

Derived equivalence classification of generalized multifold extensions of piecewise hereditary algebras of tree type

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ABSTRACT. We give a derived equivalence classification of algebras of the form $\hat{A}/\langle\phi\rangle$ for some piecewise hereditary algebra A of tree type and some automorphism ϕ of \hat{A} such that $\phi(A^{[0]}) = A^{[n]}$ for some positive integer n .

Introduction

Throughout this paper we fix an algebraically closed field \mathbb{k} , and assume that all algebras are basic and finite-dimensional \mathbb{k} -algebras and that all categories are \mathbb{k} -categories.

The classification of algebras under derived equivalences seems to be first explicitly investigated by Rickard in [9], which gave the derived equivalence classification of Brauer tree algebras (implicitly there exists an earlier work [4] by Assem–Happel giving the classification of gentle tree algebras). After that the first named author gave the classification of representation-finite self-injective algebras (see also [1] and Membrillo–Hernández [7] for type A_n). The technique used there (a covering technique for derived equivalences developed in [1]) is applicable also for representation-infinite

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algebras; it requires that the algebras in consideration have the form of orbit categories (usually of repetitive categories of some algebras having no oriented cycles in their ordinary quivers). In fact, it was applied in [3] to give the classification of twisted multifold extensions of piecewise hereditary algebras of tree type by giving a complete invariant. Here an algebra is called a *twisted multifold extension* of an algebra A if it has the form

$$T_{\psi}^n(A) := \hat{A}/\langle \hat{\psi}\nu_A^n \rangle \quad (0.1)$$

for some positive integer n and some automorphism ψ of A , where \hat{A} is the repetitive algebra of A , ν_A is the Nakayama automorphism of \hat{A} and $\hat{\psi}$ is the automorphism of \hat{A} naturally induced from ψ (see Definition 1.1 and Lemma 1.2 for details); and an algebra A is called a *piecewise hereditary algebra of tree type* if A is an algebra derived equivalent to a hereditary algebra whose ordinary quiver is an oriented tree. In this paper we extend this classification to a wider class of algebras. To state this class of algebras we introduce the following terminologies. For an integer n we say that an automorphism ϕ of \hat{A} has a *jump n* if $\phi(A^{[0]}) = A^{[n]}$. An algebra of the form

$$\hat{A}/\langle \phi \rangle$$

for some automorphism ϕ of \hat{A} with jump n for some positive integer n is called a *generalized multifold extension* of A . Since obviously $\hat{\psi}\nu_A^n$ is an automorphism of \hat{A} with jump n in the formula (0.1), twisted multifold extensions are generalized multifold extensions. We are now able to state our purpose. In this paper we will give the derived equivalence classification of generalized multifold extensions of piecewise hereditary algebras of tree type by giving a complete invariant. Note that most algebras in this class are wild and that the tame part of the class has a big intersection with the class of self-injective algebras of Euclidean type studied by Skowroński in [10] (see Remark 1.7).

The paper is organized as follows. After preparations in section 1 we first reduce the problem to the case of hereditary tree algebras in section 2. Then we investigate scalar multiples in the repetitive category of a hereditary tree algebras in section 3, which is a central part of the proof of the main result. In section 4 we show that any generalized multifold extension of a piecewise hereditary algebra of tree type is derived equivalent to a twisted multifold extension of the same type, which immediately yields the desired classification result.

1. Preliminaries

For a category R we denote by R_0 and R_1 the class of objects and morphisms of R , respectively. A category R is said to be *locally bounded* if it satisfies the following:

- Distinct objects of R are not isomorphic;
- $R(x, x)$ is a local algebra for all $x \in R_0$;
- $R(x, y)$ is finite-dimensional for all $x, y \in R_0$; and
- The set $\{y \in R_0 \mid R(x, y) \neq 0 \text{ or } R(y, x) \neq 0\}$ is finite for all $x \in R_0$.

A category is called *finite* if it has only a finite number of objects.

A pair (A, E) of an algebra A and a complete set $E := \{e_1, \dots, e_n\}$ of orthogonal primitive idempotents of A can be identified with a locally bounded and finite category R by the following correspondences. Such a pair (A, E) defines a category $R_{(A, E)} := R$ as follows: $R_0 := E$, $R(x, y) := yAx$ for all $x, y \in E$, and the composition of R is defined by the multiplication of A . Then the category R is locally bounded and finite. Conversely, a locally bounded and finite category R defines such a pair (A_R, E_R) as follows: $A_R := \bigoplus_{x, y \in R_0} R(x, y)$ with the usual matrix multiplication (regard each element of A as a matrix indexed by R_0), and $E_R := \{(\mathbb{1}_x \delta_{(i, j), (x, x)})_{i, j \in R_0} \mid x \in R_0\}$. We always regard an algebra A as a locally bounded and finite category by fixing a complete set A_0 of orthogonal primitive idempotents of A .

For a locally bounded category A , we denote by $\text{Mod } A$ the category of all (right) A -modules (= contravariant functors from A to the category $\text{Mod } \mathbb{k}$ of \mathbb{k} -vector spaces); by $\text{mod } A$ the full subcategory of $\text{Mod } A$ consisting of finitely presented objects; and by $\text{prj } A$ the full subcategory of $\text{Mod } A$ consisting of finitely generated projective objects. $\mathcal{K}^b(\mathcal{A})$ denotes the bounded homotopy category of an additive category \mathcal{A} .

