Algebra and Discrete Mathematics
Volume 9 **(2010)**. Number 2. pp. 106 – 112

RESEARCH ARTICLE

Thin systems of generators of groups

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Communicated by V. I. Sushchansky

Abstract. A subset T of a group G with the identity e is called k-thin ($k \in \mathbb{N}$) if $|A \cap gA| \leqslant k$, $|A \cap Ag| \leqslant k$ for every $g \in G$, $g \neq e$. We show that every infinite group G can be generated by some 2-thin subset. Moreover, if G is either Abelian or a torsion group without elements of order 2, then there exists a 1-thin system of generators of G. For every infinite group G, there exist a 2-thin subset X such that $G = XX^{-1} \cup X^{-1}X$, and a 4-thin subset Y such that $G = YY^{-1}$.

For a group G we denote by $\mathcal{F}_G$ the family of all finite subsets of G. A subset A of an infinite group G with the identity e is said to be

- *left (right) large* if there exists $F \in \mathcal{F}_G$ such that $G = FA$ ($G = AF$);
- *large* if A is left and right large;
- *left (right) small* if $G \setminus FA$ ($G \setminus AF$) is left (right) large for every $F \in \mathcal{F}_G$;
- *small* if A is left and right small;
- *left (right) P-small* if there exists an injective sequence $(g_n)_{n\in\omega}$ in G such that the subsets $\{g_nA : n \in \omega\}$ ($\{Ag_n : n \in \omega\}$) are pairwise disjoint;
- *P-small* if A is left and right P-small;
- *left (right) k-thin* for $k \in \mathbb{N}$ if $|gA \cap A| \leqslant k$ ($|Ag \cap A| \leqslant k$) for every $g \in G$. $g \neq e$;
- *k-thin*, if A is left and right k-thin.

2000 Mathematics Subject Classification: *20F05, 20F99.*
Key words and phrases: *small, P-small, k-thin subsets of groups.*

For the relationships between these types of subsets see [3]. In particular, every k-thin subset is small, but a small subset could be much more big than every k-thin subsets. For example, every k-thin subset T is a universal zero, i.e. $\mu(T) = 0$ for every Banach measure μ on G. On the other hand, for every countable amenable group G and every $\varepsilon > 0$, there exist a small subset S and Banach measure μ on G such that $\mu(S) > 1 - \varepsilon$. We note also that a subset A is left k-thin if and only if A^{-1} is right k-thin.

Answering a question from [4], I. V. Protasov [5] (see also [6, Theorem 13.1]) proved that every infinite group G can be generated by some small subset. Moreover, there exists a small and P-small generating subset of G [2].

In this paper we show (Theorem 1) that every infinite group G can be generated by some 2-thin subset. Moreover, if G is either Abelian or torsion group with no elements of order 2, then G can be generated by some 1-thin subset. By Theorem 2, for every infinite group G, there exists a 2-thin subset X such that $G = XX^{-1} \cup X^{-1}X$. Since every k-thin subset is small, this is an answer to the Question 13.2 from [6]. We show also that, in every infinite group G, there is a 4-thin subset X such that $G = XX^{-1}$.

Given a subset X of a group G, we denote by $\langle X \rangle$ the subgroup of G generated by X.

Theorem 1. *Every infinite group G has a 2-thin system of generators. Moreover, if G has no elements of order 2 and G is either Abelian or a torsion group, then there exists a 1-thin system of generators of G.*

Proof. Let $|G| = \kappa$. We construct inductively an increasing system $\{G_\alpha : \alpha < \kappa\}$ of subgroups of G and a subset $X = \{x_\alpha : \alpha < \kappa\}$ such that

(i) $G_0 = \langle e \rangle$, $G = \bigcup_{\alpha<\kappa} G_\alpha$;

(ii) $G_\alpha = \bigcup_{\alpha<\beta} G_\beta$ for every limit ordinal $\beta < \kappa$;

(iii) $G_{\alpha+1} = \langle G_\alpha, x_\alpha \rangle$ for every $\alpha < \kappa$.

