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RESEARCH ARTICLE

Representations of nodal algebras of type E

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Abstract. We define representation types of nodal algebras of type E.

Introduction

Finite dimesional nodal algebras were introduced in [7] as finite dimesional analogues of nodal orders considered in [2, 4]. In this paper and in papers [8, 12] their represention types (finite, tame or wild) were studied. Unfortunately, as the second author noticed, some results of [12] were incorrect. In this paper we improve them using the technique of *coverings* from the papers [3, 6] and a lemma on representations of quivers with relations which seems to be of independent interest. Namely, in each considered case we construct a Galois covering $\tilde{A}$ of the algebra A (in the sence of [3, 6]) with Galois group $\mathbb{Z}$. Then, according to [6], A and $\tilde{A}$ are of the same representation type. In most cases Lemma 2 reduces the description of representations of $\tilde{A}$ to those of some quiver, which gives the answer. Moreover, this Lemma implies that the supports of indecomposable representations of $\tilde{A}$ are bounded. Hence, according to [3], in finite or tame case all representations of A are just the natural images of those of $\tilde{A}$. Therefore, in these cases we also obtain a description of all representations of A.

2010 MSC: 16G60, 16G10, 16G20.
Key words and phrases: nodal algebras, representation type, quivers.

1. Definitions and results

We consider finite dimensional algebras over an algebraically closed field $\Bbbk$. Recall the definition of nodal algebras [7].

Definition 1. An algebra A is called *nodal* if there is a hereditary algebra H such that

1) $H \supset A \supset \operatorname{rad} H = \operatorname{rad} A$
2) $\operatorname{length}_A(H \otimes_A U) \leqslant 2$ for any simple left A-module U.

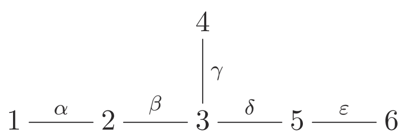
We say that the nodal algebra A *is related to the hereditary algebra H.*

As any finite dimensional algebra is Morita equivalent to its *basic algebra* [1,5] and, moreover, an algebra is nodal if and only if its basic algebra is nodal, we only consider basic algebras A, i.e. such that $A/radA \simeq \Bbbk^m$ for some m. We present such algebras by *quivers with relations* as in [1,5]. A basic nodal algebra can be obtained from

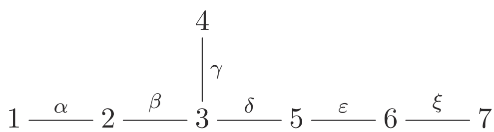
We say that a nodal algebra A is *of type* E if it is related to a hereditary algebra which is Morita equivalent to the path algebra H of a Dynkin quiver of type E or of a Euclidean quiver of type $\widetilde{\mathrm{E}}$. If A is basic, it can be obtained from H by a sequence of *glueing* and *blowing up* vertices [7,11]. Recall that glueing vertices i and j is said to be *inessential* if one of these vertices is a sink, while the other is a source. It is known that inessential glueing does not imply representation type [7].

We use the following numeration of the vertices and arrows of quivers of types E and $\widetilde{\mathrm{E}}$ (independently of their orientations):

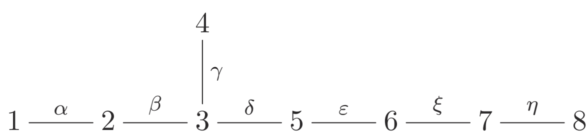
E_6 :



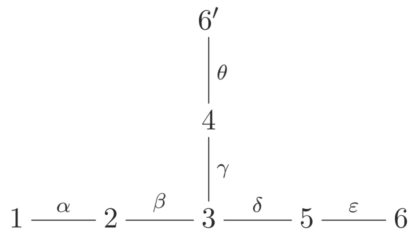
E_7 :



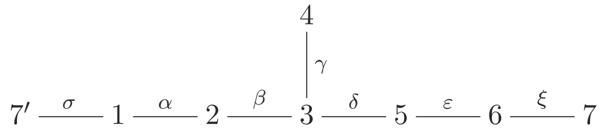
E_8 :



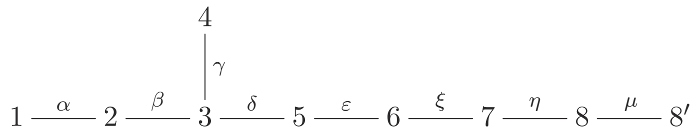
$\widetilde{E}_6$:



