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Cohomology of the categorical at zero semigroups

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Abstract. In this article we consider the relation between 0−cohomology and extended Eilenberg-MacLane cohomology of categorical at zero semigroups.

1. Introduction

The 0−cohomology theory of semigroups with zero element was introduced in the work [7] as a result of investigation devoted to projective representations of semigroups. This theory was applied also to studying Brauer monoid [9], matrix algebras [11], calculation of the Eilenberg-MacLane cohomology (EM-cohomology) of semigroups (see survey [10] and references there). Unfortunately the 0−cohomology functor is not a derived functor for all semigroups.

Nevertheless the 0−cohomology functor becomes a derived functor if we extend the category of coefficients up to the category of covariant functors from the small category to the category of Abelian groups [5].

On the other hand the 0−cohomology functor is a derived functor for a certain class of semigroups. B. V. Novikov showed that the categorical at zero semigroups belong to this class.

More precisely, [7]: let S be a categorical at zero semigroup, then there is an isomorphism $H_0^n(S, A) \cong H^n(\bar{S}, A)$ for all 0−modules under semigroup S and $n \geq 0$. Here H_0^* denotes the 0−cohomology functor, and $\bar{S}$ is the *gown* of the semigroup S (see sec. 2). This theorem provides a relation between 0−cohomology of semigroups and EM-cohomology and takes an important role in the cohomology theory of semigroups.

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The aim of this article is the generalization of this theorem for the category of all natural systems.

The paper consists of five sections. The necessary definitions and theorems are given in second section. The third section is devoted to the proof of the main result of the article, see theorem 3. The examples of applications of the main theorem are considered in section 4. The relation between the theory of Baues's-Wirshing's cohomology for small categories in case of categories without inverse morphisms and $0-$cohomology of semigroups is considered in section 5.

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2. Preliminaries

By S^1 we will denote a semigroup S with an adjoint identity. A semigroup S with zero element is called *categorical at zero* [3] if $abc = 0$ implies $ab = 0$ or $bc = 0$.

A small category $\mathbf{C}$ is called *connected* if for all objects a and b there is a sequence of morphisms $a \longrightarrow c_1 \longleftarrow c_2 \longrightarrow \ldots \longrightarrow c_n \longleftarrow b$ of the category $\mathbf{C}$.

Let $F : \mathbf{C} \longrightarrow \mathbf{D}$ be a covariant functor of small categories and $d \in Ob\mathbf{D}$. The *comma category* $(F \downarrow d)$ is the category which objects are morphisms $Fc \xrightarrow{\alpha} d$, and a morphism from $Fk \xrightarrow{\alpha} d$ to $Fl \xrightarrow{\beta} d$ is a morphism $\xi : k \longrightarrow l$ such that $\beta \circ F\xi = \alpha$.

Let $\mathbf{B}$ be a category, $\mathbf{A}$ is a subcategory of $\mathbf{B}$. A functor $R : \mathbf{B} \longrightarrow \mathbf{A}$ is said to be a *reflector* [2], if for all objects $b \in \mathbf{B}$ there is a morphism $\eta_b : b \longrightarrow Rb$ such that each arrow $g : b \longrightarrow a \in \mathbf{A}$ can be represented as $g = f\eta_b$ for unique morphism $f \in \mathrm{Mor}_{\mathbf{A}}(Rb, a)$.

Let us consider the category $\mathbf{Sem}_0$ whose objects are semigroups with zero and morphisms are mappings $f : S \setminus 0 \longrightarrow T$ such that $f(xy) = f(x)f(y)$ for $xy \neq 0$. As it was shown in [8] there is a reflector $R : \mathbf{Sem}_0 \longrightarrow \mathbf{Sem}$ where $\mathbf{Sem}$ is the category of all semigroups.

The reflector's value RS is called the *gown* of a semigroup S and is denoted by $\bar{S}$ [7].

The gown $\bar{S}$ consists of tuples $< s_1, \ldots, s_n >$ with $s_i \neq 0$, $s_i s_{i+1} = 0$. In case if S is a categorical at zero semigroup, the product of elements $s =< s_1, \ldots, s_n >$ and $t =< t_1, \ldots, t_m >$ is defined by the formula:

$$st = \begin{cases} < s_1, \ldots, s_n t_1, \ldots, t_m >, & \text{if } s_n t_1 \neq 0, \\ < s_1, \ldots, s_n, t_1, \ldots, t_m >, & \text{if } s_n t_1 = 0. \end{cases}$$

For simplicity we sometime will omit brackets in products like $\alpha < u >$ or $< u > \alpha$, where $\alpha \in \bar{S}$ and $u \in S$.

By the symbol $|s|$ we will denote the number of elements in a tuple $s =< s_1, \ldots, s_n >$.

By a *nerve* of a category $\mathbf{C}$ we will call the set $N_n(\mathbf{C})$ of all tuples $(\alpha_1, \ldots, \alpha_n)$ which components are morphisms of the category $\mathbf{C}$ such that the composition $\alpha_{i+1}\alpha_i$ exists for all $1 \leq i \leq n-1$ and $n \geq 1$. The nerve $N_0(\mathbf{C})$ consists of all objects of the category $\mathbf{C}$.

Let $C_n(\mathbf{C})$ denote a free Abelian group with the set of generators $N_n(\mathbf{C})$. We define a coboundary homomorphism $d_n : C_{n+1}(\mathbf{C}) \longrightarrow C_n(\mathbf{C})$ on the generating set by the formula:

$$\begin{aligned} d_n(\alpha_1, \ldots, \alpha_{n+1}) \quad = \quad & (\alpha_2, \ldots, \alpha_{n+1}) + \\ & \sum_{i=1}^{n} (-1)^i (\alpha_1, \ldots, \alpha_i\alpha_{i+1}, \ldots, \alpha_{n+1}) + \\ & (-1)^{n+1}(\alpha_1, \ldots, \alpha_n). \end{aligned}$$

The homology of the complex $\{C_n(\mathbf{C}), d_n\}_{n=0}^{\infty}$ is called an *integral homology of the nerve of the category* $\mathbf{C}$ and is denoted by $H_n(\mathbf{C})$ (see, for example, [12]).

