History of the conferences on Radical Theory

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In the seventies and eighties, the cold war separated the scientists of East and West, and practically the only contact between them was reading the other side's publications. This was so even in pure mathematics, in particular, in algebra, which had no connotation with politics whatsoever. Besides, in the East travel was not only an issue of money. Scientific conferences, colloquia, symposia and workshops were in general rare, although there was a strong demand for personal meeting and cooperation.

Organizers of conferences – even of non-international ones – might run into unexpected difficulties. In Kishinëv, the capital city of the Moldavian Soviet Socialist Republic, an All-Union Symposium was organized in 1968. At the railway station two organizers were meeting the arriving participants, and held a banner "Rings and Modules". As the arriving mathematicians gathered around the banner, the situation became suspicious. Some people might think of "wedding rings" and get the impression of illegal jewellery business. The police intervened and arrested the two organizers who were soon released upon intervention of V. A. Andrunakievich, President of the Academy of Sciences of the Moldavian Soviet Socialist Republic – a highly respected position.[1]

International conferences tacitly aimed also at bringing together scientists from East and West and from countries on the 'black list' of the communist countries (Israel, South Africa, South Korea, Taiwan). Sometimes this goal could be more or less achieved.

Before turning to a more detailed account of the conferences, we have to stress that the choice of names and talks cited here is certainly highly

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The second author has the priviledge of being the only person participating in all the conferences on radicals. The next placed 'frequent participants' are Rainer Mlitz, who missed only the Hobart meeting, and the first author, who was absent in Hobart and Sendai.

[1]Thanks are due to Professor V. I. Arnautov for the story, which was not at all humorous in those days.

subjective, many other talks were also interesting and significant. And a further warning: the account which follows shall not be considered as the history of radical theory in the given period of time. We discuss only the history of conferences on radicals. Many excellent results have not been presented at them.

1. **International Colloquium on Associative Rings, Modules and Radicals**, Keszthely, Hungary, August 9–13, 1971.

Time has come to organize an international conference on ring theory with special emphasis on radicals. In Hungary the political situation was relatively relaxed, and it was anticipated that the conference would provide a good opportunity for a meeting of researchers from East and West. This conception worked well with one exception: S. A. Amitsur, as an Israeli citizen, did not get the entry visa. The Russian A. G. Kurosh passed away not much before the conference, so regretfully the founders of general radical theory could never meet. 63 participants from 16 countries were present on the conference, 10 from the US and 9 from West-Germany. The Soviet delegation consisted of 4 persons. Among the talks, V. A. Andrunakievich and Yu. M. Ryabukhin lectured on generalized nil radicals, P. M. Cohn spoke on free radical rings, and K. A. Zhevlakov, who passed away only half a year after the conference at the age of 33, presented his fundamental results on the Jacobson radical of alternative rings. During the conference, as expected, a very good scientific atmosphere developed in spite of the language barrier. At the beginning the leading experts of general radical theory, the American W. G. Leavitt and the Soviet-Moldavian V. A. Andrunakievich had some problems in mutual communication, but during the visit of a wine cellar in Badacsony, both became fluent in German. In the friendly summer weather guests could enjoy sightseeing in the historical town Keszthely and take refreshing swims in the lake Balaton. The success of the conference appeared also in its proceedings *Rings, Modules and Radicals*, edited by A. Kertész, Colloquia Mathematica Societatis János Bolyai, vol. 6, North-Holland, 1973. The 520 page long proceedings consists of 45 papers, including ones of the absent Amitsur and N. H. McCoy, as well as English translations of Kurosh's pioneering papers on radicals of rings and of groups, which at that time were available only in Russian.

2. **Radical Theory**, Eger, Hungary, August 1 – 7, 1982.