$\widetilde{E}_7$:



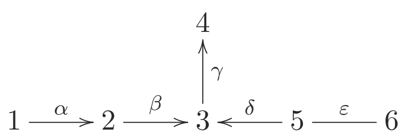
$\widetilde{E}_8$:



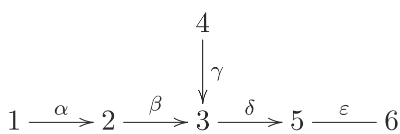
The following theorems describe representations types of nodal algebras of type E. If the orientaions of arrows are not prescribed on the pictures, they are arbitrary and do not imply the reprsentations type.

Theorem 1.1. *Let a nodal algebgra A be isomorphic or anti-isomorphic to an algebra obtained from a quiver of type* E *with one of the following operations and some inessential glueings:*

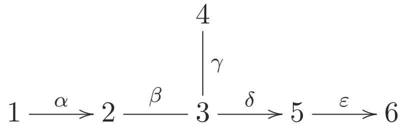
1) *glueing verices* 1 *and* 3 *in the quiver* E_6 *of the form*



or



2) *glueing vertices* 1 *and* 5 *in the quiver* E_6 *of the form*



3) *glueing vertices* 2 *and* 6 *in the quiver* E_6 *of the form*

$$\begin{array}{ccccccccc} & & & & 4 & & & & \\ & & & & \Big| {\scriptstyle\gamma} & & & & \\ 1 & \xrightarrow{\alpha} & 2 & \xrightarrow{\beta} & 3 & \overset{\delta}{\text{-----}} & 5 & \xrightarrow{\varepsilon} & 6 \end{array}$$

4) *glueing vertices* 2 *and* 7 *in the quiver* E_7 *of the form*

$$\begin{array}{ccccccccccc} & & & & 4 & & & & & & \\ & & & & \Big| {\scriptstyle\gamma} & & & & & & \\ 1 & \xrightarrow{\alpha} & 2 & \xrightarrow{\beta} & 3 & \overset{\delta}{\text{-----}} & 5 & \overset{\varepsilon}{\text{-----}} & 6 & \xrightarrow{\xi} & 7 \end{array}$$

5) *glueing vertices* 3 *and* 7 *in the quiver* E_7 *of the form*

$$\begin{array}{ccccccccccc} & & & & 4 & & & & & & \\ & & & & \Big\downarrow {\scriptstyle\gamma} & & & & & & \\ 1 & \overset{\alpha}{\text{-----}} & 2 & \xleftarrow{\beta} & 3 & \xrightarrow{\delta} & 5 & \overset{\varepsilon}{\text{-----}} & 6 & \xrightarrow{\xi} & 7 \end{array}$$

Then A *is reresentation finite.*

Theorem 1.2. *Let a nodal algebgra* A *be isomorphic or anti-isomorphic to an algebra obtained from a quiver of type* E *with one of the following operations and some inessential glueings:*

1) *glueing vertices* 1 *and* 3 *in the quiver* E_7 *of the form*

$$\begin{array}{ccccccccccc} & & & & 4 & & & & & & \\ & & & & \Big\uparrow {\scriptstyle\gamma} & & & & & & \\ 1 & \xrightarrow{\alpha} & 2 & \xrightarrow{\beta} & 3 & \xleftarrow{\delta} & 5 & \overset{\varepsilon}{\text{-----}} & 6 & \overset{\xi}{\text{-----}} & 7 \end{array}$$

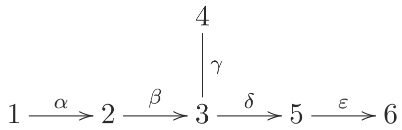
2) *glueing vertices* 1 *and* 6 *in the quiver* E_7 *of the form*

$$\begin{array}{ccccccccccc} & & & & 4 & & & & & & \\ & & & & \Big| {\scriptstyle\gamma} & & & & & & \\ 1 & \xrightarrow{\alpha} & 2 & \overset{\beta}{\text{-----}} & 3 & \overset{\delta}{\text{-----}} & 5 & \xrightarrow{\varepsilon} & 6 & \xrightarrow{\xi} & 7 \end{array}$$