By the symbol $\Delta : \mathbf{Ab} \longrightarrow \mathbf{Ab}^{\mathbf{C}}$ we denote the *diagonal functor*, which maps an Abelian group A to the constant functor $\Delta A : \mathbf{C} \longrightarrow \mathbf{Ab}$. For an object $c \in Ob\mathbf{C}$ the value of the functor ΔA is the Abelian group A and $(\Delta A)(f) = \mathrm{id}_A$ for all morphisms f of the category $\mathbf{C}$.

The right adjoint functor to the functor Δ is called an *inverse limit* $\varprojlim_{\mathbf{C}} : \mathbf{Ab}^{\mathbf{C}} \longrightarrow \mathbf{Ab}$.

Theorem 1. *[6] Let $\tau : \mathbf{C} \longrightarrow \mathbf{E}$ be a functor of small categories. Then the following conditions are equivalent:*
a) The category $(\tau \downarrow e)$ is connected and $H_n(\tau \downarrow e) = 0$ for all $n > 0,\ e \in \mathbf{E}$;
b) For every functor $F : \mathbf{E} \longrightarrow \mathbf{Ab}$ the canonical morphism:

$$\varprojlim\nolimits^n_{\mathbf{E}} F \longrightarrow \varprojlim\nolimits^n_{\mathbf{C}} F\tau$$

is an isomorphism for all $n > 0$.

Let $\mathbf{C}$ be a small category. The *category of factorization* $\mathbb{F}\mathbf{C}$ [4] is the category whose objects are all morphisms of $\mathbf{C}$ and the set $\mathrm{Mor}_{\mathbf{C}}(f, g)$ consists of three-tuples (α, f, β) such that $\alpha f \beta = g$. A covariant functor $D : \mathbb{F}\mathbf{C} \longrightarrow \mathbf{Ab}$ is called a *natural system on category* $\mathbf{C}$.

If we consider semigroup S^1 as a category with a single object, we obtain the correspondent definitions for the *category of factorizations* $\mathbb{F}S^1$ *in semigroup* S and *natural system* $\mathbb{F}S^1 \longrightarrow \mathbf{Ab}$ *on* S.

The *cohomology of a category* $\mathbf{C}$ *with coefficients in the natural system* D [4] is the Abelian groups $H^n(\mathbf{C}, D) = \mathrm{Ext}^n(Z, D)$ for all $n \geq 0$ where symbol Ext denotes the derived functor of Hom-functor and $Z : \mathbb{F}\mathbf{C} \longrightarrow \mathbf{Ab}$ is the constant functor: $Z(c) \cong \mathbb{Z}$.

Analogously to [13], [4] we define the *category of 0-factorizations in the semigroup* S *with zero* $\mathbb{F}_0 S^1$, whose objects are elements from $S \setminus 0$, and morphism's set $\mathrm{Mor}(a, b)$ consists of three-tuples (α, a, β) where α, β are elements from S, such that $\alpha a \beta = b$ [5].

A *0-natural system* on semigroup S with zero is a covariant functor $D : \mathbb{F}_0 S^1 \longrightarrow \mathbf{Ab}$. For simplicity we will denote the value of a functor D on an object $a \in \mathrm{Ob}\mathbb{F}_0 S^1$ by D_a. Let us denote $\alpha_* = D(\alpha, a, 1)$ and $\beta^* = D(1, a, \beta)$, then $D(\alpha, a, \beta) = \alpha_* \beta^*$ for each morphism (α, a, β).

For given natural number $n \geq 1$ let us denote by $N_n S^1$ the set of all n-tuples $(a_1, \ldots, a_n)$ of elements from S^1 such that $a_1 \cdots a_n \neq 0$ (the *nerve* of semigroup S^1). In case $n = 0$ let $N_0 S^1 = \{1\}$. The map with the domain on the nerve of S^1 that sends each $a = (a_1, \ldots, a_n)$ to an element from $D_{a_1 \cdots a_n}$ is called a *n-cochain.* The set of all n-cochains is an Abelian group $C_0^n(S^1, D)$ with respect to pointwise addition. For $n = 0$ let $C_0^0(S^1, D) = D_1$.

Let us define the *coboundary homomorphism* $\delta^n : C_0^n(S^1, D) \longrightarrow C_0^{n+1}(S^1, D)$ by the formula $(n \geq 1)$

$$
\begin{aligned}
&(\delta f)(a_1, \ldots, a_{n+1}) = a_{1*} f(a_2, \ldots, a_{n+1}) \\
&+ \sum_{i=1}^{n} (-1)^i f(a_1, \ldots, a_i a_{i+1}, \ldots, a_{n+1}) + (-1)^{n+1} a_{n+1}^* f(a_1, \ldots, a_n).
\end{aligned}
$$

For the case $n = 0$ we set $(\delta f)(x) = x_* f - x^* f$ for $f \in D_1$, $x \in S^1 \setminus 0$. It is simple to prove that $\delta^n \delta^{n-1} = 0$. The *group of cocycles* $Z_0^n(S^1, D)$ is the kernel of the coboundary δ^n. Let $B_0^n(S^1, D)$ denote the *group of coboundaries* which is the image of δ^{n-1}. The $0-$*cohomology of a semigroup* S *with coefficients in* $0-$*natural system* D is the Abelian groups

$$
H_0^n(S^1, D) = Z_0^n(S^1, D) / B_0^n(S^1, D).
$$

Theorem 2. *[5] Let* S *be a semigroup with zero,* $D : \mathbb{F}_0 S^1 \longrightarrow \mathbf{Ab}$ *is a* 0-*natural system. Then there is an isomorphism*

$$
H_0^n(S^1, D) \cong \mathrm{Ext}^n(Z, D),
$$

which is natural in D, *where* Z *is the constant* $0-$*natural system* $Z(s) \cong \mathbb{Z}$.

3. Main theorem

Let S be a categorical at zero semigroup and $\bar{S}$ be the grown of S. Consider the embedding functor $i : \mathbb{F}_0 S^1 \longrightarrow \mathbb{F}\bar{S}^1$ which is defined by the formula:

$$\begin{aligned} i(l) &= <l>, \\ i(s,l,p) &= (<s>,<l>,<p>), \end{aligned} \tag{1}$$

for all objects $l \in Ob\mathbb{F}_0 S^1$ and morphisms $(s,l,p) \in \mathrm{Mor}\mathbb{F}_0 S^1$.