This was the very first conference devoted entirely to radical theory. The conference provided a suitable place for an encounter of algebraists both from East and West. 64 participants came from 22 countries representing all the five continents. Since the '67 middle-east war this was the first occasion that Israelis might attend a scientific conference in Hungary. In the late seventies, L. Márki got most useful informations from Soviet colleagues on the ways how Soviet scientists could get the permission of going abroad (to other communist countries, of course), and before this conference, L. A. Skornyakov from Moscow and L. A. Bokut' from Novosibirsk gave hints to him whom one could invite and in which way. Due to this, 14 excellent algebraists attended the conference from the Soviet Union but, strangely, nobody could come from the "radical centre" Kishinëv. (Several Soviet participants came upon personal invitations of Hungarian colleagues who had to state in an official letter stamped by Hungarian authorities that they would cover all local expenses of their invitee, although these Hungarian colleagues knew that, after all, they would not be financially involved. And, from an official Soviet viewpoint, people who came in this way were not entitled to attend a conference and meet foreign colleagues.) A strong team of radical theorists had grown up in South Africa, the organizers sent them invitation letters (put in the mail in a third country) but the Hungarian authorities did not issue entry visas for them. The list of participants included S. A. Amitsur and N. Jacobson, classics of radical theory, the '94 Fields medalist E. I. Zel'manov (Novosibirsk) who had completed the classification of simple Jordan algebras not very long before, and K. I. Beidar (Moscow) who had already solved the long-standing Suliński-Anderson-Divinsky problem on the termination of the lower radical construction. As the news spread that Zel'manov might be present at the conference, a strong group of specialists in Jordan algebras from North America and Western Europe (among them N. Jacobson) decided to attend the conference. The conference featured a large number of highly interesting talks. S. A. Amitsur gave a talk on radicals and the Gel'fand-Kirillov dimension. K. I. Beidar and N. I. Dubrovin presented cute examples: Dubrovin of simple radical chain rings, while Beidar, beside his example on the arbitrary finite-step termination of the lower radical construction, gave also examples solving four other open problems in radical theory, one of them by the above example of Dubrovin. In his survey lecture,

J. Golan focussed on the problem of choosing the most appropriate topology for the torsion-theoretic spectrum of a non-commutative ring to generalize the Zariski topology in the commutative case. J. Krempa's talk on radicals and derivations included his famous example of a radical in a class of Lie algebras for which the ADS property fails to hold although its semisimple class is hereditary. K. McCrimmon spoke on the invariance of Jordan radicals, and E. I. Zel'manov lectured on radicals and the structure theory of Jordan algebras. The town Eger with its castle and many historical buildings provided a pleasant environment for the conference. The surrounding vineyards are famous for good wine, especially the red wine "Bull's Blood", which also contributed to the success of the conference. The 753 page long proceedings *Radical Theory*, edited by L. Márki and R. Wiegandt, Colloquia Mathematica Societatis János Bolyai, vol. 38, North-Holland, 1985, consists of 30 papers, including also a research paper of the South-African S. Veldsman who was not permitted to be present.

3. **Radical Theory and Applications**, Krems, Austria, August 16 – 23, 1985.

Rainer Mlitz came up with the idea to establish a series, and continue the conferences on radical theory every three years. For the next one in '85 he chose the lovely historical town Krems on the river Danube. 51 participants from 23 countries representing the five continents emphasized the high interest in radical theory. After the cultural revolution scientific work was resumed in the P. R. China, and Liu Shao-Xue surveyed the recent research done on radicals in China. For the first time in this series of conferences, a strong group of 6 participants arrived from South Africa, but only one colleague could come from the Soviet Union. Due to the South African speakers and to K. Kaarli, radicals of near-rings made a strong appearance at this series of conferences. The program also featured a three-part presentation of a very general radical theory, by L. Márki, R. Mlitz, and R. Wiegandt, which contains as special cases the general radical theories of Ω-groups (hence also of rings), the connectedness-disconnectedness theories of topological spaces and graphs, and the greatest semilattice decomposition of semigroups. J. Okniński presented a survey of radicals of group and semigroup rings. There was a memorable excursion by boat through the Wachau, a picturesque part of the Danube, up to the

stately abbey in Melk, followed by a "Heuriger" (dinner with tasting local wine in a vineyard). The proceedings of the conference, *Radical Theory and Applications*, edited by R. Mlitz, Contributions to General Algebra 4, B. G. Teubner, Stuttgart and Hölder-Pichler-Tempsky, Wien, 1987, pp 208, contains 14 papers.