1.1. Repetitive categories

Definition 1.1. Let A be a locally bounded category.

(1) The *repetitive category* \hat{A} of A is a \mathbb{k} -category defined as follows (\hat{A} turns out to be locally bounded again):

- $\hat{A}_0 := A_0 \times \mathbb{Z} = \{x^{[i]} := (x, i) \mid x \in A_0, i \in \mathbb{Z}\}$.

$$\bullet \hat{A}(x^{[i]}, y^{[j]}) := \begin{cases} \{f^{[i]} \mid f \in A(x, y)\} & \text{if } j = i, \\ \{\phi^{[i]} \mid \phi \in DA(y, x)\} & \text{if } j = i + 1, \\ 0 & \text{otherwise,} \end{cases}$$

for all $x^{[i]}, y^{[j]} \in \hat{A}_0$.

- For each $x^{[i]}, y^{[j]}, z^{[k]} \in \hat{A}_0$ the composition $\hat{A}(y^{[j]}, z^{[k]}) \times \hat{A}(x^{[i]}, y^{[j]}) \rightarrow \hat{A}(x^{[i]}, z^{[k]})$ is given as follows.

- (i) If $i = j, j = k$, then this is the composition of A $A(y, z) \times A(x, y) \rightarrow A(x, z)$.
- (ii) If $i = j, j + 1 = k$, then this is given by the right A -module structure of DA : $DA(z, y) \times A(x, y) \rightarrow DA(z, x)$.
- (iii) If $i + 1 = j, j = k$, then this is given by the left A -module structure of DA : $A(y, z) \times DA(y, x) \rightarrow DA(z, x)$.
- (iv) Otherwise, the composition is zero.

(2) We define an automorphism ν_A of \hat{A} , called the *Nakayama automorphism* of \hat{A} , by $\nu_A(x^{[i]}) := x^{[i+1]}$, $\nu_A(f^{[i]}) := f^{[i+1]}$, $\nu_A(\phi^{[i]}) := \phi^{[i+1]}$ for all $i \in \mathbb{Z}, x \in A_0, f \in A_1, \phi \in \bigcup_{x, y \in A_0} DA(y, x)$.

(3) For each $n \in \mathbb{Z}$, we denote by $A^{[n]}$ the full subcategory of \hat{A} formed by $x^{[n]}$ with $x \in A$, and by $\mathbf{1}^{[n]} : A \xrightarrow{\sim} A^{[n]} \hookrightarrow \hat{A}, x \mapsto x^{[n]}$, the embedding functor.

We cite the following from [3, Lemma 2.3].

Lemma 1.2. *Let $\psi : A \rightarrow B$ be an isomorphism of locally bounded categories. Denote by $\psi_x^y : A(y, x) \rightarrow B(\psi y, \psi x)$ the isomorphism defined by ψ for all $x, y \in A$. Define $\hat{\psi} : \hat{A} \rightarrow \hat{B}$ as follows.*

- For each $x^{[i]} \in \hat{A}$, $\hat{\psi}(x^{[i]}) := (\psi x)^{[i]}$;
- For each $f^{[i]} \in \hat{A}(x^{[i]}, y^{[i]})$, $\hat{\psi}(f^{[i]}) := (\psi f)^{[i]}$; and
- For each $\phi^{[i]} \in \hat{A}(x^{[i]}, y^{[i+1]})$, $\hat{\psi}(\phi^{[i]}) := (D((\psi_x^y)^{-1})(\phi))^{[i]} = (\phi \circ (\psi_x^y)^{-1})^{[i]}$.

Then

- (1) $\hat{\psi}$ is an isomorphism.
- (2) Given an isomorphism $\rho : \hat{A} \rightarrow \hat{B}$, the following are equivalent.
 - (a) $\rho = \hat{\psi}$;
 - (b) ρ satisfies the following.

- (i) $\rho\nu_A = \nu_B\rho$;
- (ii) $\rho(A^{[0]}) = A^{[0]}$;
- (iii) *The diagram*

$$\begin{array}{ccc} A & \xrightarrow{\psi} & B \\ \mathbb{1}^{[0]} \downarrow & & \downarrow \mathbb{1}^{[0]} \\ A^{[0]} & \xrightarrow{\rho} & B^{[0]} \end{array}$$

is commutative; and

- (iv) $\rho(\phi^{[0]}) = (\phi \circ (\psi_x^y)^{-1})^{[0]}$ for all $x, y \in A$ and all $\phi \in DA(y, x)$.

Let R be a locally bounded category with the Jacobson radical J and with the ordinary quiver Q . Then by definition of Q there is a bijection $f: Q_0 \rightarrow R_0, x \mapsto f_x$ and injections $\bar{a}_{y,x}: Q_1(x, y) \rightarrow J(f_x, f_y)/J^2(f_x, f_y)$ such that $\bar{a}_{y,x}(Q_1(x, y))$ forms a basis of $J(f_x, f_y)/J^2(f_x, f_y)$, where $Q_1(x, y)$ is the set of arrows from x to y in Q for all $x, y \in Q_0$. For each $\alpha \in Q_1(x, y)$ choose $a_{y,x}(\alpha) \in J(f_x, f_y)$ such that $a_{y,x}(\alpha) + J^2(f_x, f_y) = \bar{a}_{y,x}(\alpha)$. Then the pair (f, a) of the bijection f and the family a of injections $a_{y,x}: Q_1(x, y) \rightarrow J(f_x, f_y)$ ($x, y \in Q_0$) uniquely extends to a full functor $\Phi: \mathbb{k}Q \rightarrow R$, which is called a *display functor* for R .