Clearly, $G = \langle X \rangle$. We suppose that X is not left 2-thin and choose $g \in G$, $g \neq e$, distinct ordinals $\alpha_1, \alpha_2, \alpha_3$ such that $gx_{\alpha_1}, gx_{\alpha_2}, gx_{\alpha_3} \in X$. Let $gx_{\alpha_1} = x_{\beta_1}$, $gx_{\alpha_2} = x_{\beta_2}$, $gx_{\alpha_3} = x_{\beta_3}$. By the pigeonhole principle, there exist distinct $k, l \in \{1, 2, 3\}$ such that either $\alpha_k < \beta_k$, $\alpha_l < \beta_l$ or $\alpha_k > \beta_k$, $\alpha_l > \beta_l$. Let $\alpha_k < \beta_k$, $\alpha_l < \beta_l$. Then $x_{\beta_k} x_{\alpha_k}^{-1} \in G_{\beta_k+1} \setminus G_{\beta_k}$, $x_{\beta_l} x_{\alpha_l}^{-1} \in G_{\beta_l+1} \setminus G_{\beta_l}$ and $g = x_{\beta_k} x_{\alpha_\kappa}^{-1} = x_{\beta_l} x_{\alpha_l}^{-1}$, which is impossible because $(G_{\beta_k+1} \setminus G_{\beta_k}) \cap (G_{\beta_l+1} \setminus G_{\beta_l}) = \varnothing$. Hence, X is left 2-thin. The same arguments show that X is right 2-thin.

To prove the second statement, we assume that the constructed above subset X is not left 1-thin. Then there exist distinct $\alpha, \beta < \kappa$ and $g \neq e$ such that $gx_\alpha, gx_\beta \in X$. Let $gx_\alpha = x_{\alpha'}$, $gx_\beta = x_{\beta'}$. We choose the minimal $\lambda < \kappa$ such that $\alpha, \beta, \alpha', \beta' < \lambda$. Clearly, $\lambda = \gamma + 1$ for some $\gamma < \kappa$. Replacing g by g^{-1}, we may suppose that $\alpha = \gamma$ so $x_\alpha \in G_{\gamma+1} \setminus G_\gamma$. Since $x_{\alpha'} = gx_\alpha$ and $\alpha' < \alpha$ then $g \in G_{\alpha+1} \setminus G_\alpha$. It follows that $gx_\beta = x_\alpha$ and

(*) $g^2 x_\beta = x_{\alpha'}$;

(**) $x_\alpha^2 = x_{\alpha'} x_\beta$ if G is Abelian.

Let G be a torsion group with no elements or order 2. Then (*) is impossible because $g \in G_{\alpha+1} \setminus G_\alpha$ and $g^2 \in G_\alpha$. It follows that X is left 1-thin. The same arguments show that X is right 1-thin.

Let G be an Abelian group with no elements of order 2. We choose a system $\{G_\alpha : \alpha < \kappa\}$ of subgroups of G satisfying (i), (ii), (iii) and

(iv) $G_{\alpha+1}/G_\alpha \simeq \mathbb{Z}$ or $G_{\alpha+1}/G_\alpha \simeq \mathbb{Z}_p$ for some prime number p.

We construct $X = \{x_\alpha : \alpha < \kappa\}$ inductively by the following rule. If $G_{\alpha+1}/G_\alpha$ is not isomorphic to $\mathbb{Z}_2$, we choose an arbitrary element $x_\alpha \in G_{\alpha+1} \setminus G_\alpha$. Let $G_{\alpha+1}/G_\alpha \simeq \mathbb{Z}_2$ and $G_{\alpha+1} = \langle G_\alpha, y_\alpha \rangle$. If $y_\alpha^2 \neq x_{\alpha'} x_\beta$ for all distinct $\alpha', \beta < \alpha$, we put $x_\alpha = y_\alpha$. If $y_\alpha^2 = x_{\alpha'} x_\beta$ for some distinct $\alpha', \beta < \alpha$, $\beta < \alpha'$, we put $x_\alpha = y_\alpha x_\beta^{-1}$. Then $x_\alpha^2 = x_{\alpha'} x_\beta^{-1}$. If $x_\alpha^2 = x_{\alpha''} x_{\beta'}$ for some distinct $\alpha'', \beta' < \alpha$, $\beta' < \alpha''$ then $x_{\alpha'} x_\beta^{-1} = x_{\alpha''} x_{\beta'}$. Since $\beta < \alpha'$ and $\beta' < \alpha''$, we have $\alpha' = \alpha''$. Hence, $x_\beta^{-1} = x_{\beta'}$, but it is impossible, so $x_\alpha^2 \neq x_{\alpha''} x_{\beta'}$ for all distinct $\alpha'', \beta' < \alpha$. If X is not 1-thin, by (**), we get a contradiction with construction of X. □

