groups. In particular, we prove that a finite group $G = AB$ is supersoluble provided A, B are supersolube subgroups of G such that every primary cyclic subgroup of A X-permutes with every Sylow subgroup of B and if in return every primary cyclic subgroup of B X-permutes with every Sylow subgroup of A where $X = F(G)$ is the Fitting subgroup of G.

Introduction

Throughout this paper, all groups are finite. By well-known Fitting's theorem [2, III, 4.1] the produkt of any two normal nilpotent subgroups is nilpotent as well. It is known however, that supersoluble groups do not have such a property [3], [4]. It was observed by R.Baer [5] that the product $G = AB$ of two normal supersoluble subgroups A and B is supersoluble if G' is nilpotent. Another important results were obtained by M.Asaad and A. Shaalan in [4], where it was proved that a product $G = AB$ of supersoluble groups A and B is supersoluble if every subgroup of A is permutable with every subgroup of B. Later on the observations from [4] were extended in various papers (see for example [6], [7], [8], [9], [10], [11]). In this paper we prove some new results in this direction.

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Recall that a subgroup A of a group G is permutable with a subgroup B if $AB = BA$. In many cases we meet the situation when $AB \neq BA$ but $AB^x = B^xA$ for some $x \in G$. For example, when G is soluble and H and T are Sylow subgroups of G, then $HT^x = T^xH$, for some $x \in G$ ([2, VI, 3.1]). Another example is that if $G = HT$ and H_p, T_p are Sylow p-subgroups in H and T respectively, then $H_pT_p \neq T_pH_p$ in general but there exists an element $x \in G$ such that $H_pT_p^x = T_p^xH_p$. In the analizing of the situations of this kind it is convenient to use the following natural concepts which were introduced in [1].

Definition. Let A, B be subgroups of a group G and $\emptyset \neq X \subseteq G$. Then:

(1) A is *X-permutable* with B if there exists some $x \in X$ such that $AB^x = B^xA$;

(2) A is *X-permutable in* G if A is X-permutable with all subgroups of G;

(3) A is *hereditarily* X-permutable with B if $AB^x = B^xA$, for some $x \in X \cap < A, B >$.

1. Preliminaries

We first cite here some properties of factorizations of groups. The following two lemmas are well known.

Lemma 1.1. *Let A, B be proper subgroups of a group G with $G = AB$. Then $G = AB^x$ and $G \neq AA^x$ for all $x \in G$.*

Lemma 1.2. *If $G = AB$ and p be a prime, then there exist some Sylow p-subgroups A_p, B_p and G_p in A, B and G respectively such that $G_p = A_pB_p$.*

We shall often use the following fact which at first was proved in [15].

Lemma 1.3. *Let $G = AB$ be the product of its subgroups A, B. If L is a normal subgroup of A and $L \leq B$, then $L \leq B_G$.*

Lemma 1.4. *[14, 1.7.11]. If H/K is a chief factor of a group G and if p is a prime divisor of $|H/K|$, then $O_p(G/C_G(H/K)) = 1$.*

We shall also need the following well known facts about supersoluble and soluble groups.

Lemma 1.5. *Let G be a group. Then the following statements hold:*

(i) if G is supersoluble, then $G' \subseteq F(G)$ and G is p-closed for the largest prime divisor p of $|G|$;

(ii) if $L \trianglelefteq G$ and $G/\Phi(L)$ is supersoluble, then G is supersoluble;

(iii) G is supersoluble if and only if $|G : M|$ is a prime for every maximal subgroup M of G.

Lemma 1.6. *[14, 2.4.3]. Let M_1, M_2 be maximal subgroups of a soluble group G such that $(M_1)_G = (M_2)_G$. Then M_1 and M_2 are conjugate.*

Lemma 1.7. *[15]. Let A, B be subgroups of a group G. Assume that A permutes with B^x for every $x \in G$. If $AB \neq G$, then G is not simple.*

Now we cite some properties of X-permutable subgroups.