A path μ from y to x in a quiver with relations (Q, I) is called *maximal* if $\mu \notin I$ but $\alpha\mu, \mu\beta \in I$ for all arrows $\alpha, \beta \in Q_1$. For a \mathbb{k} -vector space V with a basis $\{v_1, \dots, v_n\}$ we denote by $\{v_1^*, \dots, v_n^*\}$ the basis of DV dual to the basis $\{v_1, \dots, v_n\}$. In particular if $\dim_{\mathbb{k}} V = 1, v^* \in DV$ is defined for all $v \in V \setminus \{0\}$.

An algebra is called a *tree algebra* if its ordinary quiver is an oriented tree.

Lemma 1.3. *Let A be a tree algebra and $\Phi: \mathbb{k}Q \rightarrow A$ a display functor with $I := \text{Ker } \Phi$. Then*

- (1) Φ uniquely induces the display functor $\hat{\Phi}: \mathbb{k}\hat{Q} \rightarrow \hat{A}$ for \hat{A} , where

- (i) $\hat{Q} = (\hat{Q}_0, \hat{Q}_1, \hat{s}, \hat{t})$ is defined as follows:

- $\hat{Q}_0 := Q_0 \times \mathbb{Z} = \{x^{[i]} := (x, i) \mid x \in Q_0, i \in \mathbb{Z}\}$,
- $Q_1 \times \mathbb{Z} := \{\alpha^{[i]} := (\alpha, i) \mid \alpha \in Q_1, i \in \mathbb{Z}\}$,
- $\hat{Q}_1 := (Q_1 \times \mathbb{Z}) \sqcup \{\mu^{*[i]} \mid \mu \text{ is a maximal path in } (Q, I), i \in \mathbb{Z}\}$,
- $\hat{s}(\alpha^{[i]}) := s(\alpha)^{[i]}, \hat{t}(\alpha^{[i]}) := t(\alpha)^{[i]}$ for all $\alpha^{[i]} \in Q_1 \times \mathbb{Z}$, and if μ is a maximal path from y to x in (Q, I) then, $\hat{s}(\mu^{*[i]}) := x^{[i]}, \hat{t}(\mu^{*[i]}) := y^{[i+1]}$.

- (ii) $\hat{\Phi}$ is defined by $\hat{\Phi}(x^{[i]}) := (\Phi x)^{[i]}$, $\hat{\Phi}(\alpha^{[i]}) := (\Phi \alpha)^{[i]}$, and $\hat{\Phi}(\mu^{*[i]}) := (\Phi(\mu^*))^{[i]}$ for all $i \in \mathbb{Z}$, $x \in Q_0$, $\alpha \in Q_1$ and maximal paths μ in (Q, I) .
- (2) We define an automorphism ν_Q of \hat{Q} by $\nu_Q(x^{[i]}) := x^{[i+1]}$, $\nu_Q(\alpha^{[i]}) := \alpha^{[i+1]}$, $\nu_Q(\mu^{*[i]}) := \mu^{*[i+1]}$ for all $i \in \mathbb{Z}$, $x \in Q_0$, $\alpha \in Q_1$, and maximal paths μ in (Q, I) .
- (3) $\text{Ker } \hat{\Phi}$ is equal to the ideal \hat{I} defined by the full commutativity relations on \hat{Q} and the zero relations $\mu = 0$ for those paths μ of \hat{Q} for which there is no path $\hat{i}(\mu) \rightsquigarrow \nu_Q(\hat{s}(\mu))$. (Therefore note that if a path $\alpha_n \cdots \alpha_1$ is in I , then $\alpha_n^{[i]} \cdots \alpha_1^{[i]}$ is in \hat{I} for all $i \in \mathbb{Z}$.)

Let R be a locally bounded category. A morphism $f: x \rightarrow y$ in R_1 is called a *maximal nonzero morphism* if $f \neq 0$ and $fg = 0$, $hf = 0$ for all $g \in \text{rad } R(z, x)$, $h \in \text{rad } R(y, z)$, $z \in R_0$.

Lemma 1.4. *Let A be an algebra and $x^{[i]}, y^{[j]} \in \hat{A}_0$. Then there exists a maximal nonzero morphism in $\hat{A}(x^{[i]}, y^{[j]})$ if and only if $y^{[j]} = \nu_A(x^{[i]})$.*

Proof. This follows from the fact that $\hat{A}(-, x^{[i+1]}) \cong D\hat{A}(x^{[i]}, -)$ for all $i \in \mathbb{Z}$, $x \in A_0$. \square

Lemma 1.5. *Let A be an algebra. Then the actions of $\phi\nu_A$ and $\nu_A\phi$ coincide on the objects of \hat{A} for all $\phi \in \text{Aut}(\hat{A})$.*

Proof. Let $x^{[i]} \in \hat{A}_0$. Then there is a maximal nonzero morphism in $\hat{A}(x^{[i]}, \nu_A(x^{[i]}))$ by Lemma 1.4. Since ϕ is an automorphism of \hat{A} , there is a maximal nonzero morphism in $\hat{A}(\phi(x^{[i]}), \phi(\nu_A(x^{[i]})))$. Hence $\phi(\nu_A(x^{[i]})) = \nu_A(\phi(x^{[i]}))$ by the same lemma. \square

The following is immediate by the lemma above.