Question 1. Let G be an infinite group with no elements of order 2. Does there exist a 1-thin system of generators of G?

Theorem 2. *For every infinite group G, there exists a 2-thin subset X such that $G = XX^{-1} \cup X^{-1}X$.*

Proof. Let $|G| = \kappa$, $\{g_\alpha : \alpha < \kappa\}$ be a numeration of G. We construct inductively a family $\{X_\alpha : \alpha < \kappa\}$ of 2-thin subsets of G of the form $X_\alpha = \{x_\beta, y_\beta x_\beta : \beta < \alpha\}$ so that $\{g_\beta : \beta < \alpha\} \subseteq X_\alpha X_\alpha^{-1}$ and put $X = \bigcup_{\alpha<\kappa} X_\alpha$.

We put $X_0 = \{e, g_0\}$ and assume that we have chosen the 2-thin subsets X_α for all $\alpha < \gamma$. Let $\gamma = \beta + 1$. We find the first element g in the numeration $\{g_\alpha : \alpha < \kappa\}$ such that $g \notin X_\beta X_\beta^{-1} \cup X_\beta^{-1} X_\beta$ and put $y_\beta = g$. To choose x_β, we use the following observation.

Let A be a subset of G, $g \in G$. If $|A| < \kappa$ and $g \notin A$ then $|\{x \in G : x^{-1}gx \notin A\}| = \kappa$. Indeed, $|\{x^{-1}gx : x \in G\}| = |G : Z_g|$, where $Z_g = \{x \in G : x^{-1}gx = g\}$, and either $|Z_g| = \kappa$ or $|G : Z_g| = \kappa$.

We choose x_β to satisfy the following conditions

(i) $x_\beta^{-1} y_\beta x_\beta \notin X_\beta^{-1} X_\beta$;

(ii) $\{x_\beta, y_\beta x_\beta\} \cap X_\beta X_\beta^{-1} X_\beta = \varnothing$;

(iii) $\{y_\beta, y_\beta^{-1}\}\{x_\beta, y_\beta x_\beta\} \cap X_\beta = \varnothing$.

Suppose that $X_{\beta+1} = X_\beta \cup \{x_\beta, y_\beta x_\beta\}$ is not left 2-thin and choose $g \in G$, $g \neq e$ and distinct $a, b, c \in X_{\beta+1}$ such that $ga, gb, gc \in X_{\beta+1}$. If $g \in X_\beta X_\beta^{-1}$ then, by (ii) and the choice of y_β, $\{a, b, c\} \subseteq X_\beta$ and $\{ga, gb, gc\} \subseteq X_\beta$ which is impossible because X_β is left 2-thin. Let $g \notin X_\beta X_\beta^{-1}$. Replacing if necessary a, b, c to ga, gb, gc and g to g^{-1}, we may suppose that $a = x_\beta$, $b = y_\beta x_\beta$, $c \in X_\beta$. If $ga \in X_\beta$ and $gb \in X_\beta$ then $X_\beta x_\beta^{-1} \cap X_\beta x_\beta^{-1} y_\beta^{-1} \neq \varnothing$ so we get a contradiction with (i). Thus, $g \in \{y_\beta, y_\beta^{-1}\}$ and $gc \in \{x_\beta, y_\beta x_\beta\}$. Hence, $\{y_\beta, y_\beta^{-1}\} \cap \{x_\beta, y_\beta x_\beta\} X_\beta^{-1} \neq \varnothing$ and we get a contradiction with (iii).