Lemma 1. *The embedding i is full.*

Proof. Let us consider the arrow $<l> \xrightarrow{(\sigma,\tau)} <t>$ for some $\sigma, \tau \in \bar{S}^1$ and $l, t \in Ob\mathbb{F}_0 S^1$. Then $<t> = \sigma <l> \tau$, what implies the following equality $|\sigma <l> \tau| = 1$. This is possible only if $\sigma, \tau \in S^1$ and $\sigma l \tau \neq 0$, i.e. $(\sigma, l, \tau) \in \mathrm{Mor}\mathbb{F}_0 S^1$. $\square$

Let us formulate the main result of the article.

Theorem 3. *Let S be a categorical at zero semigroup, $i : \mathbb{F}_0 S^1 \longrightarrow \mathbb{F}\bar{S}^1$ is the embedding functor which was defined in (1), $D : \mathbb{F}\bar{S}^1 \longrightarrow$ **Ab** is a natural system on $\bar{S}^1$. Then the functor i induces an isomorphism of cohomology*

$$H^n(\bar{S}^1, D) \cong H_0^n(S^1, Di)$$

for all $n \geq 0$.

Let us sketch the main steps of the proof. In the first step we consider cohomology functor of semigroup as a derived functor of the limit functor under some factorization category. Further, we explore the integral complex of the nerve of comma category $(i \downarrow s)$ for $s \in \bar{S}^1$. For completion of the proof, due to the theorem 1, it is sufficient to show that the comma category $(i \downarrow s)$ is connected for all $s \in \bar{S}^1$ and that the integral homology of nerve $(i \downarrow s)$ is acyclic. The lemmas 2 and 3 are devoted to this aim.

Proof. In [4] it was shown that the cohomology of a small category $\mathbf{C}$ with coefficients in natural systems is the derived functor of the inverse limit functor:

$$H^n(\mathbf{C}, E) = \varprojlim\nolimits^n_{\mathbb{F}\mathbf{C}} E, \ n \geq 0,$$

where $E : \mathbb{F}\mathbf{C} \longrightarrow \mathbf{Ab}$ is a natural system. If we consider the monoid $\bar{S}^1$ as a category with a single object, then we have the following isomorphism: $H^n(\bar{S}^1, D) \cong \varprojlim^n_{\mathbb{F}\bar{S}^1} D$ for all $n \geq 0$.

Since the $0-$cohomology functor is a derived functor (see theorem 2), it is easy to show that $H_0^n(S^1, D) \cong \mathrm{Ext}^n(Z, D) \cong \varprojlim^n_{\mathbb{F}_0 S^1} D$ for all $n \geq 0$.

The embedding functor $i : \mathbb{F}_0 S^1 \longrightarrow \mathbb{F}\bar{S}^1$ induces the natural homorphism:

$$\Psi : \varprojlim^n_{\mathbb{F}\bar{S}^1} D \longrightarrow \varprojlim^n_{\mathbb{F}_0 S^1} Di, \ n \geq 0.$$

By the theorem 1 let us show that under conditions of the theorem 3 the homorphism Ψ is an isomorphism of cohomologies.

Let us adduce auxiliary notations which will be useful in future. Let $s =< s_1, \ldots, s_m >$ be an element of the gown $\bar{S}$. Then define

$$e_k = (< s_1, \ldots, s_k >, 1, < s_{k+1}, \ldots, s_m >) \in (i \downarrow s), \ 0 \leq k \leq m,$$
$$\bar{s}_k = (< s_1, \ldots, s_{k-1} >, < s_k >, < s_{k+1}, \ldots, s_m >) \in (i \downarrow s), \ 1 \leq k \leq m.$$

Lemma 2. *Let S be a categorical at zero semigroup, $\bar{S}$ is the gown of S, $i : \mathbb{F}_0 S^1 \longrightarrow \mathbb{F}\bar{S}^1$ is the embedding functor. Then the comma category $(i \downarrow s)$ is connected for each $s \in \bar{S}^1$.*

Proof. Consider the elements $(\alpha, l, \beta),\ (\gamma, t, \delta) \in Ob(i \downarrow s)$. We define the path which connects these objects. Let $s =< s_1, \ldots, s_n >$.

a) If $n = 1$, then $(1, s, 1) \in Ob(i \downarrow s)$ and the commutative diagram

$$\begin{tikzcd}
 & s \arrow[dd] & \\
l \arrow[ur, "(\alpha,\beta)"] \arrow[dr, "(\alpha,\beta)"'] & & t \arrow[ul, "(\gamma,\delta)"'] \arrow[dl, "(\gamma,\delta)"] \\
 & s &
\end{tikzcd}$$

gives the required path.

b) Let $n \geq 2$. It is obvious that the diagram

$$\bar{s}_p \xleftarrow{(s_p, 1)} e_p \xrightarrow{(1, s_{p+1})} \bar{s}_{p+1}$$

shows us that the set $\{e_k\} \bigcup \{\bar{s_p}\} \subset Ob(i \downarrow s)$ is connected.

Suppose $(\alpha, l, \beta) = (< s_1, \ldots, s_{k-1}, \alpha_k >, l, < \beta_1, s_{k+1}, \ldots, s_n >) \in (i \downarrow s)$ does not belong to the set $\{e_k\} \bigcup \{\bar{s_p}\}$. It is easy to check that the morphism $(u, l, v) : (\alpha, l, \beta) \longrightarrow \bar{s}_k$ of the comma category $(i \downarrow s)$ which is defined by the equation

$$(u, l, v) = \begin{cases} (\alpha_k, l, \beta_1), & \text{if } \alpha_k l \beta_1 \neq 0, \\ (\alpha_k, l, 1), & \text{if } \alpha_k l \neq 0 \text{ and } l\beta_1 = 0, \\ (1, l, \beta_1), & \text{if } \alpha_k l = 0 \text{ and } l\beta_1 \neq 0, \end{cases}$$

for some $1 \leq k \leq n$, defines the path between objects (α, l, β) and $\bar{s}_k$. $\square$

We start the proof of the fact that the complex $\{C_n(i\downarrow s), d_n\}_{n=0}^{\infty}$, $s\in Ob\mathbb{F}\bar{S}^1$ is acyclic.

By the definition $C_n(i\downarrow s),\ n\geq 1$, is a free Abelian group which is generated by the set of all tuples $(f_1,\dots,f_n)$ of morphisms from the category $(i\downarrow s)$, such that the composition $f_{i+1}f_i$ exists and $C_0(i\downarrow s)=\mathbb{Z}Ob(i\downarrow s)$.