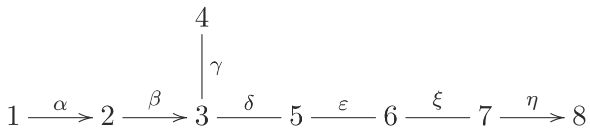
3) *glueing vertices* 2 *and* 4 *in the quiver* E_6 *of the form*

$$\begin{array}{ccccccccc} & & & & 4 & & & & \\ & & & & \Big\uparrow {\scriptstyle\gamma} & & & & \\ 1 & \xrightarrow{\alpha} & 2 & \xrightarrow{\beta} & 3 & \overset{\delta}{\text{-----}} & 5 & \overset{\varepsilon}{\text{-----}} & 6 \end{array}$$

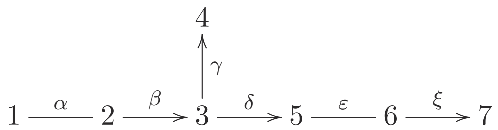
4) *glueing vertices* 2 *and* 5 *in the quiver* E_6 *of the form*



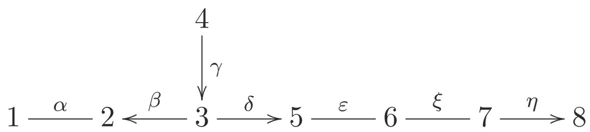
5) *glueing vertices* 2 *and* 8 *in the quiver* E_8 *of the form*



6) *glueing vertices* 3 *and* 7 *in the quiver* E_7 *of the form*



7) *glueing vertices* 3 *and* 8 *in the quiver* E_8 *of the form*



Then A *is tame (of infinite representation type).*

Theorem 1.3. *If a nodal algebra* A *of type* E *is neither isomorphic nor anti-isomorphic to an algebra occuring in Theorems 1.1 or 1.2, it is wild.*

2. Main lemma

The proof of the theorems from Section 1 is substantially based upon the following result.

Lemma 2. *Let the quiver* Γ *of an algebra* A *be a union of three parts* Γ_+, Γ_- *and* L *such that:*

1) *Every intersection* $\Gamma_+ \cap \Gamma_-$, $\Gamma_+ \cap \mathsf{L}$ *and* $\Gamma_- \cap \mathsf{L}$ *consists of a unique vertex* o *which is a sink in the quiver* Γ_+ *and the source in the quiver* Γ_-.

2) L *is a chain (a quiver of type* A_k*) and has at most one arrow starting at* o *and at most one arrow ending at* o *(for instance,* o *is a source or a sink in* L*).*
3) *If* α *is an arrow of* Γ_+ *ending at* o *and* β *is an arrow of* Γ_- *starting at* o*, then* $\beta\alpha = 0$ *in* A*.*
4) *If* λ_+ *is an arrow of* L *ending at* o *and* λ_- *is an arrow of* L *starting at* o*, then* $\lambda_-\lambda_+ = 0$ *in* A*.*

If M *be an indecomposable representation of* A*, then either* $M(\alpha) = 0$ *for every arrow* α *from* Γ_+ *ending at* o *or* $M(\beta) = 0$ *for every arrow* β *from* Γ_- *starting at* o*.*

Proof. Note that L is of the shape

$$a \,\text{———}\, \cdots \,\text{———}\, o_+ \xrightarrow{\lambda_+} o \xrightarrow{\lambda_-} o_- \,\text{———}\, \cdots \,\text{———}\, b$$

Let $M(o) = M_1 \oplus M_0$, where $M_1 = \sum_\alpha \operatorname{Im}\alpha$, where α runs through all arrows of Γ_+ ending at o. Then $\beta(M_1) = 0$ for all arrows β of Γ_- starting at o. With respect to this decomposition $\lambda_+ = \begin{pmatrix} \lambda_+^1 \\ \lambda_+^0 \end{pmatrix}$ and $\lambda_- = \begin{pmatrix} \lambda_-^1 & \lambda_-^0 \end{pmatrix}$ so that $\lambda_-^1\lambda_+^1 + \lambda_-^0\lambda_+^0 = 0$. If we decompose the restrictions of M onto the parts of L between a and o_+ and between o_- and b, the columns of λ_+ and the rows of λ_- split onto several blocks:

$$\lambda_+ = \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline \end{array} \qquad \lambda_- = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array}$$