4. **Rings, Modules and Radicals**, Hobart, Australia, August 17 – 21, 1987.

Not waiting for another three years, Barry Gardner took the opportunity and organized a conference right after a conference on Abelian Groups, Perth, Australia (the distance was negligible, only some 2500 km). Most of the participants (who represented all the five continents) attended both conferences. After Keszthely, 1971, this was the second conference with that title. At the meeting, affirmative results were presented on fundamental properties of radicals in certain classes of non-associative rings (for properties which do not hold in general), based on results of K. I. Beidar, E. R. Puczyłowski, and S. Veldsman (although the first two of these authors could not be present). J. Golan discussed the frame of torsion theories, with a strong reference to works of H. Simmons. A. D. Sands gave a survey on dependence and independence among heredity and strongness properties of radicals of associative rings, presenting an almost complete picture of them. Beside the talks, participants enjoyed the special beauty of Australia. At a visit in a zoo, they got acquainted with Australian birds and mammals, including the local Tasmanian devil. The conference proceedings *Rings, Modules and Radicals*, edited by B. J. Gardner, Pitman Research Notes in Mathematics 204, Longman Scientific & Technical, 1989, pp 194, contains a total of 17 survey and research articles.

5. **International Conference on Radicals**, Sendai, Japan, July 24 – 30, 1988.

The abbreviation ICOR '88 was invented by the organizer Shoji Kyuno, and has been adopted also for subsequent conferences on radicals. Once again, South African citizens could not be present because of visa restrictions. With 40 participants from 12 countries and 20 lectures and discussions, the conference provided an advanced, comprehensive and relaxed forum in the field of radical theory. B. J. Gardner reported on considerable progress in the theory

of special radicals, introducing the notion of extraspecial radicals and raising several important problems. R. Mlitz discussed interpolation properties appearing in generalizations of Jacobson's density theorem, with connections to a universal algebraic notion of independence. In a joint paper with E. Roszkowska, E. R. Puczyłowski gave an account of results on atoms in various lattices of radicals, a question strongly linked with the investigation of important concrete radicals. Ladies took part on a short ikebana course and also on a brief (one-hour only) tea ceremony. The very generously organized conference culminated in a one-day excursion to Matsushima, cruising by a boat in the cliff forest on the sea, spending the night in a brand new Japanese styled luxorious hotel, and having an authentic Japanese dinner with geishas. In the proceedings, *Radical theory*, edited by S. Kyuno, Uchida Rokakuho, Tokyo, 1989, of 161 pages, 12 papers are published.

6. **ICOR '91**, Szekszárd, Hungary, July 9 – 13, 1991.

The end of the cold war provided a relaxed international political situation, 49 mathematicians from 20 countries representing all the five continents attended the conference. One of the most important results was presented by K. I. Beidar, characterizing special radicals in terms of the so-called intersection property. R. Göbel reported on the impact of set-theoretical methods in the radical theory of abelian groups. E. Jespers gave a survey on results about radicals, especially the Jacobson radical, of various kinds of graded rings. E. R. Puczyłowski discussed the state of a number of open problems on concrete radicals of associative rings. The success of the meeting could be well seen from the fact that in spite of the dog-days there were very few participants only who sometimes chose the near-by swimming pool instead of the hot lecture-room. The participants and the accompanying persons got acquainted with the historical town of Szekszárd and its surroundings. Beside Eger, also Szekszárd produces the famous red wine "Bull's Blood". This and other local wine sorts were tasted in a conference dinner. The proceedings *Theory of Radicals*, edited by L. Márki and R. Wiegandt, Colloquia Mathematica Societatis János Bolyai 61, North-Holland 1993, of 310 pages contains 19 papers, some of them are survey articles by invited speakers, the rest consists of research papers with full proofs.