Let $\Omega=((u_1,v_1),\dots,(u_n,v_n))$ be an element of the nerve $N_n(i\downarrow s),\ n\geq 1,\ s\in Ob\mathbb{F}\bar{S}^1$. This means that the diagram

$$\begin{tikzcd}
l_0 \arrow[r, "{(u_1,v_1)}"] \arrow[drrr, "{(\alpha_0,\beta_0)}"'] & l_1 \arrow[r, "{(u_2,v_2)}"] \arrow[drr] & l_2 \arrow[r] \arrow[dr] & \dots \arrow[r] & l_{n-1} \arrow[r, "{(u_n,v_n)}"] \arrow[dl] & l_n \arrow[dll, "{(\alpha_n,\beta_n)}"] \\
& & & s & &
\end{tikzcd}$$

is commutative where $l_i\in S^1$ and $\alpha_i,\beta_i\in\bar{S}^1$. In other words the following equalities

$$\begin{cases} l_{i+1}=u_{i+1}l_iv_{i+1}, \\ \alpha_i=\alpha_{i+1}u_{i+1}, \\ \beta_i=v_{i+1}\beta_{i+1}, \end{cases}$$

exist for all $0\leq i\leq n-1$. It implies that the nerve's element Ω is defined by the cortege

$$[\alpha_n,u_n,\dots,u_1,l_0,v_1,\dots,v_n,\beta_n].$$

In such a way the basis of the group $C_n(i\downarrow s)$ is the set $N_n(i\downarrow s)$ of elements $[\alpha,u_n,\dots,u_1,l,v_1,\dots,v_n,\beta]$, such that $u_nu_{n-1}\cdots l\cdots v_{n-1}v_n\in S^1, \alpha u_nu_{n-1}\dots l\cdots v_{n-1}v_n\beta=s$ for some $\alpha,\beta\in\bar{S}^1,\ n\geq 1$.

On basis elements of $C_n(i\downarrow s)$ the coboundary $d_{n-1}:C_n(i\downarrow s)\longrightarrow C_{n-1}(i\downarrow s),\ n\geq 1$ acts by the formula:

$$\begin{aligned}d_{n-1}[\alpha,u_n,\dots,u_1,l,v_1,\dots,v_n,\beta]&=[\alpha,u_n,\dots,u_1lv_1,\dots,v_n,\beta]+\\ \sum_{i=1}^{n-1}(-1)^i[\alpha,u_n,\dots,u_{i+1}u_i,&\dots,u_1,l,v_1,\dots,v_iv_{i+1},\dots,v_n,\beta]+\\ &(-1)^n[\alpha u_n,\dots,u_1,l,v_1,\dots,v_n\beta].\end{aligned}$$

It is simple to prove that the groups $C_n(i\downarrow s)$ with their coboundary maps form the simplicial object in the category of Abelian groups. By the theorem of normalization [1] it is sufficient to prove that the normalized complex $\{\hat{C}_n(i\downarrow s),d_n\}_{n=0}^{\infty}$ is acyclic. Here $\hat{C}_n(i\downarrow s),\ n\geq 1$ is a free Abelian group with the set of generators:

$$\begin{aligned}\hat{N}_n(i\downarrow s)\ &=\ \{[\alpha,u_n,\dots,l,\dots,v_n,\beta]\in N_n(i\downarrow s)|\\ &\quad (u_k,v_k)\neq(1,1),\ \alpha,\beta\in\bar{S}^1,\ \forall k\}.\end{aligned}$$

The group $\hat{C}_0$ coincides with C_0 by the definition.

Lemma 3. *The complex* $\{\hat{C}_n(i \downarrow s), d_n\}_{n=0}^{\infty}$ *is acyclic.*

Proof. Let us construct a contracting homotopy $\varepsilon_{n+1} : \hat{C}_n \longrightarrow \hat{C}_{n+1}$, $n \geq 1$. Let $\Omega = [\alpha, u_n, \ldots, u_1, l, v_1, \ldots, v_n, \beta] \in \hat{N}_n(i \downarrow s)$ for elements $\alpha =< \alpha_1, \ldots, \alpha_k >$, $\beta =< \beta_1, \ldots, \beta_p >$. For all elements $s =< s_1, \ldots, s_m >\in \bar{S}$ let us denote $l(s) =< s_2, \ldots, s_m >$ and $r(s) =< s_1, \ldots, s_{m-1} >$. Denote by ω the product $u_n \cdots u_1 l v_1 \cdots v_n$. Define the homotopy by the formula:

$$\varepsilon_{n+1}\Omega = \begin{cases} 0, & \text{if } \alpha_k\omega = \omega\beta_1 = 0, \\ (-1)^{n+1}[r(\alpha), \alpha_k, u_n, \ldots, v_n, \beta_1, l(\beta)], & \text{if } \alpha_k\omega\beta_1 \neq 0, \\ (-1)^{n+1}[r(\alpha), \alpha_k, u_n, \ldots, v_n, 1, \beta], & \text{if } \alpha_k\omega \neq 0,\ \omega\beta_1 = 0, \\ (-1)^{n+1}[\alpha, 1, u_n, \ldots, v_n, \beta_1, l(\beta)], & \text{if } \alpha_k\omega = 0,\ \omega\beta_1 \neq 0, \end{cases}$$

for all $n \geq 1$.

Let us prove that in the case $n \geq 2$ the diagram

$$\hat{C}_{n+1} \underset{\varepsilon_{n+1}}{\overset{d_n}{\rightleftarrows}} \hat{C}_n \underset{\varepsilon_n}{\overset{d_{n-1}}{\rightleftarrows}} \hat{C}_{n-1}$$

yields the following equation:

$$\varepsilon_n d_{n-1} + d_n \varepsilon_{n+1} = \mathrm{id}_{\hat{C}_n}. \tag{2}$$

Consider three possible cases which correspond to the definition of ε_{n+1}.