The number of columns in λ_+ and the number of rows in λ_- equal the lengths of the corresponding parts of L. The horizontal subdivision of λ_+ and the vertical subdivision of λ_- correspond, as above, to the decomposition $M(o) = M_1 \oplus M_0$. As one can replace any vector v from a basis of M_0 by $v + u$, where $u \in M_1$, one can replace any row (say, i-th) from the upper part of λ_+ by a sum of this row and a multiple of some row (say, j-th) from the lower part. Doing it, one also has to subtract the same multiple of the i-th column of the matrix λ_- from its j-th column. Moreover, one can arrange the columns of λ_+ (and the rows of λ_-) so that one can add a multiple of any column from a vertical block to any column of the block which is on the right (respectively, add a multiple of any row from a horizontal block to any row from a block below). Using these transformations and the condition $\lambda_-\lambda_+ = 0$, one can decompose

$M_1 = M_{11} \oplus M_{10}$ and $M_0 = M_{01} \oplus M_{11}$ so that the matrices λ_+ and λ_- become of the form:

$$\lambda_+ = \begin{pmatrix} \lambda_+^{11} \\ 0 \\ \hline \lambda_+^{01} \\ 0 \end{pmatrix} \qquad \lambda_- = \begin{pmatrix} 0 & \lambda_-^{10} & \mid & 0 & \lambda_-^{00} \end{pmatrix},$$

where the subdivision of the rows of λ_+ is the same as the subdivision of the columns of λ_- and the first two rows (columns) are from λ_+^1 (respectively, from λ_-^1), while the other two are from λ_+^0 (respectively, from λ_-^0). Evidently, if M is indecomposable, at most one of the spaces M_{ij} can be non-zero. It implies the claim of the lemma. □

3. Proofs

We prove Theorems 1.1–1.3 simultaneously, considering all sorts of glueing vertices in quivers of types E and $\widetilde{\mathrm{E}}$. One can easily see that any blowing up vertices in these quivers gives a wild algebra.

Recall that a *Galois covering* [3, 6] of an algebra A given by a quiver with relations (Q, R) consists of an algebra $\tilde{A}$ given by a quiver with relations $(\tilde{Q}, \tilde{R})$, a homomorphism of algebras $\tilde{A} \to A$ given by a homomorphism of quivers $\phi : \tilde{Q} \to Q$ preserving relations and a free action of a group G (the *Galois group* of this covering) on the quiver $\tilde{Q}$ such that the preimages of vertices and arrows of the quiver Q under the map ϕ coincide with the orbits of this action. For any representation M of the algebra $\tilde{A}$ one can consider the induced representation $\phi_* M = A \otimes_{\tilde{A}} M$ of the algebra the A. The Galois group G also acts on the category of representations of $\tilde{A}$ and $\phi_* M \simeq \phi_* N$ if and only if $N \simeq M^g$ for some $g \in G$. Note that usually the algebra $\tilde{A}$ is infinite dimensional and the quiver $\tilde{Q}$ is infinite. They say that $\tilde{A}$ is *representation support bounded* if there is a number C such that $\#\{v \in \operatorname{Ver} \tilde{Q} \mid M(v) \neq 0\} \leqslant C$ for every finite dimensional representation M of the algebra $\tilde{A}$.

Recall the main results about representation types of Galois coverings from [3, 6].

Theorem 3.1. *Let $\tilde{A}$ is a Galois covering of an algebra A with Galois group G*

1) *If $\tilde{A}$ is representation finite, so is A and vice versa. Moreover, in this case every indecomposable representation of A is isomorphic to $\phi_* M$ for some indecomposable representation M of $\tilde{A}$.*

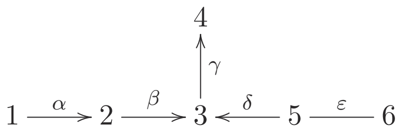
2) *Let the group G be torsion free. If $\tilde{A}$ is tame, so is A and vice versa. Moreover, if $\tilde{A}$ is tame and representation support bounded, every indecomposable finite dimensional representation of A is isomorphic to $\phi_* M$ for some indecomposable representation M of $\tilde{A}$.*

We only consider several typical cases of glueings including one which seems the most complicated. All other cases are similar (as a rule, simpler).

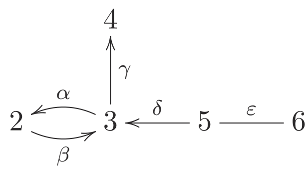
Case 1. Glueing vertices 1 and 3.