Proposition 1.6. *Let A be an algebra, n an integer, and ϕ an automorphism of \hat{A} . Then the following are equivalent:*

- (1) ϕ is an automorphism with jump n ;
- (2) $\phi(A^i) = A^{[i+n]}$ for some integer i ;
- (3) $\phi(A^j) = A^{[j+n]}$ for all integers j ; and
- (4) $\phi = \sigma\nu_A^n$ for some automorphism σ of \hat{A} with jump 0.

Remark 1.7. Let A be an algebra.

- (1) In Skowroński [10, 11] an automorphism ϕ of \hat{A} is called *rigid* if $\phi(A^{[j]}) = A^{[j]}$ for all $j \in \mathbb{Z}$. Hence ϕ is rigid if and only if it is an automorphism with jump 0 by the proposition above. Therefore for an integer n , ϕ is an automorphism with jump n if and only if $\phi = \sigma\nu_A^n$ for some rigid automorphism σ of \hat{A} .
- (2) Noting this fact we see by [11, Theorem 4.7] that the class of self-injective algebras of Euclidean type contains a lot of generalized multifold extensions of piecewise hereditary algebras of tree type.

In the sequel, we always assume that n is a positive integer when we consider a morphism with jump n .

1.2. Derived equivalences and tilting subcategories

Let R be a locally bounded category and $\phi \in \text{Aut}(R)$. Then ϕ induces an equivalence $\phi(-) : \text{mod } R \rightarrow \text{mod } R$ defined by $\phi M := M \circ \phi^{-1} : R \rightarrow \text{mod } \mathbb{k}$ for all $M \in \text{mod } R$. In particular for $R(-, x)$ with $x \in R$, we have $\phi(R(-, x)) = R(\phi^{-1}(-), x) \cong R(-, \phi x)$, and the last isomorphism is given by ϕ itself. Thus the identification $\phi(R(-, x)) = R(-, \phi x)$ depends on ϕ , and the subset $\{R(-, x) \mid x \in R\}$ of $\text{prj } R$ is not $\langle \phi(-) \rangle$ -stable in a strict sense. This makes it difficult to give explicitly a complete set of representatives of isoclasses of indecomposable objects of $\mathcal{K}^b(\text{prj } R)$ which is $\langle \mathcal{K}^b(\phi(-)) \rangle$ -stable. To avoid this difficulty we used in [2] the formal additive hull $\text{add } R$ ([5, 2.1 Example 8]) of R defined below instead of $\text{prj } R$.

Definition 1.8. Let R be a locally bounded category. The *formal additive hull* $\text{add } R$ of R is a category defined as follows.

- $(\text{add } R)_0 := \{\bigoplus_{i=1}^n x_i := (x_1, \dots, x_n) \mid n \in \mathbb{N}, x_1, \dots, x_n \in R_0\}$;
- For each $x = \bigoplus_{i=1}^m x_i, y = \bigoplus_{j=1}^n y_j \in (\text{add } R)_0$,

$$\begin{aligned}
 (\text{add } R)(x, y) := & \{(\mu_{j,i})_{j,i} \mid \mu_{j,i} \in R(x_i, y_j) \\
 & \text{for all } i = 1, \dots, m, j = 1, \dots, n\}; \text{ and}
 \end{aligned}$$

- The composition is given by the matrix multiplication.

We regard that R is contained in $\text{add } R$ by the embedding $(f : x \rightarrow y) \mapsto ((f) : (x) \rightarrow (y))$ for all f in R .

Remark 1.9. Let R and ϕ be as above.

- (1) Define a functor $\eta_R: \text{add } R \rightarrow \text{prj } R$ by $(x_1, \dots, x_n) \mapsto R(-, x_1) \oplus \dots \oplus R(-, x_n)$ and $(\mu_{ji})_{j,i} \mapsto (R(-, \mu_{ji}))_{j,i}$. Then η_R is an equivalence, called the *Yoneda equivalence*.
- (2) Let $F: R \rightarrow S$ be a functor of locally bounded categories. Then F naturally induces functors $\text{add } F: \text{add } R \rightarrow \text{add } S$ and $\tilde{F} := \mathcal{K}^b(\text{add } F): \mathcal{K}^b(\text{add } R) \rightarrow \mathcal{K}^b(\text{add } S)$, which are isomorphisms if F is an isomorphism. Namely, $\text{add } F$ is defined by $(x_1, \dots, x_n) \mapsto (Fx_1, \dots, Fx_n)$ and $(\mu_{ji}) \mapsto (F\mu_{ji})$ for all objects (x_1, \dots, x_n) and all morphisms (μ_{ji}) in $\text{add } R$; and \tilde{F} is defined by $\text{add } F$ component-wise. Further if $G: S \rightarrow T$ is a functor of locally bounded categories, then we have $(GF)^\sim = \tilde{G}\tilde{F}$.
- (3) The automorphism ϕ acts on $\mathcal{K}^b(\text{add } R)$ as $\tilde{\phi}$, and ${}^\phi\mathcal{K}^b(\eta_R)(X^\bullet) \cong \mathcal{K}^b(\eta_R)(\tilde{\phi}(X^\bullet))$ for all $X^\bullet \in \mathcal{K}^b(\text{add } R)$.

We cite the following from [2, Proposition 5.1] which follows from Keller [6] (Cf. Rickard [8], [1, Proposition 1.1]).