Case 1. $\alpha_k\omega = \omega\beta_1 = 0$. Then $d_n\varepsilon_{n+1}\Omega = 0$ and

$$\begin{aligned} \varepsilon_n d_{n-1}\Omega \ = \ & \varepsilon_n[\alpha, u_n, \ldots, u_1 l v_1, \ldots, v_n, \beta] + \\ & \sum_{i=1}^{n-1}(-1)^i \varepsilon_n[\alpha, u_n, \ldots, u_{i+1}u_i, \ldots, u_1, l, v_1, \ldots, v_i v_{i+1}, \ldots, v_n, \beta] + \\ & (-1)^n \varepsilon_n[\alpha u_n, \ldots, u_1, l, v_1, \ldots, v_n\beta]. \end{aligned} \tag{3}$$

By associativity of S^1 all the summands of equality (3) except the last one are equal to zero.

Let $u_n = 1$ then $v_n \neq 1$. Since S is categorical at zero we conclude that $v_n\beta_1 = 0$. Thus we have

$$\begin{aligned} \varepsilon_n[\alpha u_n, u_{n-1}, \ldots, v_{n-1}, v_n\beta] = \varepsilon_n[\alpha, u_{n-1}, \ldots, v_{n-1}, < v_n, \beta_1 >] = \\ (-1)^n[\alpha, 1, u_{n-1}, \ldots, v_{n-1}, v_n, \beta] = (-1)^n\Omega \end{aligned}$$

The proof of the equation (2) in the case when $u_n \neq 1$ and $v_n = 1$ is dual with the given one. Now let $u_n \neq 1$ and $v_n \neq 1$. Since S is categorical at zero we have $\alpha_k u_n = v_n \beta_1 = 0$. Hence

$$\varepsilon_n[\alpha u_n, u_{n-1}, \dots, v_{n-1}, v_n\beta] =$$
$$\varepsilon_n[< \dots, \alpha_k, u_n >, u_{n-1}, \dots, v_{n-1}, < v_n, \beta_1, \dots >] = (-1)^n\Omega.$$

So the proof of the equation (2) is finished for the first case.

Case 2. Let $\Omega \in \hat{N}_n(i \downarrow s)$ such that $\alpha_k\omega\beta_1 \neq 0$. Then

$$\begin{gathered} d_n\varepsilon_{n+1}\Omega = (-1)^{n+1}d_n[r(\alpha), \alpha_k, u_n, \dots, v_n, \beta_1, l(\beta)] = \\ (-1)^{n+1}([r(\alpha), \alpha_k, u_n, \dots, u_1 l v_1, \dots, v_n, \beta_1, l(\beta)] + \\ \sum_{i=1}^{n-1}(-1)^i[r(\alpha), \alpha_k, \dots, u_{i+1}u_i, \dots, v_i v_{i+1}, \dots, \beta_1, l(\beta)] + \\ (-1)^n[r(\alpha), \alpha_k u_n, u_{n-1}, \dots, v_{n-1}, v_n\beta_1, l(\beta)] + (-1)^{n+1}\Omega) \end{gathered} \tag{4}$$

Evidently the sum of the first n summands of (3) equals to the correspondent sum of (4) with the opposite sign. By the definition of the product in $\bar{S}^1$ the last summand of the equality (3) has the form:

$$(-1)^n\varepsilon_n[\alpha u_n, u_{n-1}, \dots, v_{n-1}, v_n\beta] =$$
$$(-1)^n\varepsilon_n[< \dots, \alpha_{k-1}, \alpha_k u_n >, u_{n-1}, \dots, v_{n-1}, < v_n\beta_1, \beta_2, \dots >] =$$
$$(-1)^n[r(\alpha), \alpha_k u_n, \dots, v_n\beta_1, l(\beta)].$$

This statement ends the proof of the second case.

Case 3. In case when $\alpha_k\omega = 0$ and $\omega\beta_1 \neq 0$ (the case when $\alpha_k\omega \neq 0$ and $\omega\beta_1 = 0$ is dual) the proof of (2) is similar with the second case.

Now we start the proof of equation (2) for $n = 1$. Let $s =< s_1, \dots, s_m >$, $m \geq 2$ (the case when $m = 1$ is evident).

Define the homorphism $\varepsilon_1 : C_0(i \downarrow s) \longrightarrow \hat{C}_1(i \downarrow s),\ m \geq 2$. Introduce the following auxiliary notations:

$$s_k^l = [< s_1, \dots, s_{k-1} >, s_k, 1, 1, < s_{k+1}, \dots, s_m >] \in \hat{C}_1(i \downarrow s),\ 1 \leq k \leq m,$$
$$s_k^r = [< s_1, \dots, s_{k-1} >, 1, 1, s_k, < s_{k+1}, \dots, s_m >] \in \hat{C}_1(i \downarrow s),\ 1 \leq k \leq m.$$

We give ε_1 by the formulas:

$$\varepsilon_1 e_k = \sum_{i=1}^{k} s_i^r + \sum_{i=k+1}^{m} s_i^l, \quad 0 \leq k \leq m,$$
$$\varepsilon_1 \bar{s}_k = \sum_{i=1}^{k} s_i^r + \sum_{i=k}^{m} s_i^l, \quad 1 \leq k \leq m.$$

Now consider the case when the element $\gamma = [\alpha, u, \beta] \in C_0(i \downarrow s)$ does not belong to the set $\{e_k, \bar{s}_l\}_{k,l}$. Then define

$$\varepsilon_1(\gamma) = \begin{cases} -[r(\alpha), \alpha_k, u, 1, 1, \beta] + \varepsilon_1 \bar{s}_k, & \text{if } u \neq 1,\ \alpha_k u \neq 0, u\beta_1 = 0, \\ -[\alpha, 1, u, \beta_1, l(\beta)] + \varepsilon_1 \bar{s}_{k+1}, & \text{if } u \neq 1,\ \alpha_k u = 0, u\beta_1 \neq 0, \\ -[r(\alpha), \alpha_k, u, \beta_1, l(\beta)] + \varepsilon_1 \bar{s}_k, & \text{if } \ \alpha_k u \beta_1 \neq 0. \end{cases}$$

Let $\Omega = [\alpha, u_1, l, v_1, \beta] \in \hat{N}_1(i \downarrow s)$. Denote $\omega = u_1 l v_1$. Consider three cases.