We can suppose that the arrow α starts at the vertex 1. Depending on the orientation, we have several possibilities of essential glueings which do not give an algebra that is evidently wild. They are (for E_6):

1.1.

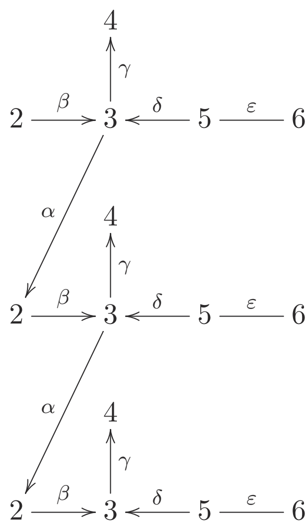


Then A is given by the quiver with relations



$$\alpha\beta = 0, \alpha\delta = 0.$$

There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver

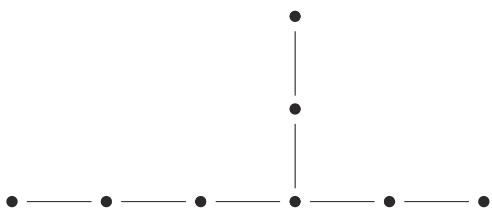


with the same relations $\alpha\beta = 0, \alpha\delta = 0$ for all α, β, δ. Here and later on we denote vertices and arrows of $\tilde{Q}$ by the same letters as their images in Q and suppose that the described quiver repeats infinitely up and down.

Using Lemma 2 in two subsequent vertices 3, we can make zero the arrows β, δ, then α. We obtain the quiver of type E_7 which is representation finite. Hence, $\tilde{A}$ is representation finite and so is A.

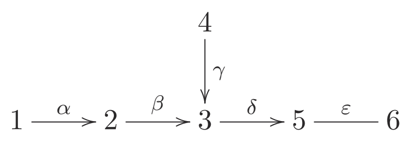
The same observations for the quiver E_7 leads to a quiver of type $\widetilde{\mathrm{E}}_7$ which is tame. Moreover, they show that $\tilde{A}$ is representation support bounded.

In case of the quiver $\widetilde{\mathrm{E}}_6$, $\tilde{Q}$ has a wild subquiver without relations

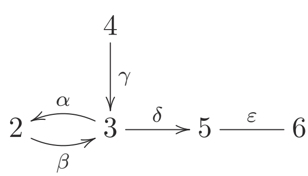


Therefore, in this case $\tilde{A}$ and A are wild. The same is the case of other Dynkin and Euclidean quivers.

1.2.

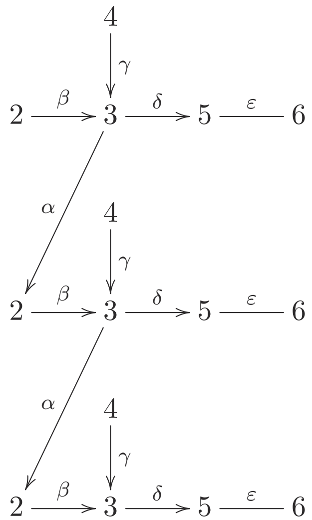


Then A is given by the quiver with relations



$\alpha\beta = 0, \alpha\gamma = 0.$

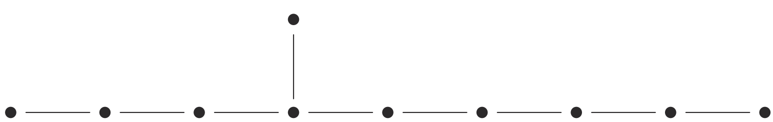
There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver



with relations $\alpha\beta = 0, \alpha\gamma = 0$.

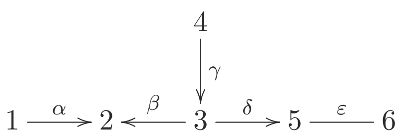
Using Lemma 2 in two subsequent vertices 3, we can make zero the arrows β, γ, then α. We obtain the quiver of type E_8 which is representation finite. Hence, $\tilde{A}$ is representation finite and so is A.

In case of the quiver E_7 we obtain, as a subquiber of $\tilde{Q}$ the wild quiver without relations.