Lemma 1.8. *[1]. Let A, B, X be subgroups of G and $K \trianglelefteq G$. Then the following statements hold:*

(1) If A is (hereditarily) X-permutable with B, then B is (hereditarily) X-permutable with A.

(2) If A is (hereditarily) X-permutable with B, then A^x is (hereditarily) X^x-permutable with B^x for all $x \in G$.

(3) If $K \leq A$, then A/K is (hereditarily) XK/K-permutable with BK/K in G/K if and only if A is (hereditarily) X-permutable with B in G.

(4) If $A, B \leq M \leq G$ and A is hereditarily X-permutable with B, then A is hereditarily $(X \cap M)$-permutable with B.

Lemma 1.9. *Let p be a prime, $G = Z_pB$ where $|Z_p| = p \nmid |B|$, B is a soluble group and Z_p X-permutes with every Sylow subgroup of B where $X = F(G)$ is the Fitting subgroup of G. Then G is a soluble group.*

Proof. Assume that this lemma is false and let G be a counterexample with minimal order. Then:

(1) *G is not simple.*

Assume that G is a simple group. Then $X = 1$. Let B_q be a Sylow q-subgroup of B. Then by hypotheses, $Z_pB_q = B_qZ_p$. Besides, because for every $x \in G = Z_pB$ we have

$$Z_pB_q^x = Z_pB_q^{ba} = Z_p(B_q^b)^a = Z_pB_q^b = B_q^x Z_p$$

where $b \in B$ and $a \in Z_p$. Then by Lemma 1.7, G is not simple.

(2) *G/N is soluble for every normal subgroup N of G.*

Indeed, let N be a normal subgroup of G. If $Z_p \subseteq N$, then $G/N = NB/N \simeq B/N \cap B$ is a soluble group.

Let $Z_p \nsubseteq N$. Then $G/N = (Z_pN/N)(BN/N)$ is the product of the subgroup Z_pN/N with order p and the soluble subgroup BN/B. Let D/N be a Sylow q-subgroup of BN/N. Then $D = B_qN$ for some Sylow q-subgroup B_q of B, and so by hypotheses,

$$(Z_pN/N)(D/N)^{xN} = (D/N)^{xN}(Z_pN/N)$$

for some $xN \in XN/N \leq F(G/N)$. Thus the hypotheses are true for G/N. Since $|G/N| < |G|$ and by the choice of G, the subgroup G/N is a soluble group.

(3) *Final contradiction.*

If $X \neq 1$ then in view of (2), G/X is soluble and so G is a soluble group, a contradiction. Hence $X = 1$. Let N be a minimal normal subgroup of G. Then in view of (1), $N \neq G$. First assume that $p \nmid |N|$. Then evidently $N \subseteq B$. Since by hypotheses, B is a soluble group, N is soluble and so in view of (2), G is a soluble group, a contradiction. Hence $p \mid |N|$. Since B is a Hall p'-subgroup of G, so $B \cap N$ is a Hall p'-subgroup of N . It is clear that $Z_p \subseteq N$, and so by Dedekind Law, we have $N = N \cap Z_pB = Z_p(N \cap B)$. Let Q be a Sylow q-subgroup of $N \cap B$, B_q be a Sylow q-subgroup of B such that $B_q \cap N = Q$. Then by hypotheses, $B_qZ_p = Z_pB_q$, and hence $N \cap B_qZ_p = Z_p(N \cap B_q) = Z_pQ = QZ_p$. Thus the hypotheses are true for N and N is a soluble group. It follows that G is soluble, contrary to the choice of G.

□

2. Main results

N.M. Kurnosenko has proved in [9] that the product $G = AB$ of two supersoluble subgroups A and B having coprime orders is supersoluble if A and every cyclic subgroup of A permutes with every Sylow subgroup of B and if in return B and every cyclic subgroup of B permutes with every Sylow subgroup of A.

The following theorem is a local analog of this result in the case when G is soluble.