7. **ICOR '94**, Shijiazhuang, P. R. China, August 20 – 26, 1994.

The double R means rings and radicals. The conference was attended by 58 mathematicians from 11 countries. The most important progress connected with radicals and reported at the conference was due to W. G. Leavitt and related to matric-extensible radicals, giving easily testable criteria especially for lower and upper radicals to have this property. There were excursions to places of interest in the region surrounding Shijiazhuang (about 300 km to the South-West from Beijing): Zheng Ding, Longzing Monastery (one afternoon) and Bao Du Mountains (one full day). At the conclusion of the conference there was an optional three-day tour of Beijing with visits to the Great Wall and the Ming Tombs. The proceedings *Rings and Radicals*, edited by B. J. Gardner, Liu Shaoxue and R. Wiegandt, Pitman Research Notes in Mathematics, 346, Longman 1996, pp 260, contains 21 articles (8 survey and 13 research papers) based on talks given at the conference.

8. **ICOR '97**, Port Elizabeth, South Africa, July 7 – 11, 1997.

Thirty-six mathematicians attended this conference with a very familiar atmosphere in the mild winter of Port Elizabeth. G. F. Birkenmeier gave an account of the results on essential covers of radical classes with a special emphasis on their eventual semisimplicity, posing also some problems. In the radical theory of near-rings, K. Kaarli emphasized the role of non-zero-symmetric objects. J. E. van den Berg's survey investigated how much information relating to the internal structure of a ring is carried by its torsion preradicals. R. Wiegandt presented a carefully compiled selection of associative rings and ring constructions distinguishing a large number of radical properties of rings which had evolved in investigations into the structure of rings, and finding the exact position of these radicals in the lattice of radicals. The conference saw a celebration of the 65$^{\text{th}}$ birthday of R. Wiegandt. There was a one-day excursion to Addo Elephant Park, followed by a city tour and a visit to a gallery and craft center where a folk dance program was shown. The journal *Quaestiones Mathematicae* issued vol. 22(3) (1999), 299–474, as the proceedings of ICOR '97, consisting of 13 articles by the participants.

9. **ICOR '00**, Innsbruck, Austria, July 30 – August 5, 2000.

The conference took place in the historical capital of Tyrol surrounded by beautiful mountains. 35 mathematicians attended the conference, among them the still active prominents H. J. Hoehnke, F. Kasch, and W. G. Leavitt. The scientific highlights of the conference were linked with Köthe's problem. Agata Smoktunowicz, as a keynote speaker, reported on her outstanding result: polynomial rings over nil rings need not be nil. She answered also Levitzki's problem: simple nil rings do exist, and so the nil and the antisimple radicals are not comparable. A good approximation of Köthe's problem was provided by the theorem of K. I. Beidar, Y. Fong, and E. R. Puczyłowski stating that polynomial rings over nil rings cannot be mapped homomorphically onto rings with nonzero idempotents. V. I. Arnautov presented a survey on the radical theory of topological rings. L. Márki explained how radicals of arbitrary rings (or, more generally, radicals in semi-abelian categories) can be obtained from a kind of generalized Galois connections. H. J. Weinert gave an overview on the general radical theory of semirings. As side programs there was a sight-seeing in Innsbruck, an ascent by cable car to the Hafelekar (2234 m), and an excursion to South Tyrol (Italy) through the Brenner pass, offering a visit to Brixen (Bressanone) followed by a Tyrolian dinner in Neustift. An unexpected difficulty arose during the drive on the way to Brixen: on the old road, right under the Europabrücke, some 50 m ahead of the conference bus, a truck loaded with paper burned down, blocking the traffic for some 3 hours.

