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RESEARCH ARTICLE

On the structure of skew groupoid rings which are Azumaya

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Abstract. In this paper we present an intrinsic description of the structure of an Azumaya skew groupoid ring, having its center contained in the respective ground ring, in terms of suitable central Galois algebras and commutative Galois extensions.

1. Introduction

Groupoids are usually presented as small categories whose morphisms are invertible.

The notion of a groupoid action that we use in this paper arose from the notion of a partial groupoid action [3], which is a natural extension of the notion of a partial group action [5]. First, partial ordered groupoid actions on sets were introduced in the literature, as ordered premorphisms, by N. Gilbert [8]. After, partial ordered groupoid actions on rings were considered by D. Bagio and the authors [2] as a generalization of partial group actions, as introduced by M. Dokuchaev and R. Exel in [5]. And in [3] this notion was extended to the general context of groupoids.

Accordingly [3], an action of a groupoid G on a K-algebra R is a pair $\beta = (\{E_g\}_{g\in G}, \{\beta_g\}_{g\in G})$, where for each $g \in G$, $E_g = E_{r(g)}$ is an ideal of R and $\beta_g : E_{g^{-1}} \to E_g$ is an isomorphism of K-algebras satisfying some suitable conditions of compatibility (see the subsection 2.3).

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The notion of groupoid action given by Caenepeel and De Groot in [4] is equivalent to the above one in the case that the set G_0 of all identities of G is finite, each ideal E_e $(e \in G_0)$ is unital and $R = \bigoplus_{e\in G_0} E_e$, [7, Proposition 2.2]. This is the groupoid action we will deal with throughout all this paper.

Given such an action β of G on R, we can consider the skew groupoid ring $A = R \star_\beta G$, similarly to the construction given in [2], which is an associative and unital K-algebra.

Our aim is to present an intrinsic structural description of A in the case that G is finite, A is Azumaya and its center is contained in R (see Theorem 3.3).

This paper is organized as follows. In the next section we give preliminaries about groupoids, groupoid actions, skew groupoid rings, and separability, Hirata-separability, Galois, and Azumaya properties. In that section we will be concerned only with the results strictly necessary to construct the appropriate conditions to prove our main theorem, whose proof we set in the section 3.

Throughout, unless otherwise specified, rings and algebras are associative and unital.

2. Definitions and basic results

2.1. Groupoids

The axiomatic version of groupoid that we adopt in this paper was taken from [13]. A *groupoid* is a non-empty set G equipped with a partially defined binary operation, that we will denote by concatenation, for which the usual axioms of a group hold whenever they make sense, that is:

(i) For every $g, h, l \in G$, $g(hl)$ exists if and only if $(gh)l$ exists and in this case they are equal.

(ii) For every $g, h, l \in G$, $g(hl)$ exists if and only if gh and hl exist.

(iii) For each $g \in G$ there exist (unique) elements $d(g), r(g) \in G$ such that $gd(g)$ and $r(g)g$ exist and $gd(g) = g = r(g)g$.

(iv) For each $g \in G$ there exists an element $g^{-1} \in G$ such that $d(g) = g^{-1}g$ and $r(g) = gg^{-1}$.

We will denote by G^2 the subset of the pairs $(g, h) \in G \times G$ such that the element gh exists.

An element $e \in G$ is called an *identity* of G if $e = d(g) = r(g^{-1})$, for some $g \in G$. In this case e is called the *domain identity* of g and the *range identity* of g^{-1}. We will denote by G_0 the set of all identities of G and we will denote by G_e the set of all $g \in G$ such that $d(g) = r(g) = e$. Clearly, G_e is a group, called the *isotropy (or principal) group associated to e.*

The assertions listed in the following lemma are straightforward from the above definition. Such assertions will be freely used along this paper.