Proposition 1.10. *Let R and S be locally bounded categories. Then the following are equivalent:*

- (1) *There is a triangle equivalence $\mathcal{D}(\text{Mod } S) \rightarrow \mathcal{D}(\text{Mod } R)$; and*
- (2) *There is a full subcategory E of $\mathcal{K}^b(\text{add } R)$ such that*
 - (a) *$\mathcal{K}^b(\text{add } R)(T, U[n]) = 0$ for all $T, U \in E$ and all $n \neq 0$;*
 - (b) *R is contained in the smallest full triangulated subcategory of $\mathcal{K}^b(\text{add } R)$ containing E that is closed under direct summands and isomorphisms; and*
 - (c) *E is isomorphic to S .*

Definition 1.11. We say that locally bounded categories R and S are *derived equivalent* if one of the equivalent conditions above holds. In (2) the triple (R, E, S) is called a *tilting triple* and $E \subseteq \mathcal{K}^b(\text{add } R)$ is called a *tilting subcategory* for R .

Theorem 1.5 in [1] is interpreted as follows.

Theorem 1.12. *If (A, E, B) is a tilting triple of locally bounded categories with an isomorphism $\psi: E \rightarrow B$, then $(\hat{A}, \hat{E}, \hat{B})$ is also a tilting triple with the isomorphism $\hat{\psi}: \hat{E} \rightarrow \hat{B}$, where \hat{E} is isomorphic to (and identified with) the full subcategory of $\mathcal{K}^b(\text{add } \hat{A})$ consisting of the $(\mathbb{1}^{[n]})^\sim(T)$ with $T \in E$, $n \in \mathbb{Z}$.*

For a group G acting on a category S we say that a subclass E of the objects of S is G -stable (resp. G -stable up to isomorphisms) if $gx \in E$ (resp. if gx is isomorphic to some object in E) for all $g \in G$ and $x \in E$.

Proposition 1.13. *Let (A, E, B) be a tilting triple of locally bounded categories with an isomorphism $\psi: E \rightarrow B$, g an automorphism of \hat{A} and h an automorphism of \hat{B} . Then $\hat{A}/\langle g \rangle$ is derived equivalent to $\hat{B}/\langle h \rangle$ if*

- (1) g is of infinite order and $\langle g \rangle$ acts freely on \hat{A} ;
- (2) \hat{E} is $\langle \tilde{g} \rangle$ -stable; and
- (3) The following diagram commutes:

$$\begin{array}{ccc}
 \hat{E} & \xrightarrow{\hat{\psi}} & \hat{B} \\
 \tilde{g} \downarrow & & \downarrow h \\
 \hat{E} & \xrightarrow{\quad} & \hat{B}. \\
 & \hat{\psi} &
 \end{array}$$

Remark 1.14. Let E be a tilting subcategory for a locally bounded category R and G a group acting on R . If E is G -stable up to isomorphisms, then there exists a tilting subcategory E' for R such that $E \cong E'$ and E' is G -stable (see [1, Remark 3.2] and [2, Lemma 5.3.3 and Remark 5.3(2)]).

2. Reduction to hereditary tree algebras

Let Q be a quiver. We denote by \bar{Q} the underlying graph of Q , and call Q finite if both Q_0 and Q_1 are finite sets. Each automorphism of Q is regarded as an automorphism of \bar{Q} preserving the orientation of Q , thus $\text{Aut}(Q)$ can be regarded as a subgroup of $\text{Aut}(\bar{Q})$. Suppose now that Q is a finite oriented tree. Then it is also known that $\text{Aut}(Q) \leq \text{Aut}_0(\bar{Q}) := \{f \in \text{Aut}(\bar{Q}) \mid f(x) = x \text{ for some } x \in Q_0\}$. We say that Q is an *admissibly oriented tree* if $\text{Aut}(Q) = \text{Aut}_0(\bar{Q})$. We quote the following from [3, Lemma 4.1]:

Lemma 2.1. *For any finite tree T there exists an admissibly oriented tree Q with a unique source such that $\bar{Q} = T$.*

We cite the following from [3, Lemma 5.4].

Lemma 2.2. *Let A be a piecewise hereditary algebra of type Q for an admissibly oriented tree Q . Then there is a tilting triple (A, E, kQ) such that E is $\langle \tilde{\phi} \rangle$ -stable up to isomorphisms for all $\phi \in \text{Aut}(A)$.*

By the following proposition we can reduce the derived equivalence classification of generalized multifold extensions of *piecewise hereditary* algebras of tree type to the corresponding problem of generalized multifold extensions of *hereditary* tree algebras. The second statement also enables us to compare the generalized multifold extension and a twisted version corresponding to it using the repetitive category of the common hereditary algebra.