10. **ICOR '03**, Chişinău, Moldova, August 11 – 16, 2003.

Chişinău has an international reputation in research in radical theory. The conference was dedicated to the memory of V. A. Andrunakievich (1917 – 1997), founder of the Moldavian algebra school and one of the most prominent researchers in radical theory. 55 participants from all the five continents presented and discussed the most recent developments in radical theory and related topics. M. Ferrero surveyed results on various notions of strong primeness, posing also some open problems. B. J. Gardner gave a survey on radical reflections, a way of transferring radicals from one context to another. E. R. Puczyłowski pointed out that Köthe's problem is equivalent to Andrunakievich's problem raised in 1969, whether an arbitrary ring modulo the sum of all its nil left ideals (which is an

ideal) cannot contain non-zero nil left ideals. S. Veldsman initiated the study of radicals of convolution rings, a construction which generalizes many familiar constructions (including polynomial rings, matrix rings, incidence algebras). R. Wisbauer explained how torsion theory of modules over a ring can be applied to understand the structure of comodules of a coalgebra. There was a full day excursion to the old fort Soroca, and one afternoon to the excavations of the town Orhei. At the end of the conference guests visited places where the famous Russian poet Pushkin spent his time in exile and got inspiration for his work. The *Buletinul Academiei de Ştiinţe a Republicii Moldova, Matematica,* 1(44) 2004 published 14 papers based on talks given at the conference.

11. **ICOR '06**, Kyiv, Ukraine, July 30 – August 5, 2006.

Information about the participants, the talks, and the side programs of this conference can be found in the rest of this issue.

After evoking the past, is there anything to say about the present or even the future of radical theory? We think the answer is yes, but to give it, we must take another look back into the past. Radicals of rings appeared first in G. Köthe's celebrated paper from 1930, then many concrete radicals came up in investigations into the structure of associative rings. In the early fifties, Amitsur and Kurosh independently defined the same general notion of radicals in order to have a framework for a common treatment of all these concrete radicals. Indeed, this approach has led to a general decomposition theorem for semisimple rings with respect to the so-called special radicals. In any case, there followed a boom of general radical theory from the early sixties, concentrating first on associative rings, extending stepwise to 'close-to-associative' and then general non-associative rings, then going over to further and further kinds of algebraic, topological, and other structures, and summing up all these in the language of categories. By the late eighties, a well-established theory was in place. Since then, progress of the general theory has slowed down, except some novel insight coming from category theory. Does this mean that radical theory is fading away?

We claim that this is not the case. Indeed, throughout the past fifty years, many important (sometimes very difficult) results on concrete radicals of rings have seen the light, and this line of development does not make the impression of getting exhausted. In spite of the spectacular

results of Smoktunowicz during the last decade, Köthe's famous problem from 1930 is still unsolved. This problem admits many seemingly different equivalent formulations. Generalizations and looking for analogies have led to a number of results or still open problems in different directions. Moreover, a good part of the recent papers on rings (mainly associative rings) concern concrete radicals or uses radicals. Sometimes authors are even not aware that they in fact study a kind of a new radical, and if they recognized that, they would immediately see a wider context of their studies. There are many interesting and hard open problems concerning radicals (both concrete and general ones) of rings, and people work on them, often without knowing that they develop the theory of radicals. For instance, far too little is known about the possibility of transferring properties between a ring R and the polynomial ring $R[x]$. Many questions of this kind are rooted in radical theory, and answering them may contribute also to further progress in the general theory. Also, the torsion theory initiated by S. E. Dickson in the sixties first in abelian groups and then in abelian categories, was in its original form a special case of Kurosh–Amitsur radical theory. Torsion theory and an outcome of this which also bears the name radicals, has developed in a different direction. This line of research is still active today, also on a categorical level, and its connection to Kurosh–Amitsur radical theory is still to be established.

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