Theorem 2.1. *Let G be a soluble group and $G = AB$ be a product of p-supersoluble subgroups A, B having coprime orders. Assume that p divides $|A|$ and*

(1) if $p > 2$ then A and every its subgroup with prime order p permutes with every Sylow subgroup of B;

(2) if $p = 2$ then A and every its subgroup with order 2 or 4 permutes with every Sylow subgroup of B.

Then G is a p-supersoluble group.

Proof. Assume that the result is false and let G be a counterexample with minimal order. Let $\mathfrak{F}$ be the class of all p-supersoluble group.

Let M be a $\mathfrak{F}$-abnormal maximal in G subgroup. Then $|G : M| = p^\alpha$ for some $\alpha \in \mathbb{N}/\{1\}$ or $|G : M| = q^\beta$ for some $\beta \in \mathbb{N}$ and $q \in \mathbb{P}, q \neq p$. First assume that $|G : M| = p^\alpha$. Since p divides $|A|$ and since by Hall's theorem [13, 1, 3.3], G has an element x such that $B \subseteq M^x$, then without loss of generality we may assume that $B \subseteq M$. Now we shall show that for M the hyposeses are true. Indeed, by using the Dedekind Law, we have $M = M \cap AB = (M \cap A)B$ where $M \cap A$ and B are p-supersoluble subgroups of M having coprime orders. If $M \cap A$ is a p'-group then M is a p'-group and so M is a p-supersoluble group. Now suppose that $p \mid M \cap A$. Let T be a subgroup of $M \cap A$ with prime order p (or 4 in the case when $p = 2$). And let B_q be a Sylow q-subgroup of B. By hypotheses $B_qT = TB_q$. Since $AB_q = B_qA$, so $AB_q \cap M = (M \cap A)B_q = B_q(M \cap A)$. So the hypotheses are true for M and its subgroups $M \cap A$ and B. Hence by the choice of G, the subgroup M is p-supersoluble. Now let $|G : M| = q^\beta$ where $q \neq p$. Same as above, we can see that M is p-supersoluble. Thus every $\mathfrak{F}$-abnormal maximal subgroup of G is a p-supersoluble group.

Since G is soluble, so by [12, VI, 24.2] G has a normal p-subgroup P satisfying the following conditions:

(i) G/P is p-supersoluble and P is the smallest normal subgroup of G with p-supersoluble quotient;

(ii) if $p > 2$, then the exponent of P is p; if $p = 2$, then the exponent of P is 2 or 4;

(iii) $P/\Phi(P)$ is a chief factor of G.

It is clear that $P \subseteq A$. Let $\Phi = \Phi(P)$ and let q be a prime such that $q \nmid |A|$, G_q be a Sylow q-subgroup of G. Denote by $G_{q'}$ some Hall q'-subgroup of G such that $A \leq G_{q'}$. Then $P \subseteq G_{q'}$. Using the same argument as above, we see that $G_{q'}$ is p-supersoluble. Hence we see that $G_{q'}/\Phi$ has a normal subgroup H/Φ such that $|H/\Phi| = p$ and so $H =< a > \Phi$ where $< a >\subseteq P$. It is clear that $| < a > | = p$ or $| < a > | = 4$. For some $x \in G$, we have $G_q^x \leq B$. Then by hypotheses, $< a > G_q^x = G_q^x < a >$. Since $< a >$ is subnormal in G and $(| < a > |, q) = 1$, so $G_q^x \subseteq N_G(< a >)$, and therefore $H/\Phi \trianglelefteq G/\Phi$. Then we have $P/\Phi = H/\Phi$ is a cyclic group. It is clear that $G/P \simeq (G/\Phi)/(P/\Phi)$ is p-supersoluble and so G/Φ is a p-supersoluble group, a contradiction.

□

By extending the results [9] we prove the following two theorems.