Suppose that X_β is not right 2-thin and choose $g \in G$, $g \neq e$ and distinct $a, b, c \in X_{\beta+1}$ such that $ag, bg, cg \in X_{\beta+1}$. Let $g \in X_\beta^{-1} X_\beta$. If either $a = x_\beta$ or $a = y_\beta x_\beta$ then, by (ii), either $g = x_\beta^{-1} y_\beta x_\beta$ or $g = x_\beta^{-1} y_\beta^{-1} x_\beta$, and in both cases we get a contradiction with (i). Hence, $a, b, c \in X_\beta$ and $ag, bg, cg \in X_\beta$ so X_β is not right 2-thin. Let $g \notin X_\beta^{-1} X_\beta$. Replacing if necessary a, b, c to ag, bc, cg and g to g^{-1}, we may suppose that $a = x_\beta$, $b = y_\beta x_\beta$, $c \in X_\beta$. If $ag \in X_\beta$ and $bg \in X_\beta$ then $y_\beta \in X_\beta X_\beta^{-1}$ contradicting the choice of y_β. Thus, we have

$$\{x_\beta, y_\beta x_\beta\} g \cap \{x_\beta, y_\beta x_\beta\} \neq \varnothing,$$

$$X_\beta g \cap \{x_\beta, y_\beta x_\beta\} \neq \varnothing.$$

It follows that

$$\{x_\beta, y_\beta x_\beta\}^{-1} \{x_\beta, y_\beta x_\beta\} \cap X_\beta^{-1} \{x_\beta, y_\beta x_\beta\} \neq \varnothing,$$

so $\{y_\beta, y_\beta^{-1}, e\}\{x_\beta, y_\beta x_\beta\} \cap X_\beta \neq \varnothing$ and we get a contradiction with (i) and (ii).

□

Corollary 1. *For every infinite Abelian group G, there exists a 2-thin subset X such that $G = XX^{-1}$.*

Remark 1. Let a group G be defined to have a *small square roots* if for any subset $A \subseteq G$ with $|A| < |G|$ the set $\sqrt{A} = \{x \in G : x^2 \in A\}$ has cardinality $|\sqrt{A}| < |G|$. Taras Banakh proved that if an infinite group G with identity e has small square roots, then it contains a 1-thin subset X such that $G = \sqrt{\{e\}} \cup XX^{-1} \cup X^{-1}X$. By this theorem, for every Abelian group G with no elements of order 2 there exists a 1-thin subset X such that $G = XX^{-1}$.

By the Chou's lemma [1], for every infinite group G there exists a 4-thin subset X such that $|X| = |G|$.

Corollary 2. *For every infinite group G, there exists a 2-thin subset X such that $|X| = |G|$.*

Theorem 3. *For every infinite group G, there exists a 4-thin subset X such that $G = XX^{-1}$.*

Proof. Let $|G| = \kappa$, $\{g_\alpha : \alpha < \kappa\}$ be a numeration of G. We construct inductively a family $\{X_\alpha : \alpha < \kappa\}$ of 4-thin subsets of G of the form $X_\alpha = \{x_\beta, y_\beta x_\beta : \beta < \alpha\}$. Also we demand the fulfilment of the condition $|X_\alpha \cap X_\alpha g| \leqslant 2$ for all $g \notin X_\alpha X_\alpha^{-1}$. Observe that $\{y_\beta : \beta < \alpha\} \subseteq X_\alpha X_\alpha^{-1}$ and put $X = \bigcup_{\alpha<\kappa} X_\alpha$.

We put $X_0 = \{e, g_0\}$ and assume that we have chosen subsets X_α for all $\alpha < \gamma$ such that

(1) $|X_\alpha \cap gX_\alpha| \leqslant 4$ for all $g \in G \setminus \{e\}$;

(2) $|X_\alpha \cap X_\alpha g| \leqslant 2$ for $g \notin X_\alpha X_\alpha^{-1} \cup \{e\}$;

(3) $|X_\alpha \cap X_\alpha g| \leqslant 4$ for $g \in X_\alpha X_\alpha^{-1} \setminus \{e\}$.