Case 1: $\alpha_k \omega \beta_1 \neq 0$. Then we have:

$$\varepsilon_1 d_0 \Omega = \varepsilon_1[\alpha, u_1 l v_1, \beta] - \varepsilon_1[\alpha u_1, l, v_1\beta] = -[r(\alpha), \alpha_k, u_1 l v_1, \beta_1, l(\beta)] + \\ \varepsilon_1 \bar{s}_k + [r(\alpha), \alpha_k u_1, l, v_1\beta_1, l(\beta)] - \varepsilon_1 \bar{s}_k.$$

In the same time

$$d_1 \varepsilon_2 \Omega = d_1[r(\alpha), \alpha_k, u_1, l, v_1, \beta_1, l(\beta)] = [r(\alpha), \alpha_k, u_1 l v_1, \beta_1, l(\beta)] - \\ [r(\alpha), \alpha_k u_1, l, v_1\beta_1, l(\beta)] + \Omega.$$

This ends the exploration of the first case.

Case 2: $\alpha_k \omega = 0$ and $\omega\beta_1 \neq 0$.

a) $l \neq 1$. Then

$$\varepsilon_1 d_0 \Omega = -[\alpha, 1, u_1 l v_1, \beta_1, l(\beta)] + \varepsilon_1 \bar{s}_{k+1} + [\alpha, u_1, l, v_1\beta_1, l(\beta)] - \varepsilon_1 \bar{s}_{k+1},$$
$$d_1 \varepsilon_2 \Omega = d_1[\alpha, 1, u_1, l, v_1, \beta_1, l(\beta)] = [\alpha, 1, u_1 l v_1, \beta_1, l(\beta)] - \\ [\alpha, u_1, l, v_1\beta_1, l(\beta)] + \Omega.$$

b) $l = 1,\ u_1 = 1,\ v_1 \neq 1$. In this case we have:

$$\varepsilon_1 d_0 \Omega = \varepsilon_1[\alpha, v_1, \beta] - \varepsilon_1[\alpha, 1, < v_1\beta_1, \dots >] = -[\alpha, 1, v_1, \beta_1, l(\beta)] + \\ \varepsilon_1 \bar{s}_{k+1} - \varepsilon_1 e_k = -[\alpha, 1, v_1, \beta_1, l(\beta)] + s^r_{k+1},$$
$$d_1 \varepsilon_2 \Omega = d_1[\alpha, 1, 1, 1, v_1, \beta_1, l(\beta)] = [\alpha, 1, v_1, \beta_1, l(\beta)] - s^r_{k+1} + \Omega.$$

c) $l = 1,\ u_1 \neq 1,\ v_1 = 1$. Then we obtain

$$\varepsilon_1 d_0 \Omega = \varepsilon_1[\alpha, u_1, \beta] - \varepsilon_1[< \dots, \alpha_k, u_1 >, 1, \beta] = -[\alpha, 1, u_1, \beta_1, l(\beta)] + \\ \varepsilon_1 \bar{s}_{k+1} + [\alpha, u_1, 1, \beta_1, l(\beta)] - \varepsilon_1 \bar{s}_{k+1},$$
$$d_1 \varepsilon_2 \Omega = [\alpha, 1, u_1, \beta_1, l(\beta)] - [\alpha, u_1, 1, \beta_1, l(\beta)] + \Omega.$$

This ends our consideration of the second case. It is clear that the proof in case when $\alpha_k \omega \neq 0$ and $\omega\beta_1 = 0$ is similar to the considered one.

Case 3: $\alpha_k \omega = 0$ and $\omega\beta_1 = 0$. Then $(\alpha, u_1 l v_1, \beta) = \bar{s}_{k+1} \in C_0(i \downarrow s)$, $\varepsilon_2 \Omega = 0$. We have

$$\varepsilon_1 d_0 \Omega = \varepsilon_1 \bar{s}_{k+1} - \varepsilon_1(\alpha u_1, l, v_1\beta).$$

a) $l \neq 1$ or $l = 1$ when $u_1 \neq 1$ and $v_1 \neq 1$. Then $\varepsilon_1 d_0 \Omega = \varepsilon_1 \bar{s}_{k+1} + \Omega - \varepsilon_1 \bar{s}_{k+1} = \Omega$.

b) $l = 1$ and $u_1 = 1$. Then $\varepsilon_1 d_0 \Omega = \varepsilon_1 \bar{s}_{k+1} - \varepsilon_1 e_k = s^r_{k+1} = \Omega$.

c) $l = 1$ and $v_1 = 1$. In this case $\varepsilon_1 d_0 \Omega = \varepsilon_1 \bar{s}_{k+1} - \varepsilon_1 e_{k+1} = s^l_{k+1} = \Omega$.

In such a way the equation (2) for the case $n = 1$ is proven. The proof of the lemma as well as the main theorem is completed. □

□

Remark. There is another form of the described isomorphism. Let $i^* : \mathbf{Ab}^{\mathbb{F}\bar{S}^1} \longrightarrow \mathbf{Ab}^{\mathbb{F}_0 S^1}$ be the restriction functor which is induced by i. Let us denote by Lan_i left Kan extension along the functor i ($Lan_i \dashv i^*$). Since i is full, we get from [2] (Ch. X)

$$i^*(Lan_i G) = G,$$

for each functor $G : \mathbb{F}_0 S^1 \longrightarrow \mathbf{Ab}$. Replacing D by $Lan_i G$ in the theorem 3, we obtain the other form of the isomorphism:

$$H^n_0(S, G) \cong H^n(\bar{S}, Lan_i G).$$

Corollary. *[7]. Let S be a categorical at zero semigroup, $\bar{S}$ its grown. Then*

$$H^n(\bar{S}, A) \cong H^n_0(S, A), \; n \geq 0,$$

for each $0-$module A over S.

4. Examples

Example 1. Let us compute the cohomology of the semigroup W with the representation: $\langle a_i, b_j, w \; | a_i b_i = w, \; 1 \leq i, j \leq n \rangle$. Let the ideal $I = W \setminus \{a_i, b_j, w | \; 1 \leq i, j \leq n\}$. It is simple to check that Rees factor semigroup $S = W/I$ is categorical at zero and the gown $\bar{S} = W$.

Using theorem 3 for computing the cohomology of W, it is sufficient to calculate $0-$cohomology of the semigroup S.