Lemma 2.1. *Let G be a groupoid. Then,*

(i) *for every $g \in G$, the element g^{-1} is unique satisfying $g^{-1}g = d(g)$ and $gg^{-1} = r(g)$,*

(ii) *for every $g \in G$, $d(g^{-1}) = r(g)$ and $r(g^{-1}) = d(g)$,*

(iii) *for every $g \in G$, $(g^{-1})^{-1} = g$,*

(iv) *for every $g, h \in G$, $(g, h) \in G^2$ if and only if $d(g) = r(h)$,*

(v) *for every $g, h \in G$, $(h^{-1}, g^{-1}) \in G^2$ if and only if $(g, h) \in G^2$ and, in this case, $(gh)^{-1} = h^{-1}g^{-1}$,*

(vi) *for every $(g, h) \in G^2$, $d(gh) = d(h)$ and $r(gh) = r(g)$,*

(vii) *for every $e \in G_0$, $d(e) = r(e) = e$ and $e^{-1} = e$,*

(viii) *for every $(g, h) \in G^2$, $gh \in G_0$ if and only if $g = h^{-1}$,*

(ix) *for every $g, h \in G$, there exists $l \in G$ such that $g = hl$ if and only if $r(g) = r(h)$,*

(x) *for every $g, h \in G$, there exists $l \in G$ such that $g = lh$ if and only if $d(g) = d(h)$.*

2.2. Groupoid actions and skew groupoid rings

Let G be a groupoid and R an algebra over a commutative ring K. Following [3], an action of G on R is a pair

$$\beta = (\{E_g\}_{g\in G}, \{\beta_g\}_{g\in G}),$$

where for each $g \in G$, $E_g = E_{r(g)}$ is an ideal of R, $\beta_g : E_{g^{-1}} \to E_g$ is an isomorphism of K-algebras (not necessarily unital), and the following conditions hold:

(i) β_e is the identity map I_{E_e} of E_e, for all $e \in G_0$,

(ii) $\beta_g\beta_h(r) = \beta_{gh}(r)$, for all $(g, h) \in G^2$ and $r \in E_{h^{-1}} = E_{(gh)^{-1}}$.

In particular, β induces an action of the group G_e on E_e, for every $e \in G_0$.

Accordingly [2, Section 3], the skew groupoid ring $R\star_\beta G$ corresponding to β is defined as the direct sum

$$R \star_\beta G = \bigoplus_{g\in G} E_g\delta_g$$

in which the δ_g's are symbols, with the usual addition, and multiplication determined by the rule

$$(x\delta_g)(y\delta_h) = \begin{cases} x\beta_g(y)\delta_{gh} & \text{if } (g,h) \in G^2 \\ 0 & \text{otherwise,} \end{cases}$$

for all $g, h \in G$, $x \in E_g$ and $y \in E_h$.

This multiplication is well defined. Indeed, if $(g,h) \in G^2$ then $d(g) = r(h)$ (see Lemma 2.1(iv)). So, $E_{g^{-1}} = E_{r(g^{-1})} = E_{d(g)} = E_{r(h)} = E_h$, $\beta_g(y)$ makes sense, and $x\beta_g(y) \in E_g = E_{r(g)} \overset{(\star)}{=} E_{r(gh)} = E_{gh}$, where the equality $(\star)$ is ensured by Lemma 2.1(vi).

By a routine calculation one easily sees that $A = R\star_\beta G$ is associative, and by [2, Proposition 3.3] it is unital if G_0 is finite and E_e is unital, for all $e \in G_0$. In this case, the identity element of A is $1_A = \sum_{e\in G_0} 1_e\delta_e$, where 1_e denotes the identity element of E_e, for all $e \in G_0$.

2.3. Invariants, Galois extensions and Galois algebras

Let R, G and β as in the preceding subsection.

For our purposes we will assume hereafter that G is finite, $R = \bigoplus_{e\in G_0} E_e$ and every ideal E_g is unital, with its identity element denoted by 1_g (in particular, each 1_g is a central idempotent of R).

We will denote by R^β the subalgebra of the *invariant elements* of R under the action β, that is,

$$R^\beta = \{r \in R \mid \beta_g(r1_{g^{-1}}) = r1_g,\ \textit{for all}\ g \in G\}.$$

For any non-empty subset X of R, any subring Y of R containing X and any (Y,Y)-bimodule V we will denote by $C_V(X)$ the centralizer of X in V, that is, the set of all $v \in V$ such that $xv = vx$ for all $x \in X$. If, in particular, $X = Y = V = R$, then $C_V(X)$ is the center of R and we will denote it simply $C(R)$.