If γ is a limit ordinal, we put $X_\gamma = \bigcup_{\alpha<\gamma} X_\alpha$. Let $\gamma = \beta + 1$. We find the first element g in the numeration $\{g_\alpha : \alpha < \kappa\}$ such that $g \notin X_\beta X_\beta^{-1}$ and put $y_\beta = g$. Then we choose x_β to satisfy the following conditions

(i) $\{x_\beta, y_\beta x_\beta\} \cap X_\beta X_\beta^{-1} X_\beta = \varnothing$;

(ii) $\{e, y_\beta, y_\beta^{-1}\}\{x_\beta, y_\beta x_\beta\}\{e, y_\beta, y_\beta^{-1}\} \cap X_\beta = \varnothing$;

(iii) $x_\beta^{-1} y_\beta x_\beta \notin (X_\beta^{-1} X_\beta \cup X_\beta X_\beta^{-1}) \setminus \{y_\beta, y_\beta^{-1}\}$.

We put $X_{\beta+1} = X_\beta \cup \{x_\beta, y_\beta x_\beta\}$. Now it is necessary to show the fulfilment of (1)–(3) for $\alpha = \beta + 1$. First we show that $|X_{\beta+1} \cap gX_{\beta+1}| \leqslant 4$ for all $g \in G \setminus \{e\}$. Since $X_{\beta+1} = X_\beta \cup \{x_\beta, y_\beta x_\beta\}$, for every $g \in G \setminus \{e\}$, we have

$$X_{\beta+1} \cap gX_{\beta+1} = (X_\beta \cup \{x_\beta, y_\beta x_\beta\}) \cap (gX_\beta \cup g\{x_\beta, y_\beta x_\beta\}) =$$

$$= (X_\beta \cap gX_\beta) \cup Y_1 \cup Y_2 \cup Y_3,$$

where $Y_1 = X_\beta \cap \{gx_\beta, gy_\beta x_\beta\}$, $Y_2 = \{x_\beta, y_\beta x_\beta\} \cap gX_\beta$, $Y_3 = \{x_\beta, y_\beta x_\beta\} \cap \{gx_\beta, gy_\beta x_\beta\}$. We consider two cases:

Case 1: $g \in X_\beta X_\beta^{-1}$. By (i), $Y_1 = \varnothing$ and $Y_2 = \varnothing$. Since $y_\beta \notin X_\beta X_\beta^{-1}$, $Y_3 = \varnothing$. Then $X_{\beta+1} \cap gX_{\beta+1} = X_\beta \cap gX_\beta$ and, by the inductive assumption, $|X_{\beta+1} \cap gX_{\beta+1}| \leqslant 4$.

Case 2: $g \notin X_\beta X_\beta^{-1}$. Then $X_\beta \cap gX_\beta = \varnothing$. Since $Y_1 \cup Y_2 \cup Y_3 \subseteq \{x_\beta, y_\beta x_\beta, gx_\beta, gy_\beta x_\beta\}$, we have $|X_{\beta+1} \cap gX_{\beta+1}| = |Y_1 \cup Y_2 \cup Y_3| \leqslant 4$.

Now we show that $|X_{\beta+1} \cap X_{\beta+1}g| \leqslant 2$ for all $g \notin X_{\beta+1}X_{\beta+1}^{-1}$ and $|X_{\beta+1} \cap X_{\beta+1}g| \leqslant 4$ for all $g \in G \setminus \{e\}$. Since $X_{\beta+1} = X_\beta \cup \{x_\beta, y_\beta x_\beta\}$, for every $g \in G \setminus \{e\}$, we have

$$X_{\beta+1} \cap X_{\beta+1}g = (X_\beta \cup \{x_\beta, y_\beta x_\beta\}) \cap (X_\beta g \cup \{x_\beta g, y_\beta x_\beta g\}) =$$

$$= (X_\beta \cap X_\beta g) \cup Z_1 \cup Z_2 \cup Z_3' \cup Z_3'',$$

where $Z_1 = \{x_\beta g, y_\beta x_\beta g\} \cap X_\beta$, $Z_2 = \{x_\beta, y_\beta x_\beta\} \cap X_\beta g$, $Z_3' = \{x_\beta\} \cap \{y_\beta x_\beta g\}$, $Z_3'' = \{y_\beta x_\beta\} \cap \{x_\beta g\}$. We consider three cases.