Let $\mathbb{F}_0 S^1$ be the category of $0-$factorizations of S^1 and $D : \mathbb{F}_0 S^1 \longrightarrow \mathbf{Ab}$ is a natural system. Consider the normalized complex:

$$\hat{C}^0_0(S^1, D) \longrightarrow \hat{C}^1_0(S^1, D) \longrightarrow \hat{C}^2_0(S^1, D) \longrightarrow \ldots \longrightarrow \hat{C}^n_0(S^1, D) \longrightarrow \ldots$$

It is obvious that $\hat{C}^p_0(S^1, D) = 0$ if $p \geq 3$. Let $f \in \hat{Z}^2_0(S^1, D) = \hat{C}^2_0(S^1, D)$. Then f is defined by its values on the elements of the set $\{(a_k, b_k)\}^n_{k=1} \subset S \times S$. In that way we have: $\hat{Z}^2_0(S^1, D) \cong \bigoplus^n_{i=1} D_w$.

Now let $f \in \hat{B}_0^2(S^1, D)$. It means that there is a function $h \in \hat{C}_0^1(S^1, D)$ such that

$$f(a_i, b_i) = D(a_i, 1)h(b_i) - h(w) + D(1, b_i)h(a_i),\ 1 \le i \le n.$$

Denote by M_i an Abelian group which consists of elements $D(a_i, 1)x + D(1, b_i)y$ for all $x \in D_{b_i},\ y \in D_{a_i},\ 1 \le i \le n.$

Let K be a subset of $\bigoplus_{i=1}^n D_w$ of the tuples $(m_1, \dots, m_n)$ where $m_i \in t + M_i$ for some $t \in D_w,\ 1 \le i \le n$. Then $H_0^2(S^1, D) \cong (\bigoplus_{i=1}^n D_w)/K$. Let us compute this factor group.

Consider the map

$$\eta : D_w/M_1 \oplus D_w/M_2 \oplus \cdots \oplus D_w/M_n \longrightarrow D_w^n/K, \tag{5}$$

which is given by the rule:

$$\eta(x_1 + M_1, x_2 + M_2, \dots, x_n + M_n) = (x_1, x_2, \cdots, x_n) + K.$$

We now verify that η is a correctly defined map. Let $x_i' \in x_i + M_i,\ 1 \le i \le n$. Then $\eta(x_1 + M_1, x_2 + M_2, \dots, x_n + M_n) - \eta(x_1' + M_1, x_2' + M_2, \dots, x_n' + M_n) = (x_1 - x_1', \dots, x_n - x_n') + K = 0$ since $x_i - x_i' \in M_i$, and the map η is correctly defined.

Obviously η is a surjective map. Let us calculate the kernel L of η. Let $\eta(x_1 + M_1, x_2 + M_2, \dots, x_n + M_n) = (t + m_1, \dots, t + m_n)$ for some $t \in M,\ m_i \in M_i,\ 1 \le i \le n$. It follows that

$$x_i = t + m_i,\ 1 \le i \le n.$$

Thus L consists of tuples of cosets $(t + M_1, \dots, t + M_n)$ generated by an element $t \in D_w$. By the Noeter Theorem we conclude that

$$H^2(S^1, D) \cong (\bigoplus_{i=1}^{n} D_w/M_i)/L,\ L = \{(t + M_1, \dots, t + M_n) |\ t \in D_w\} \tag{6}$$

Example 2. Let us compute the second cohomology group of the semigroup S from the example 1 for more specific case. We will need this example in section 5.

Let $S = W/I$ be the semigroup from example for $n = 3$, A be an Abelian finite group of odd order, $D : \mathbb{F}_0 S^1 \longrightarrow \mathbf{Ab}$ be a $0-$natural system. For each nonzero element $s \in S$ the value of the functor D is the Abelian group $D_s = A \bigoplus A$. If (s, u, t) be a morphism from $\mathbb{F}_0 S^1$, $a = (a_1, a_2) \in D_u$ then:

$$D(s, u, t)a = s_* t^* a = \begin{cases} a, & \text{if } s = t = 1, \\ (a_1 + a_2, a_1 + a_2), & \text{if } s = a_1 \text{ and } t = 1, \\ (a_1 - a_2, a_2 - a_1), & \text{if } s \in \{a_2, a_3\} \text{ and } t = 1, \\ 0, & \text{otherwise.} \end{cases}$$

It is easy to check that D is a covariant functor.

Now compute the second $0-$cohomology group $H_0^2(S, D)$.

Using notation from example 1 we have:

$$\begin{aligned} M_1 &= a_{1*}D_{b_1} = \{(x,x)|\ x \in A\}, \\ M_i &= a_{i*}D_{b_i} = \{(x,-x)|\ x \in A\},\ i = 2,3, \\ K &= \{(v+m_1, v+m_2, v+m_3)|\ v \in D_w, m_i \in M_i\}. \end{aligned}$$

Define the map $\varphi : \bigoplus_{i=1}^{3} D_w/M_i \longrightarrow A^3$ by the formula

$$\varphi(l+M_1, p+M_2, k+M_3) = (l_1-l_2, p_1+p_2, k_1+k_2),$$

where $l = (l_1, l_2),\ p = (p_1, p_2)$ and $k = (k_1, k_2)$.

Lemma 4. *The map φ is an isomorphism of Abelian groups. The inverse map φ^{-1} is defined by the formula $\varphi^{-1}(a_1, a_2, a_3) = ((a_1, 0) + M_1, (a_2, 0) + M_2, (a_3, 0) + M_3)$.*

Consider the epimorphism $\psi = \eta \circ \varphi^{-1} : A^3 \longrightarrow D_w^3/K$ where

$$\eta : D_w/M_1 \bigoplus D_w/M_2 \bigoplus D_w/M_3 \longrightarrow D_w^3/K$$

is the homomorphism which was defined in (5). Using the formula (6) we obtain $H_0^2(S^1, D) = A^3/Ker\psi$.

Let us compute the kernel ψ. Let $(a_1, a_2, a_3) \in Ker\psi$. Then

$$\begin{aligned} &((a_1,0),(a_2,0),(a_3,0)) \in K \quad \Leftrightarrow \\ &\qquad ((a_1,0),(a_2,0),(a_3,0)) \quad = \quad (t+(l_1,l_2), t+(l_2,-l_2), t+(l_3,-l_3)) \end{aligned}$$

for some $t_i,\ l_i \in A$. It follows that $a_1 = T - P$ and $a_2 = a_3 = T + P$ where $T, P \in A$. Since A has odd order, $Ker\psi = \{(a,b,b),\ a,b \in A\}$. Thus we obtain

$$H_0^2(S^1, D) \cong A. \tag{7}$$

5. Cohomology of categories without inverse morphisms

A small category $\mathbf{C}$ is called a *category without inverse morphisms* if for all morphisms f, g from $f \circ g = \mathrm{id}_x$ it follows $f = g = \mathrm{id}_x,\ x \in Ob\mathbf{C}$.