Theorem 2.2. *Let $G = AB$ be a product of supersoluble subgroups A, B having coprime orders and $X = F(G)$ the Fitting subgroup of G. Assume that A and every its subgroup with prime order or with order dividing 4 is hereditarily X-permutable with every subgroup of B and in return B and every its subgroup with prime order or with order dividing 4 is hereditarily X-permutable with every subgroup of A. Then G is a supersoluble group.*

Proof. Assume that this theorem is false and let G be a counterexample with minimal order. Then:

(1) *Some maximal subgroup of G is not supersoluble.*

Assume that every maximal subgroup of G is supersoluble. Then G is soluble [15] and by [13, 7, 6.18] it has a normal Sylow p-subgroup P satisfying the following conditions:

(i) G/P is supersoluble and P is the smallest normal subgroup of G with supersoluble quotient;

(ii) if $p > 2$, then the exponent of P is p; if $p = 2$, then the exponent of P is 2 or 4;

(iii) $P/\Phi(P)$ is a chief factor of G.

Using the same arguments in the proof of Theorem 2.1, we can prove (1).

(2) *G is not soluble.*

Assume that G is soluble and let M be a maximal in G subgroup. Then $|G : M| = p^\alpha$ for some prime p. Without loss of generality one can suppose that $p \mid |B|$. By Hall's theorem [13, 1, 3.3], G has an element x such that $A \subseteq M_1 = M^x$. Now we shall prove that M_1 is supersoluble. Indeed, by using the Dedekind Law, we have $M_1 = M_1 \cap AB = A(M_1 \cap B)$ where A and $M_1 \cap B$ are supersoluble subgroups of M_1 having coprime orders. Let T be a subgroup of A with prime order or with order dividing 4. And let B_1 be a subgroup of $M_1 \cap B$. Then by hypotheses $TB_1^x = B_1^x T$ for some $x \in X \cap < T, B_1 > \leq M_1$. Since $X \cap M_1 \leq F(M_1)$, so $x \in M_1 \cap < T, B_1 >$. So the hypotheses are true for M_1 and its subgroups $A \cap M_1$ and B. Since $|M_1| < |G|$ and by the choice of G, the subgroup M_1 is supersoluble, and so M is supersoluble too. Thus every maximal subgroup of G is supersoluble, contrary (1). Thus we have conclude that (2) is true.

(3) *G has a normal Sylow subgroup.*

Let p be the largest prime divisor of $|G|$. Without loss of generality we may assume that $p \mid |A|$. Let A_p be a Sylow p-subgroup of A. Since by hypotheses A is supersoluble, by Lemma 1.5 we have $A_p \trianglelefteq A$. Now let B_q be a Sylow q-subgroup of B where $q \neq p$. By hypothesis, $D = AB_q^x = B_q^x A$ for some $x \in X \cap < A, B_q >$. Assume that $D = G$. By Lemma 1.1 $G = AB_q$ and so $B_q = B$. Then by hypotheses $A_p^x B_q = B_q A_p^x$ for some $x \in X \cap < A_p, B_q >$. If $A_p^x B_q = G$, then by Burnside's $p^a q^b$-theorem G is soluble, contrary (2). Hence $A_p^x B_q \neq G$. It is evident that the hypotheses are true for the group $A_p^x B_q$, and so by the choice of G, $A_p^x B_q$ is supersoluble. That implies $A_p^x \trianglelefteq A_p^x B_q$. Thus $A_p \trianglelefteq G$.