Proposition 2.3. *Let A be a piecewise hereditary algebra of tree type \bar{Q} for an admissibly oriented tree Q , and n a positive integer. Then we have the following:*

- (1) *For any $\phi \in \text{Aut}(\hat{A})$ with jump n , there exists some $\psi \in \text{Aut}(\widehat{\mathbb{k}Q})$ with jump n such that $\hat{A}/\langle\phi\rangle$ is derived equivalent to $\widehat{\mathbb{k}Q}/\langle\psi\rangle$; and*
- (2) *If we set $\phi' := \nu_A^n \hat{\phi}_0 \in \text{Aut}(\hat{A})$, where $\phi_0 := (\mathbf{1}^{[0]})^{-1} \nu_A^{-n} \phi \mathbf{1}^{[0]}$, then there exists some $\psi' \in \text{Aut}(\widehat{\mathbb{k}Q})$ with jump n such that $\hat{A}/\langle\phi'\rangle$ is derived equivalent to $\widehat{\mathbb{k}Q}/\langle\psi'\rangle$, and that the actions of ψ and ψ' coincide on the objects of $\widehat{\mathbb{k}Q}$.*

Proof. (1) We set $\nu := \nu_A$ and $\phi_i := (\mathbf{1}^{[i]})^{-1} \nu_A^{-n} \phi \mathbf{1}^{[i]} \in \text{Aut}(A)$ for all $i \in \mathbb{Z}$. By Lemma 2.2, there exists a tilting triple $(A, E, \mathbb{k}Q)$ with an isomorphism $\zeta: E \rightarrow \mathbb{k}Q$ such that E is $\langle\tilde{\eta}\rangle$ -stable up to isomorphisms for all $\eta \in \text{Aut}(A)$. In particular, E is $\langle\tilde{\phi}_i\rangle$ -stable up to isomorphisms for all $i \in \mathbb{Z}$. Then $(\hat{A}, \hat{E}, \widehat{\mathbb{k}Q})$ is a tilting triple with the isomorphism $\hat{\zeta}$ by Theorem 1.12 and the following holds.

Claim 1. *\hat{E} is $\langle\tilde{\phi}\rangle$ -stable up to isomorphisms.*

Indeed, for each $T \in E_0$ and $i \in \mathbb{Z}$, we have

$$\begin{aligned} \tilde{\phi}(\mathbf{1}^{[i]})^\sim(T) &= (\nu^n \nu^{-n} \phi \mathbf{1}^{[i]})^\sim(T) \\ &= (\nu^n \mathbf{1}^{[i]} \phi_i)^\sim(T) \\ &= (\mathbf{1}^{[i+n]})^\sim \tilde{\phi}_i(T). \end{aligned} \tag{2.1}$$

Since E is $\langle\tilde{\phi}_i\rangle$ -stable up to isomorphisms, we have $\tilde{\phi}_i(T) \cong T'$ for some $T' \in E$, and hence $\tilde{\phi}((\mathbf{1}^{[i]})^\sim(T)) \cong (\mathbf{1}^{[i+n]})^\sim(T') \in \hat{E}$, as desired.

By Remark 1.14, we have a $\langle\tilde{\phi}\rangle$ -stable tilting subcategory \hat{E}' and an isomorphism $\theta: \hat{E}' \xrightarrow{\sim} \hat{E}$. Therefore by Proposition 1.13 $\hat{A}/\langle\phi\rangle$ and $\hat{E}'/\langle\tilde{\phi}\rangle$ are derived equivalent. If we set $\psi := (\hat{\zeta}\theta)\tilde{\phi}(\hat{\zeta}\theta)^{-1}$, then (2.1) shows that ψ is an automorphism with jump n , and that $\hat{E}'/\langle\tilde{\phi}\rangle \cong \widehat{\mathbb{k}Q}/\langle\psi\rangle$. Hence $\hat{A}/\langle\phi\rangle$ and $\widehat{\mathbb{k}Q}/\langle\psi\rangle$ are derived equivalent.

(2) Note that ϕ' is also an automorphism with jump n . By the same argument we see that \hat{E} is also $\langle \tilde{\phi}' \rangle$ -stable up to isomorphisms; there exists a $\langle \tilde{\phi}' \rangle$ -stable tilting subcategory \hat{E}'' and an isomorphism $\theta' : \hat{E}'' \xrightarrow{\sim} \hat{E}$; and $\hat{A}/\langle \phi' \rangle$ and $\hat{E}''/\langle \tilde{\phi}' \rangle$ are derived equivalent. Set $\psi' := (\hat{\zeta}\theta')\tilde{\phi}'(\hat{\zeta}\theta')^{-1}$, then ψ' is an automorphism with jump n , $\hat{E}''/\langle \tilde{\phi}' \rangle \cong \widehat{\mathbb{k}Q}/\langle \psi' \rangle$, and $\hat{A}/\langle \phi' \rangle$ and $\widehat{\mathbb{k}Q}/\langle \psi' \rangle$ are derived equivalent. Now for $i = 0$ the equality (2.1) shows that $\tilde{\phi}(\mathbb{1}^{[0]})^\sim(T) = (\mathbb{1}^{[n]})^\sim \tilde{\phi}_0(T)$ for all $T \in E_0$. Since $\phi'_0 = \phi_0$, the same calculation shows that $\tilde{\phi}'(\mathbb{1}^{[0]})^\sim(T) = (\mathbb{1}^{[n]})^\sim \tilde{\phi}_0(T)$ for all $T \in E_0$. Thus the actions of $\tilde{\phi}$ and $\tilde{\phi}'$ coincide on the objects of $E^{[0]}$, which shows that the actions of ψ and ψ' coincide on the objects of $\mathbb{k}Q^{[0]}$. Hence by Lemma 1.5 their actions coincide on the objects of $\widehat{\mathbb{k}Q}$. Indeed, $\psi(x^{[i]}) = \psi\nu^i(x^{[0]}) = \nu^i\psi(x^{[0]}) = \nu^i\psi'(x^{[0]}) = \psi'\nu^i(x^{[0]}) = \psi'(x^{[i]})$ for all $x \in Q_0$ and $i \in \mathbb{Z}$. \square

3. Hereditary tree algebras

Remark 3.1. Let Q be an oriented tree.