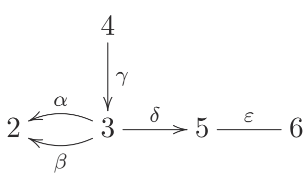


Therefore, in this case $\tilde{A}$ and A are wild. The same is the case of other Dynkin and Euclidean quivers.

1.3.

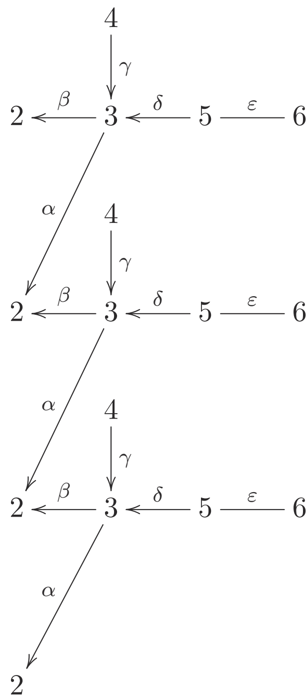


Then A is given by the quiver with relations

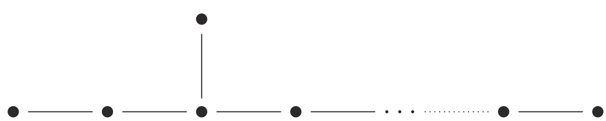


$\alpha\gamma = 0, \alpha\delta = 0.$

There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver

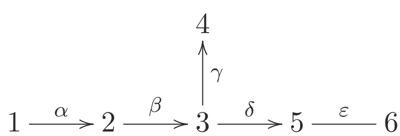


with relations $\alpha\gamma = 0, \alpha\delta = 0$. It contains a wild subquiver without relations

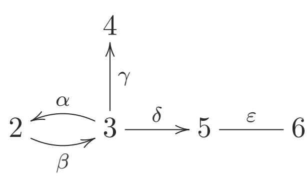


with arbitrary long branch to the right.

1.4.



Then A is given by the quiver with relations



$\alpha\beta = 0.$

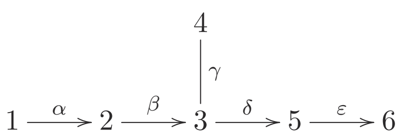
One esaily sees that $\tilde{Q}$ also has a wild subquiver without relations, so $\tilde{A}$ and A are wild. All the more so they are in other Dynkin and Euclidean cases.

Thus we see that all cases of glueing vertices 1 and 3 which are representation finite or tame are listed in Theorems 1.1 and 1.2, while all other cases give wild algebras, as claimed in Theorem 1.3. One can also easily verify that any additional essential glueing leads to a wild algebra.

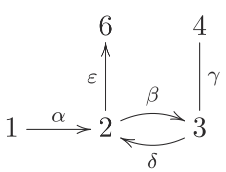
Case 2. Glueing vertices 2 and 5.

If both arrows α and β end at the vertex 2 and the glueing is essential, it gives an algebra which is evidently wild. So we can suppose that they have the orientation $1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$. Then there are two possibilities of essential glueings which do not give an algebra that is evidently wild. They are (for E_6):

2.1. (Perhaps, the most complicated case.)

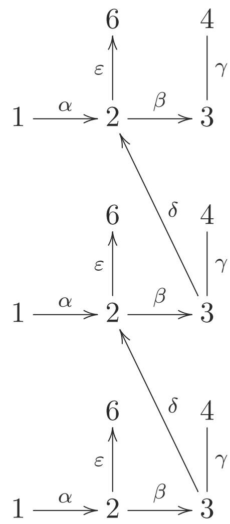


Then A is given by the quiver with relations



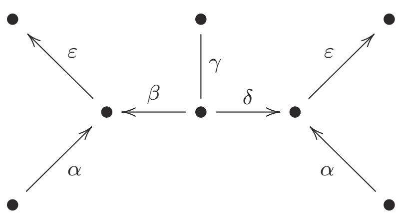
$$\varepsilon\alpha = 0, \beta\delta = 0.$$

There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver

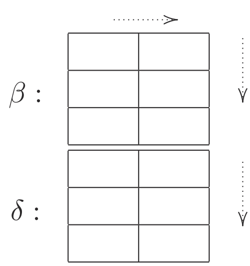


with relations $\varepsilon\alpha = 0$, $\beta\delta = 0$.