A non-empty subset X of R is called *β-invariant* if $\beta_g(E_{g^{-1}} \cap X) = E_g \cap X$, for every $g \in G$.

In particular, $C(R)$ and $C_R(R^\beta)$ are β-invariant. Indeed, if $r \in E_{g^{-1}} \cap C(R)$, then

$$\beta_g(r)x = \beta_g(r)1_g x = \beta_g(r\beta_{g^{-1}}(x1_g)) = \\ = \beta_g(\beta_{g^{-1}}(x1_g)r) = x1_g\beta_g(r) = x\beta_g(r),$$

for all $x \in R$, and so $\beta_g(r) \in E_g \cap C(R)$, for all $g \in G$. Hence, $\beta_g(E_{g^{-1}} \cap C(R)) = E_g \cap C(R)$. By similar arguments one also gets $\beta_g(E_{g^{-1}} \cap C_R(R^\beta)) = E_g \cap C_R(R^\beta)$.

Moreover, since $1_g \in C(R) \subseteq C_R(R^\beta)$, β induces by restriction an action on $X' = C_R(X)$, with $X = R$ or R^β, given by the pair $\beta|_{X'} = (\{E'_g = X'1_g\}_{g\in G}, \{\beta_g|_{E'_{g^{-1}}}\}_{g\in G})$. Notice that $E'_g = E'_{r(g)}$, for all $g \in G$, and $X' = \bigoplus_{e\in G_0} E'_e$.

We say that R is a *β-Galois extension* (resp., *G-Galois extension*, if G is a group) of R^β (resp., R^G) if there exist elements $x_i, y_i \in R$, $1 \leq i \leq m$, such that

$$\sum_{1\leq i\leq m} x_i\beta_g(y_i1_{g^{-1}}) = \delta_{e,g}1_e,$$

for all $e \in G_0$ and $g \in G$. In particular, in this case, E_e is a G_e-Galois extension of $E_e^{G_e}$ for every $e \in G_0$. The set $\{x_i, y_i \mid 1 \leq i \leq n\}$ is called a *Galois coordinate system* of R over R^β (resp., R^G).

We also say that R is a *β-Galois algebra* (resp., *β-central Galois algebra*) if R is a β-Galois extension of R^β and $R^\beta \subseteq C(R)$ (resp., $R^\beta = C(R)$). In the particular case that G is a group, we replace "β-" by "G-".

Remark 2.2. *Observe that $R^\beta \subseteq \bigoplus E_e^{G_e}$. Indeed, any element $x \in R$ is of the form $x = \sum_{e\in G_0} x_e$, with $x_e \in E_e$, and $x \in R^\beta$ if and only if $\beta_g(x1_{g^{-1}}) = x1_g$ if and only if $\beta_g(x_{d(g)}) = x_{r(g)}$, for all $g \in G$. In particular, $\beta_g(x_e) = x_e$, for all $e \in G_0$ and $g \in G_e$. Hence, each x_e belongs to $E_e^{G_e}$ and $x \in \bigoplus_{e\in G_0} E_e^{G_e}$.*

In general the inclusion $R^\beta \subseteq \bigoplus E_e^{G_e}$ is strict, as it is shown in the following example: take $G = \{g, g^{-1}, r(g), d(g)\}$, $R = Se_1 \oplus Se_2 \oplus Se_3 \oplus Se_4$, where S is a ring and e_1, e_2, e_3 and e_4 are pairwise orthogonal idempotents with sum 1_R, $E_g = E_{r(g)} = Se_1 \oplus Se_2$, $E_{g^{-1}} = E_{d(g)} = Se_3 \oplus Se_4$, $\beta_{d(g)} = I_{E_{d(g)}}$, $\beta_{r(g)} = I_{E_{r(g)}}$, $\beta_g(xe_3 + ye_4) = xe_1 + ye_2$ and $\beta_{g^{-1}}(xe_1 + ye_2) = xe_3 + ye_4$, para todo $x, y \in S$. It is immediate to check that $\beta = (\{E_l\}_{l\in G}, \{\beta_l\}_{l\in G})$ is an action of G on R, $R^\beta = S(e_1 + e_3) \oplus S(e_2 + e_4)$, $E_{d(g)}^{G_{d(g)}} = Se_3 \oplus Se_4$ and $E_{r(g)}^{G_{r(g)}} = Se_1 \oplus Se_2$. In this example we also notice that $R^\beta 1_{d(g)} = E_{d(g)}^{G_{d(g)}}$ and $R^\beta 1_{r(g)} = E_{r(g)}^{G_{r(g)}}$.