(4) *Final contradiction.*

Let Q be a normal Sylow subgroups of G. Then $|G : Q| = q^\alpha$ for some $\alpha \in \mathbb{N}$. Without loss of generality we may assume that $q \mid |A|$. Now we shall show that for $G/Q = (A/Q)(BQ/Q)$ the hypotheses are true. Indeed, A/Q and BQ/Q are supersoluble subgroups of G/Q having coprime orders. Assume that $p \mid |A/Q|$. Let H/Q be a subgroup of A/Q with prime order (or with order dividing 4 in the case when $p = 2$). Then by Schur-Zassenhaus's theorem [14, 1.7.9] G has a subgroup T such that $H = TQ$ and $|T| = |H/Q|$. Let B_1/Q be a subgroup of BQ/Q. Then by using the Dedekind Law, we have $B_1 = Q(B_1 \cap B)$. By hypotheses $T(B_1 \cap B)^x = (B_1 \cap B)^x T$ for some $x \in X$ and so $(H/Q)(B_1/Q)^{xQ} = (TQ/Q)((B_1 \cap B)^x Q/Q) = T(B_1 \cap B)^x Q/Q = (B_1 \cap B)^x T/Q = ((B_1 \cap B)^x Q/Q)(TQ/Q) = (B_1/Q)^{xQ}(H/Q)$. Since $Q \leq X$, so $xQ \in X/Q \leq F(G/Q)$. Hence the hypotheses are true for G/Q, and so G/Q is soluble. Now we obtain that G is a soluble group. This contradiction completes the proof.

□

Theorem 2.3. *Let $G = AB$ be a product of supersoluble subgroups A, B and $X = F(G)$ the Fitting subgroup of G. If every primary cyclic subgroup of A X-permutes with every Sylow subgroup of B and if in return every primary cyclic subgroup of B X-permutes with every Sylow subgroup of A, then G is a supersoluble group.*

Proof. Assume that this theorem is false and let G be a counterexample with minimal order. Then:

(1) *G/N is supersoluble for every non-identity normal subgroup N of G.*

Let N be a non-identity normal subgroup of G. First of all we note that $G/N = (AN/N)(BN/N)$ is the product of the supersoluble subgroups $AN/N \simeq A/N \cap A$ and $BN/N \simeq B/N \cap B$. Now let T/N be a cyclyc primary subgroup of AN/N. It is clear that for some cyclic primary subgroup $< b >$ of T we have $T =< b > N$. Since $T \leq AN$, $b = an$ for some element $a \in A$ having primary order and for some $n \in N$, and so $< a > N =< b > N$. Let D/N be a Sylow q-subgroup of BN/N. Hence $D/N = B_q N/N$ for some Sylow q-subgroup B_q of B. Since by hypotheses, $< a > B_q^x = B_q^x < a >$ for some $x \in X$ and so we have $(D/N)^{xN}(T/N) = (D^x N/N)(< a > N/N) = (B_q^x N/N)(< a > N/N) = B_q^x < a > N/N =< a > B_q^x N/N = (< a > N/N)(B_q^x N/N) = (< a > N/N)(D^x N/N) = (T/N)(D/N)^{xN}$. It is clear that $xN \in XN/N \leq F(G/N)$. Thus the hypotheses are true for G/N. But $|G/N| < |G|$, and so by the choice of G we have (1).

(2) *G is a soluble group.*

Assume that G is not soluble.

If $X \neq 1$ then by (1), G/X is supersoluble and so G is a soluble group, a contradiction.

Hence $X = 1$. Let p be the largest prime divisor of $|G|$. Without loss of generality we may assume that $p \mid |A|$. Let A_p be a Sylow p-subgroup of A. Then since by hypotheses A is supersoluble, from Lemma 1.5 we have $A_p \trianglelefteq A$. Thus A has a minimal normal subgroup, say H, such that $|H| = p$. If $H \leq B$, then by Lemma 1.3, $H^G \leq B$ and so a minimal normal subgroup of G contained in H^G is abelian since by hypotheses B is supersoluble as well as the subgroup A. From (1) it follows that G is soluble, a contradiction. Let $H \nsubseteq B$ and let $B = B_1 \dots B_t$ where $B_1, \dots, B_t$ are Sylow subgroups of B. Then since by hypotheses H permutes with all $B_1, \dots, B_t$, $D = HB = BH$. Assume $D \neq G$. Since the hypotheses are true for D and $|D| < |G|$, so we obtain that D is supersoluble. But $G = AD$, and so by Lemma 1.3 and in view of (1), we again have a contradiction. Now suppose that $D = G$. In view of Lemma 1.9 we may assume that $p|\,|B|$. Let B_p be a Sylow p-subgroup of B. Then $HB_p = B_pH$, and $G = HB = (HB_p)B$. Hence because $B_p \trianglelefteq B$, $B_p^G \subseteq HB_p$, and so by (1), G is a soluble group, a contradiction. That implies (2).