- (1) We may identify $\widehat{\mathbb{k}Q} = \mathbb{k}\hat{Q}/\hat{I}$ as stated in Lemma 1.3, and we denote by $\bar{\mu}$ the morphism $\mu + \hat{I}$ in $\widehat{\mathbb{k}Q}$ for each morphism μ in $\mathbb{k}\hat{Q}$.
- (2) Let $x, y \in \hat{Q}_0$. Since \hat{I} contains full commutativity relations, we have $\dim_{\mathbb{k}} \widehat{\mathbb{k}Q}(x, y) \leq 1$, and in particular \hat{Q} has no double arrows.
- (3) Let $\alpha : x \rightarrow y$ be in \hat{Q}_1 and $\phi \in \text{Aut}(\widehat{\mathbb{k}Q})$. Then there exists a unique arrow $\phi x \rightarrow \phi y$ in \hat{Q} , which we denote by $(\hat{\pi}\phi)(\alpha)$, and we have $\phi(\bar{\alpha}) = \phi_\alpha(\hat{\pi}\phi)(\alpha) \in \widehat{\mathbb{k}Q}(\phi x, \phi y)$ for a unique $\phi_\alpha \in \mathbb{k}^\times := \mathbb{k} \setminus \{0\}$. This defines an automorphism $\hat{\pi}\phi$ of \hat{Q} , and thus a group homomorphism $\hat{\pi} : \text{Aut}(\widehat{\mathbb{k}Q}) \rightarrow \text{Aut}(\hat{Q})$.
- (4) Similarly, let $\alpha : x \rightarrow y$ be in Q_1 and $\psi \in \text{Aut}(\mathbb{k}Q)$. Then there exists a unique arrow $\psi x \rightarrow \psi y$ in Q , which we denote by $(\pi\psi)(\alpha)$. This defines an automorphism $\pi\psi$ of Q , and thus a group homomorphism $\pi : \text{Aut}(\mathbb{k}Q) \rightarrow \text{Aut}(Q)$.

We cite the following from [3, Proposition 7.4].

Proposition 3.2. *Let R be a locally bounded category, and g, h automorphisms of R acting freely on R . If there exists a map $\rho : R_0 \rightarrow \mathbb{k}^\times$ such that $\rho(y)g(f) = h(f)\rho(x)$ for all morphisms $f : x \rightarrow y$ in R , then $R/\langle g \rangle \cong R/\langle h \rangle$. \square*

Definition 3.3. (1) For a quiver $Q = (Q_0, Q_1, s, t)$ we set $Q[Q_1^{-1}]$ to be the quiver

$$Q[Q_1^{-1}] := (Q_0, Q_1 \sqcup \{\alpha^{-1} \mid \alpha \in Q_1\}, s', t'),$$

where $s'|_{Q_1} := s, t'|_{Q_1} := t, s'(\alpha^{-1}) := t(\alpha)$ and $t'(\alpha^{-1}) := s(\alpha)$ for all $\alpha \in Q_1$. A *walk* in Q is a path in $Q[Q_1^{-1}]$.

(2) Suppose that Q is a finite oriented tree. Then for each $x, y \in Q_0$ there exists a unique shortest walk from x to y in Q , which we denote by $w(x, y)$. If $w(x, y) = \alpha_n^{\varepsilon_n} \cdots \alpha_1^{\varepsilon_1}$ for some $\alpha_1, \dots, \alpha_n \in Q_1$ and $\varepsilon_1, \dots, \varepsilon_n \in \{1, -1\}$, then we define a subquiver $W(x, y)$ of Q by $W(x, y) := (W(x, y)_0, W(x, y)_1, s', t')$, where $W(x, y)_0 := \{s(\alpha_i), t(\alpha_i) \mid i = 1, \dots, n\}$, $W(x, y)_1 := \{\alpha_1, \dots, \alpha_n\}$, and s', t' are restrictions of s, t to $W(x, y)_1$, respectively. Since Q is an oriented tree, $w(x, y)$ is uniquely recovered by $W(x, y)$. Therefore we can identify $w(x, y)$ with $W(x, y)$, and define a *sink* and a *source* of $w(x, y)$ as those in $W(x, y)$.

The following is a central part of the proof of the main result.

Proposition 3.4. *Let Q be a finite oriented tree and ϕ, ψ automorphisms of $\widehat{\mathbb{k}Q}$ acting freely on $\widehat{\mathbb{k}Q}$. If the actions of ϕ and ψ coincide on the objects of $\widehat{\mathbb{k}Q}$, then there exists a map $\rho: (\hat{Q}_0 =) \widehat{\mathbb{k}Q}_0 \rightarrow \mathbb{k}^\times$ such that $\rho(y)\psi(f) = \phi(f)\rho(x)$ for all morphisms $f: x \rightarrow y$ in $\widehat{\mathbb{k}Q}$. Hence in particular, $\widehat{\mathbb{k}Q}/\langle \phi \rangle$ is isomorphic to $\widehat{\mathbb{k}Q}/\langle \psi \rangle$.*