But, neither this fact occurs in general, as we see in the next example: take R *as above and* $G = \{g_1, g_2, g\}$*, with* $G_0 = \{g_1, g_2\}$*,* $g^{-1} = g$ *and* $gg = g_2$*. Setting* $E_{g_1} = Se_1 \oplus Se_4$*,* $E_{g_2} = Se_2 \oplus Se_3 = E_{d(g)} = E_{r(g)}$ *and* $\beta_g(ae_2 + be_3) = be_2 + ae_3$*, one easily sees that* $R^\beta = Se_1 \oplus S(e_2 + e_3) \oplus Se_4$ *and* $R^\beta 1_{g_2} = S(e_2 + e_3) \subset E_{g_2} = E_{g_2}^{G_{g_2}}$*.*

2.4. Azumaya, Hirata-separable and separable algebras

Let S be a subalgebra of R. We say that R is *separable over* S [11] if R is a direct summand of $R \otimes_S R$ as an R-bimodule. In this case, if $S \subseteq C(R)$ (resp., $S = C(R)$) we also say that R is a *separable* S*-algebra* (resp., *Azumaya over* S or Azumaya algebra or simply *Azumaya*). We say that R is *Hirata-separable* over S [9] if $R \otimes_S R$ is isomorphic, as an R-bimodule, to a direct summand of a finite direct sum of copies of R. It is well known that every Azumaya algebra is Hirata-separable over its center [14] as well as every Hirata-separable extension is separable [10].

3. The main results

Also in this section, R, G and β are as in the later one.

We start with the following theorem, which is a generalization of [1, Theorems 1 and 2] to the setting of groupoid actions, and shows a nice and closed relation among the notions of Azumaya algebra, Galois extension and Hirata-separability. The arguments used in the proof of the mentioned theorems in [1] are practically the same we will use here.

Theorem 3.1. *The following statements are equivalent:*

(i) A *is Azumaya and* $C(A) \subseteq R$*.*

(ii) A *is Hirata-separable over* R *and* R *is separable over* $C(R)^\beta$*.*

(iii) R *is a* β*-Galois extension of* R^β *and* R^β *is Azumaya over* $C(A)$*.*

(iv) $C_R(R^\beta)$ *is a* β*-Galois extension of* $C(A)$ *and* R^β *is Azumaya over* $C(A)$*.*

Moreover, in this case, $C(A) = C(R)^\beta = C(R^\beta)$*.*

For the proof of this theorem we need the following lema.

Lemma 3.2. $C(A) \subseteq R$ *if and only if* $C(A) = C(R)^\beta$*.*

Proof. If $C(A) \subseteq R$, then $C(A) \subseteq C(R)$ and $(\sum_{e\in G_0} x1_e\delta_e)1_g\delta_g = 1_g\delta_g(\sum_{e\in G_0} x1_e\delta_e)$, for every $x \in C(A)$ and $g \in G$. Since $(\sum_{e\in G_0} x1_e\delta_e)1_g\delta_g = (x1_{r(g)}\delta_{r(g)})(1_g\delta_g) = x1_g\delta_g$ and $1_g\delta_g(\sum_{e\in G_0} x1_e\delta_e) = (1_g\delta_g)(x1_{d(g)}\delta_{d(g)}) = \beta_g(x1_{g^{-1}})\delta_g$, it follows that $\beta_g(x1_{g^{-1}}) = x1_g$, for every $g \in G$. Therefore $x \in C(R)^\beta$. The converse is straightforward. □

Proof of Theorem 3.1.