(3) *G has the only minimal normal subgroup, say N, and $G = [N]M$ where $N = C_G(N) = O_p(G)$ for some prime p, M is a supersoluble maximal subgroup of G and $O_p(M) = 1$.*

Since the class of all supersoluble groups is closed under subdirect products, then in view of (2), G has the only minimal normal subgroup, say N. In view of (1) and by Lemma 1.5, we also have $L \nsubseteq \Phi(G)$. Let M be a maximal subgroup of G not containing N and $C = C_G(N)$. Then by the Dedekind Law, we have $C = C \cap NM = N(C \cap M)$. Since N is abelian, $C \cap M \trianglelefteq G$ and so $C \cap M = 1$. This shows that $N = O_p(G) = C_G(N)$ and $M \simeq G/N$ is a supersoluble group with $O_p(M) = 1$ by Lemma 1.4.

(4) *p is the largest prime divisor of $|G|$.*

Let T_1 and T_2 be maximal subgroups of G such that $A \leq T_1$, $B \leq T_2$. Since $G = AB = T_1T_2$, then by Lemma 1.1, $T_1 \neq T_2^x$ for all $x \in G$. Hence by Lemma 1.6, $(T_1)_G \neq (T_2)_G$, and so we have either $N \subseteq T_1$ or $N \subseteq T_2$. Let $N \subseteq T_1$. Let q be the largest prime divisor of $|T_1|$. Then a Sylow q-subgroup of T_1 is normal in T_1, and hence it contained in $C_G(N) = N$. Thus p is the largest prime divisor of $|T_1|$. If T_1 is not a Hall subgroup of G, we have (4). Let T_1 be a Hall subgroup of G and assume that $p \neq q$, where q is the largest prime divisor of $|G|$. Then $|G : T_1| = q^\alpha$ for some $\alpha \in \mathbb{N}$. Since $N \subseteq T_1$, so by (1), $|G : T_1| = q$ is the order of a Sylow q-subgroup of G. It is clear that $q|\,|B|$. Let B_q be a Sylow q-subgroup of B. By hypotheses, $AB_q^x = B_q^xA$ for some $x \in X$ and by Lemma 1.5, $B_q^x \trianglelefteq B^x$. Hence by Lemma 1.3, $N \leq AB_q^x$. So $N \subseteq A_p$, and therefore A has a normal subgroup Z with order p such that $Z \leq N$ and $ZB^y = B^y Z$ for some $y \in X$. By Lemma 1.3, $N \leq ZB^y$, and so by (2), $B_q^y \leq C_G(N) = N$. This contradiction completes the proof of (4).

(5) *N is a Sylow p-subgroup of G.*

Assume that the assertion is not true. Then, we have $p \mid |G : N|$. This means that $p \mid |M|$, and so by (4) and by Lemma 1.5, we see that $O_p(M) \neq 1$. This contradicts (3). Hence, N is a Sylow p-subgroup of G.

(6) *Final contradiction.*

Since $G = AB$ and N is a Sylow p-subgroup of G, we have either $N \cap A \neq 1$ or $N \cap B \neq 1$. Let $N \cap A = A_p \neq 1$, Z_p be a minimal normal in A subgroup contained in $N \cap A$.

By hypotheses, $D = Z_p B_q^x = B_q^x Z_p$ for some $x \in X$ where q is a prime, $q \neq p$ and B_q is a Hall q-subgroup of B. Then $Z_p = N \cap Z_p B_q^x \trianglelefteq Z_p(N \cap D) \trianglelefteq D$. Hence $Z_p \trianglelefteq NAB_q^x$ and so $Z_p \trianglelefteq G$. Therefore $Z_p = N$, and so G is a supersoluble group, contrary to the choice of G.

□

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