Using Lemma 2 in two subsequent vertices 2, we can make zero the arrow β, then δ. It gives the quiver



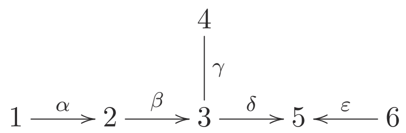
with relations $\varepsilon\alpha = 0$. If we reduce the matrices corresponding to α, ε and γ, the rows of the matrices corresponding to β and δ subdivide into 3 parts and their columns subdivide into 2 parts. Moreover, one can make elementary transformations in every horizontal stripe (independently) and in every vertical stripe simultaneously in β and δ. Moreover, one can also add columns and rows from one part tp those of another part according to the following picture:



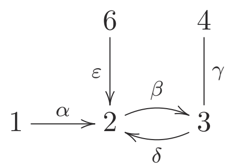
It is the problem of representations of a pair of posets (2) and $(3,3)$ in the sense of [9, 10], which is tame. Therefore, $\tilde{A}$ and A are tame and $\tilde{A}$ is representation support bounded.

If we start with the quiver $\widetilde{\mathrm{E}}_6$, we obtain in the same way representation of the pair of posets (3) and $(3,3)$, which is a wild problem. Hence $\tilde{A}$ and A are wild. Analogously, in all other Dynkin and Euclidean cases we obtain wild algebras.

2.2.

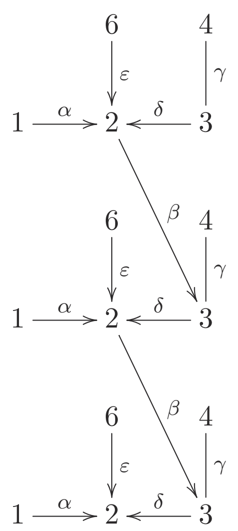


Then A is given by the quiver with relations

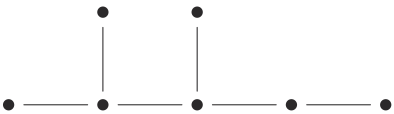


$$\beta\varepsilon = 0, \beta\delta = 0.$$

There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver



with relations $\beta\varepsilon = 0$, $\beta\delta = 0$. It contains a wild subquiver without relations



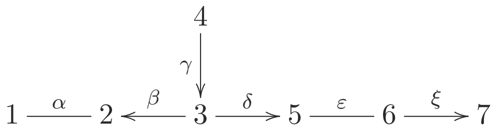
Hence $\tilde{A}$ and A are wild. All the more, it is so for all other Dynkin and Euclidean quivers.

Thus, the only case when glueing 2 and 5 is not wild is case 4 from Theorem 1.2 and all other cases are wild. One can also easily verify that any additional essential glueing leads to a wild algebra.

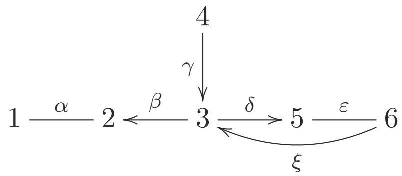
Case 3. Glueing vertices 3 and 7.

We consider the quiver E_7 and suppose that the arrow ξ ends at 7. There are the following variants of essential glueing which do not give an algebra that is evidently wild:

3.1.

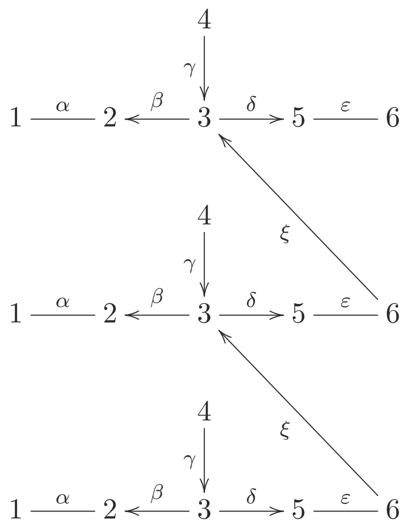


Then A is given by the quiver with relations



$\beta\xi = 0, \ \delta\xi = 0.$

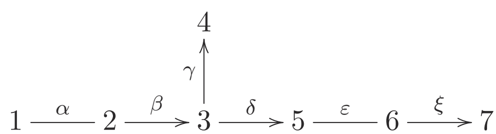
There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver



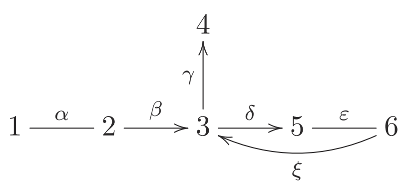
with relations $\beta\xi = 0,\ \delta\xi = 0$.