(i)⇒(ii) By Lemma 3.2 we have $C(A) = C(R)^\beta$. It follows from [12, Theorem 1] that A is Hirata-separable over R, since A is a finitely generated projective R-module. In addition, R is a direct summand of A as an R-bimodule. Hence, it follows from [14, Proposition 1.3] that $C_A(R)$ is separable over $C(A)$ and $C_A(C_A(R)) = R$. Consequently, R is separable over $C(A)$ by [6, Theorem III.4.3].

(ii)⇒(iii) We will proceed by steps.

step 1: A is Azumaya and $C(A) \subseteq R$.

Since A is Hirata-separable over R and R is a direct summand of A as an R-bimodule, it follows that $C_A(R)$ is separable over $C(A)$ and $C_A(C_A(R)) = R$, by [14, Proposition 1.3]. Furthermore, $C(A) = C(R)^\beta$ by Lemma 3.2. On the other hand, by the assumptions and [10, Theorem 2.2] we have that A is separable over R, so over $C(R)^\beta$ too.

step 2: $E = End_{C(R)^\beta}(R)$ is Azumaya over $C(R)^\beta$.

Indeed, A is Azumaya over $C(R)^\beta$ (by step 1) and so a projective finitely generated $C(R)^\beta$-module, by [6, Theorem III.3.4]. Hence, R also is a projective finitely generated $C(R)^\beta$-module, as a direct summand of A. Now, the assertion follows by [6, Theorem III.4.1].

step 3: $C_E(A) = End_A(R) \simeq (R^\beta)^{op}$ as $C(R)^\beta$-algebras.

First of all, we observe that R is a left A-module via the action given by $r_g\delta_g \cdot r = r_g\beta_g(r1_{g^{-1}})$, for all $g \in G$, $r_g \in E_g$ and $r \in R$. Consequently, E is an A-bimodule in a natural way, that is, $af(r) = a \cdot f(r)$ and $fa(r) = f(a \cdot r)$, for all $a \in A$, $f \in E$ and $r \in R$. Now, the equality of the assertion is immediate. It is also immediate that R is a $((R^\beta)^{op}, A)$-bimodule, with $(R^\beta)^{op}$ acting on R by the left via the right multiplication, that is, $x \cdot r = rx$, for all $r \in R$ and $x \in (R^\beta)^{op}$.

The claimed isomorphism of the assertion is given by the map $\theta : (R^\beta)^{op} \to End_A(R)$ defined by $\theta(x)(r) = x \cdot r$, for all $x \in (R^\beta)^{op}$ and $r \in R$, whose inverse is given by $f \mapsto f(1_R)$, for all $f \in End_A(R)$.

Now, we are in condition to prove the statement (iii).

It follows from the assumptions, step 3 and [12, Lemma 1] that R is a generator in the category of the left A-modules. Hence, R is a β-Galois extension of R^β, as well as $A \simeq End(R)_{R^\beta}$ as $C(R)^\beta$-algebras, by [3, Theorem 3]. Then, A can be seen as a $C(R)^\beta$-subalgebra of E and, by steps 1 and 2 and [6, Theorem III.4.3], we have that $C_E(A)$ is Azumaya over $C(R)^\beta = C(A)$. So, $(R^\beta)^{op}$ (and, therefore, also R^β) is Azumaya over $C(R)^\beta$ by step 3, which also ensure that $C(R^\beta) = C(R)^\beta$.

(iii)⇒(i) By [3, Theorem 3] we have that R is a projective finitely generated right R^β-module, $A \simeq End(R)_{R^\beta}$ as $C(R)^\beta$-algebras. By [6, Theorem III.3.4] we have that R^β is a projective finitely generated $C(A)(= C(R^\beta))$-module. By Lemma 3.2, $C(A) = C(R)^\beta$. Therefore, R is a projective finitely generated $C(R)^\beta$-module. Moreover, R is a generator in the category of the $C(R)^\beta$-modules. Hence, $E = End_{C(R)^\beta}(R)$ is Azumaya over $C(R)^\beta$ by [6, Theorem III.4.1], and $C_E(R^\beta)$ is Azumaya over $C(R)^\beta$ by [6, Theorem III.4.3].