Let $\mathbf{C}$ be a category without inverse morphisms. Define the semigroup $S_N\mathbf{C}$. The elements of $S_N\mathbf{C}$ are all nonidentical morphisms of $\mathbf{C}$ and a zero element. For all $f, g \in S_N\mathbf{C} \setminus \{0\}$ define the multiplication by the formula:

$$fg = \begin{cases} \bar{f} \circ \bar{g}, & \text{if the composition } \bar{f} \circ \bar{g} \text{ exists,} \\ 0, & \text{otherwise.} \end{cases}$$

Here and further on for the element $f \in S_N\mathbf{C}$ we denote by $\bar{f}$ the correspondent morphism of the category $\mathbf{C}$.

Let us define the map

$$^* : \mathrm{Mor}\mathbf{C} \longrightarrow \mathrm{Mor}\mathbf{C} \bigcup \{1\}$$

by the rule

$$f^* = \begin{cases} 1, & \text{if } f = \mathrm{id}_x,\ x \in Ob\mathbf{C}, \\ f, & \text{otherwise.} \end{cases}$$

Lemma 5. *Let* $\mathbf{C}$ *be a small category without inverse morphisms. The map* * *can be extended up to the equivalence of categories* $^* : \mathbb{F}\mathbf{C} \xrightarrow{\sim} \mathbb{F}_0 S_N \mathbf{C}^1$.

Proof. Let the value of the functor * for object $f \in Ob\mathbb{F}\mathbf{C}$ be $f^* \in Ob\mathbb{F}_0 S_N \mathbf{C}^1$ and (α^*, f^*, β^*) be the image of the morphism $(\alpha, f, \beta) \in \mathrm{Mor}\mathbb{F}\mathbf{C}$. It is simple to check that * is well-defined covariant functor.

Let us prove that the functor * is an equivalence of categories. Denote by ψ the map $\mathrm{Mor}_{\mathbb{F}\mathbf{C}}(f, g) \longrightarrow \mathrm{Mor}_{\mathbb{F}_0 S_N \mathbf{C}^1}(f^*, g^*)$ which is induced by the functor *.

Ensure that ψ is injective. Let $(\alpha^*, f^*, \beta^*) = (\alpha^*, k^*, \beta^*)$ for some morphisms $(\alpha, f, \beta), (\alpha, k, \beta) \in \mathrm{Mor}_{\mathbb{F}\mathbf{C}}(f, g)$. Let g be a nonidentical morphism. Consider two cases. If $f^* = 1$ then $f = k = \mathrm{id}_{\mathrm{dom}\alpha}$. In case if $f^* \neq 1$ the morphism $f = k = \bar{f^*}$. If g is the identical morphism, it is obvious that ψ is injective.

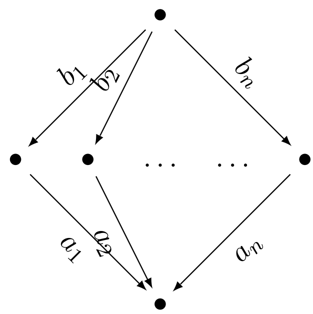
Let us check surjectivity. If (s_1, f^*, s_2) be a morphism $\mathbb{F}_0 S_N \mathbf{C}^1$ then $s_1 f^* s_2 \neq 0$ and the composition $\bar{s_1} \circ \bar{f^*} \circ \bar{s_2}$ exists. It is obvious that $(\bar{s_1}, \bar{f^*}, \bar{s_2})$ is the necessary preimage of (s_1, f^*, s_2). □

Since the equivalence of the categories implies the isomorphism of cohomologies, we have proved the following

Theorem 4. *Let* $\mathbf{C}$ *be a small category without inverse morphisms,* $D : \mathbb{F}_0 S_N \mathbf{C}^1 \longrightarrow \mathbf{Ab}$ *is a* 0*-natural system. Then there is the isomorphism of cohomologies:*

$$H^n(\mathbf{C}, D^*) \cong H^n_0(S_N \mathbf{C}^1, D),\ n \geq 0.$$

Example 3. Let $\mathbf{E}$ be a small category which is defined by the commutative diagram



with $w = a_i b_i$. Obviously $\mathbf{E}$ is the category without inverse morphisms and $S_N\mathbf{E}$ coincides with the semigroup S from example 1.

Using the result from this example and theorem 4 we get

Proposition. *Let $P : \mathbb{F}\mathbf{E} \longrightarrow \mathbf{Ab}$ be a natural system on $\mathbf{E}$, $\tau : \mathbb{F}_0 S_N \mathbf{C}^1 \xrightarrow{\sim} \mathbb{F}\mathbf{C}$ is an equivalence of categories. Then*

$$H^2(\mathbf{E}, P) = (\bigoplus_{i=1}^{n} (P\tau)w/K_i)/L, \;\; L = \{(m+K_1, \dots, m+K_n) | m \in (P\tau)w\},$$

where K_i is a subgroup of $(P\tau)w$ which consists of the elements $(P\tau)(a_i, 1)x + (P\tau)(1, b_i)y$.

Remark. The cohomology of the category $\mathbf{E}$ from example 3 was explored in [4]. In that work the following result was introduced without proof

$$H^2(\mathbf{E}, P) = P_w/I_2 \bigoplus P_w/I_3, \tag{8}$$

with $I_2 = a_{1*}P_{b_1} + a_{2*}P_{b_2}$ and $I_3 = a_{1*}P_{b_1} + a_{3*}P_{b_3}$. It is erroneous.

Indeed, let us consider the functor $P = D^*$ where D is the $0-$natural system which was defined in (7). Then $I_2 = M_1 + M_2 = D_w$, $I_3 = M_1 + M_3 = D_w$ and formula (8) becomes the form $H^2(\mathbf{E}, D^*) = 0$.

From the other hand, using the result that was received in (7) we obtain the formula $H^2(\mathbf{E}, D^*) \cong A$ which contradicts with (8).

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