Using Lemma 2 in two subsequent vertices 3, we can make zero the arrows β, δ, then ξ. We obtain the quiver of type E_8 which is representation finite. Hence, $\tilde{A}$ is representation finite and so is A. One easily sees that for all other Dynkin and Euclidean quivers the same consideration gives a wild quiver.

3.2.

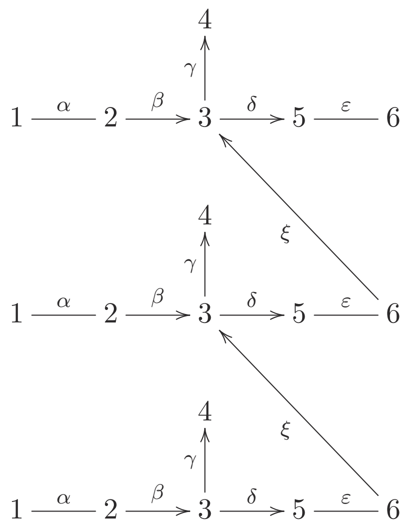


Then A is given by the quiver with relations



$\gamma\xi = 0,\ \delta\xi = 0.$

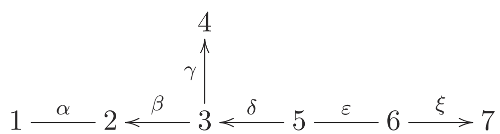
There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver



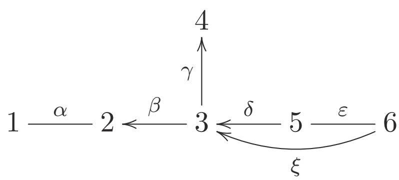
with relations $\gamma\xi = 0,\ \delta\xi = 0.$

Using Lemma 2 in two subsequent vertices 3, we can make zero the arrows γ, δ, then ξ. We obtain the quiver of type $\widetilde{\mathrm{E}}_8$ which is tame. Hence, $\tilde{A}$ is tame and so is A. Moreover, $\tilde{A}$ is representation support bounded. One easily sees that for all other Dynkin and Euclidean quivers the same consideration gives a wild quiver.

3.3.

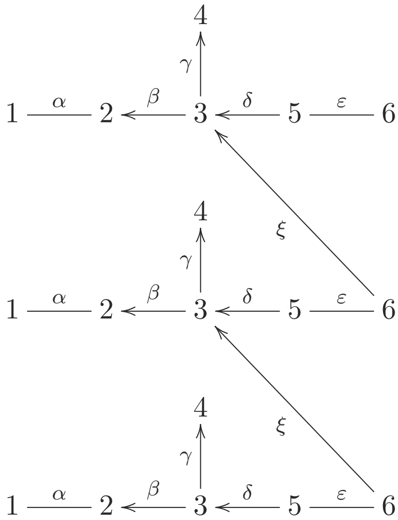


Then A is given by the quiver with relations

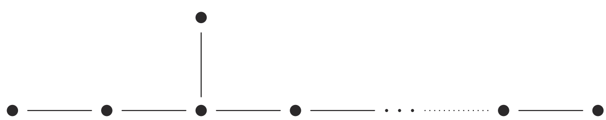


$\beta\xi = 0,\ \gamma\xi = 0.$

There is a Galois covering $\tilde{A}$ of A with Galois group $\mathbb{Z}$ given by the quiver



with relations $\beta\xi = 0$, $\gamma\xi = 0$. It contiains a wild subquiver without relations



with arbitrary long branch to the right. Hence the algebras $\tilde{A}$ and A are wild. The same is if we start from another Dynkin or Euclidean quiver.

Thus we see that all cases of glueing vertices 3 and 7 which are representation finite or tame are listed in Theorems 1.1 and 1.2, while all other cases give wild algebras, as claimed in Theorem 1.3. One can also easily verify that any additional essential glueing leads to a wild algebra.

Just in the same way we check all other cases of gluing, which accomplishes the proof of Theorems 1.1, 1.2 and 1.3.

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