Finally, $End(R)_{R^\beta} = C_E(R^\beta)$. Indeed, it is enough to notice that $End(R)_{R^\beta}$ is a subalgebra of E and a R^β-bimodule via the actions $xfy(r) = f(rx)y$, for all $f \in End(R)_{R^\beta}$, $x, y \in R^\beta$ and $r \in R$.

(iii)⇒(iv) By assumption R^β is Azumaya over $C(A)$ and it follows from the equivalence (iii)⇔(i) that A is Azumaya. By [6, Theorem III.4.3], $C_A(R^\beta)$ is Azumaya over $C(A)$.

Therefore, $A' = C_R(R^\beta) \star_\beta G$ is Azumaya and $C(A') = C(A) = C(R^\beta) \subseteq C_R(R^\beta)$. Then, it follows again from the equivalence (i)⇔(iii) that $C_R(R^\beta)$ is a β-Galois extension of $C_R(R^\beta)^\beta = C(R^\beta) = C(A)$.

(iv)⇒(iii) It is enough to observe that, by assumption, there exist elements $x_i, y_i \in C_R(R^\beta) \subseteq R$, $1 \leq i \leq m$, such that $\sum_{1\leq i\leq m} x_i\beta_g(y_i 1_{g^{-1}}) = \delta_{e,g}1_e$, for all $g \in G$ and $e \in G_0$. □

Theorem 3.3. *Assume that A is Azumaya, $C(A) \subseteq R$, and $R' = C_R(R^\beta)$ satisfies $R'^\beta 1_e = (E'_e)^{G_e}$, where $E'_e = R'1_e$, for all $e \in G_0$. Then,*

(i) *$A \simeq R^\beta \otimes_{C(A)} (R' \star_\beta G)$ as $C(A)$-algebras.*

(ii) *$R' \star_\beta G \simeq End(R')_{R'^\beta}$ as $C(A)$-algebras.*

(iii) *$R' = \bigoplus_{e\in G_0} E'_e$.*

(iv) *for each $e \in G_0$ there exist pairwise orthogonal idempotents $v_{e,i} \in C(E'_e)$, $1 \leq i \leq m_e$, and subgroups H_i of G_e such that*

– *each $E'_e v_{e,i}$ is an H_i-central Galois algebra,*

– $E'_e = \bigoplus_{1\leq i\leq m_e} E'_e v_{e,i}$ *or* $E'_e = (\bigoplus_{1\leq i\leq m_e} E'_e v_{e,i}) \bigoplus C(E'_e)v_e$, *with* $v_e = 1_e - \sum_{1\leq i\leq m_e} v_{e,i}$,
– $G_e|_{C(E'_e)v_e} \simeq G_e$ *and*
– $C(E'_e)v_e = E'_e v_e$ *is a commutative Galois extension of* $(E'_e v_e)^{G_e}$.

Proof. **(i)** It follows from Theorem 3.1 and [6, Theorem III.4.3] that $A \simeq R^\beta \otimes_{C(A)} C_R(R^\beta)$ as $C(A)$-algebras. It is routine to check that $C_A(R^\beta) = R' \star_\beta G$.

(ii) It follows from Theorem 3.1(vi) that R' is a β-Galois extension of $C(A) = C(R^\beta) = R'^\beta$. So, $R' \star_\beta G \simeq End(R')_{R'^\beta}$ as $C(A)$-algebras.

(iii) It is immediate, since $R = \bigoplus_{e\in G_0} E_e$.

(iv) It follows from Theorem 3.1 and [6, Theorem III.4.3] that $R' \star_\beta G = C_A(R^\beta)$ is Azumaya over $C(A)$, which implies that $C(R' \star_\beta G) = C(A) = C(R^\beta) = R'^\beta$ and, consequently, $R'^\beta \subseteq C(R')$. Thus, $(E'_e)^{G_e} = R'^\beta 1_e \subseteq C(R')1_e = C(R'1_e) = C(E'_e)$, that is, E'_e is a G_e-Galois algebra for all $e \in G_0$. The result follows now from [15, Theorem 3.8]. □

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