Case 1: $g \in X_\beta X_\beta^{-1}$. By (i), $Z_1 = \varnothing$ and $Z_2 = \varnothing$. Since $g \in X_\beta X_\beta^{-1}$ and $y_\beta \notin X_\beta X_\beta^{-1}$ then $g \in (X_\beta^{-1} X_\beta \cup X_\beta X_\beta^{-1}) \setminus \{y_\beta, y_\beta^{-1}\}$. So, by (iii), $Z_3' = \varnothing$ and $Z_3'' = \varnothing$. Hence, $X_{\beta+1} \cap X_{\beta+1}g = X_\beta \cap X_\beta g$ and required inequalities hold by inductive hypothesis.

Case 2: $g \in \{y_\beta, y_\beta^{-1}\}$. By (ii), $Z_1 = \varnothing$ and $Z_2 = \varnothing$. Hence, $X_{\beta+1} \cap X_{\beta+1}g = (X_\beta \cap X_\beta g) \cup Z_3' \cup Z_3''$. Since $g \notin X_\beta X_\beta^{-1}$ then $|X_\beta \cap X_\beta g| \leqslant 2$. Since $|Z_3'| \leqslant 1$ and $|Z_3''| \leqslant 1$ then $|X_{\beta+1} \cap X_{\beta+1}g| \leqslant 4$. Observe that $g \in X_{\beta+1}X_{\beta+1}^{-1}$, so we do not need to check the condition (2).

Case 3: $g \notin X_\beta X_\beta^{-1} \cup \{y_\beta, y_\beta^{-1}\}$. Since $g \notin X_\beta X_\beta^{-1}$ then, by inductive hypothesis, $|X_\beta \cap X_\beta g| \leqslant 2$. Since $y_\beta \notin X_\beta X_\beta^{-1}$ then $|Z_1| \leqslant 1$ and $|Z_2| \leqslant 1$. We consider two subcases.

Subcase 3.1: $g \in X_\beta^{-1} X_\beta$. By (i), $Z_1 = \varnothing$ and $Z_2 = \varnothing$. By (iii), $Z_3' = \varnothing$ and $Z_3'' = \varnothing$. Hence, $X_{\beta+1} \cap X_{\beta+1}g = X_\beta \cap X_\beta g$ and required inequalities hold by inductive hypothesis.

Subcase 3.2: $g \notin X_\beta^{-1} X_\beta$. Then $X_\beta \cap X_\beta g = \varnothing$, so $X_{\beta+1} \cap X_{\beta+1}g = Z_1 \cup Z_2 \cup Z_3' \cup Z_3''$. By (ii), if $Z_3' \neq \varnothing$ then $Z_2 = \varnothing$, and if $Z_3'' \neq \varnothing$ then $Z_1 = \varnothing$. Taking into account the inequalities $|Z_1| \leqslant 1$, $|Z_2| \leqslant 1$, $|Z_3'| \leqslant 1$ and $|Z_3''| \leqslant 1$ we obtain $|X_{\beta+1} \cap X_{\beta+1}g| \leqslant 2$.

So the inequalities (1)–(3) hold for $\alpha = \beta + 1$. Note that $y_\beta \in X_{\beta+1}X_{\beta+1}^{-1}$. We put $X = \bigcup_{\alpha<\kappa} X_\alpha$ and observe that, by the choice of y_β, $G = XX^{-1}$ and X is 4-thin.

□

Question 2. Which is a minimal number k_{th} such that, for every infinite group G, there exists a k_{th}-thin subset X such that $G = XX^{-1}$?

Question 3. Which is a minimal number k_{lth} such that, for every infinite group G, there exists a left k_{lth}-thin subset X such that $G = XX^{-1}$?

An infinite group G of period 2 shows that $k_{th} \geqslant 2$, $k_{lth} \geqslant 2$. By Theorem 3, $k_{th} \leqslant 4$, $k_{lth} \leqslant 4$.

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Received by the editors: 10.03.2010
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