Proof. Assume that the actions of $\phi, \psi \in \text{Aut}(\widehat{\mathbb{k}Q})$ coincides on the objects of $\widehat{\mathbb{k}Q}$. Then ϕ and ψ induce the same quiver automorphism $q = \hat{\pi}\phi = \hat{\pi}\psi$ of \hat{Q} , and there exist $(\phi_\alpha)_{\alpha \in \hat{Q}_1}, (\psi_\alpha)_{\alpha \in \hat{Q}_1} \in (k^\times)^{\hat{Q}_1}$ such that for each $\alpha \in \hat{Q}_1$ we have

$$\phi(\bar{\alpha}) = \phi_\alpha \overline{q(\alpha)}, \quad \psi(\bar{\alpha}) = \psi_\alpha \overline{q(\alpha)}.$$

For each path $\lambda = \alpha_n \cdots \alpha_1$ in \hat{Q} with $\alpha_1, \dots, \alpha_n \in \hat{Q}_1$ we set $\phi_\lambda := \phi_{\alpha_n} \cdots \phi_{\alpha_1}$. Then we have

$$\phi(\bar{\lambda}) = \phi_\lambda \overline{q(\lambda)},$$

where $q(\lambda) := q(\alpha_n) \cdots q(\alpha_1)$ because

$$\phi(\bar{\alpha_n}) \cdots \phi(\bar{\alpha_1}) = \phi_{\alpha_n} \cdots \phi_{\alpha_1} \overline{q(\alpha_n)} \cdots \overline{q(\alpha_1)}.$$

To show the statement we may assume that $\psi_\alpha = 1$ for all $\alpha \in \hat{Q}_1$. Since for each $x, y \in \hat{Q}_0$ the morphism space $\widehat{\mathbb{k}Q}(x, y)$ is at most 1-dimensional and has a basis of the form $\bar{\mu}$ for some path μ , it is enough

to show that there exists a map $\rho : \hat{Q}_0 \rightarrow \mathbb{k}^\times$ satisfying the following condition:

$$\rho(v^{[j]}) = \phi_\beta \rho(u^{[i]}) \quad \text{for all } \beta : u^{[i]} \rightarrow v^{[j]} \text{ in } \hat{Q}_1. \tag{3.1}$$

We define a map ρ as follows:

Fix a maximal path $\mu : y \rightsquigarrow x$ in Q . Then x is a sink and y is a source in Q . We can write μ as $\mu = \alpha_l \cdots \alpha_1$ for some $\alpha_1, \dots, \alpha_l \in Q_1$. First we set $\rho(x^{[0]}) := 1$. By induction on $0 \leq i \in \mathbb{Z}$ we define $\rho(x^{[i]})$ and $\rho(x^{[-i]})$ by the following formulas:

$$\rho(x^{[i+1]}) := \phi_{\mu^{[i+1]}} \phi_{\mu^{*[i]}} \rho(x^{[i]}), \tag{3.2}$$

$$\rho(x^{[i-1]}) := \phi_{\mu^{*[i-1]}}^{-1} \phi_{\mu^{[i]}}^{-1} \rho(x^{[i]}). \tag{3.3}$$

Now for each $i \in \mathbb{Z}$ and $u \in Q_0$ if $w(u, x) = \beta_m^{\varepsilon_m} \cdots \beta_1^{\varepsilon_1}$ for some $\beta_1, \dots, \beta_m \in Q_1$ and $\varepsilon_1, \dots, \varepsilon_m \in \{1, -1\}$, then we set

$$\rho(u^{[i]}) := \phi_{\beta_1^{[i]}}^{-\varepsilon_1} \cdots \phi_{\beta_m^{[i]}}^{-\varepsilon_m} \rho(x^{[i]}). \tag{3.4}$$

We have to verify the condition (3.1).

Case 1. $\beta = \alpha^{[i]} : u^{[i]} \rightarrow v^{[i]}$ for some $i \in \mathbb{Z}$, and $\alpha : u \rightarrow v$ in Q_1 . Since Q is an oriented tree, we have either $w(u, x) = w(v, x)\alpha$ or $w(v, x) = w(u, x)\alpha^{-1}$. In either case we have $\rho(v^{[i]}) = \phi_{\alpha^{[i]}} \rho(u^{[i]})$ by the formula (3.4).

Case 2. Otherwise, we have $\beta = \lambda^{*[i]} : u^{[i]} \rightarrow v^{[i+1]}$ for some maximal path $\lambda : v \rightsquigarrow u$ in Q and $i \in \mathbb{Z}$. In this case the condition (3.1) has the following form:

$$\rho(v^{[i+1]}) = \phi_{\lambda^{*[i]}} \rho(u^{[i]}). \tag{3.5}$$

Two paths are said to be *parallel* if they have the same source and the same target. We prepare the following for the proof.

Claim 2. *If ζ and η are parallel paths in \hat{Q} , then we have $\phi_\zeta = \phi_\eta$.*

Indeed, since $\zeta - \eta \in \hat{I}$, we have $\phi(\bar{\zeta}) = \phi(\bar{\eta})$, which shows

$$\phi_\zeta \overline{q(\zeta)} = \phi_\eta \overline{q(\eta)}.$$

Here we have $\overline{q(\zeta)} = \psi(\bar{\zeta}) = \psi(\bar{\eta}) = \overline{q(\eta)}$, and $\psi(\bar{\zeta}) \neq 0$ because $\bar{\zeta} \neq 0$. Hence $\phi_\zeta = \phi_\eta$, as required.

We now set $d(a, b)$ to be the number of sinks in $w(a, b)$ for all $a, b \in Q_0$. By induction on $d(y, v)$ we show that the condition (3.5) holds. Note that both v and y (resp. u and x) are sources (resp. sinks) in Q .

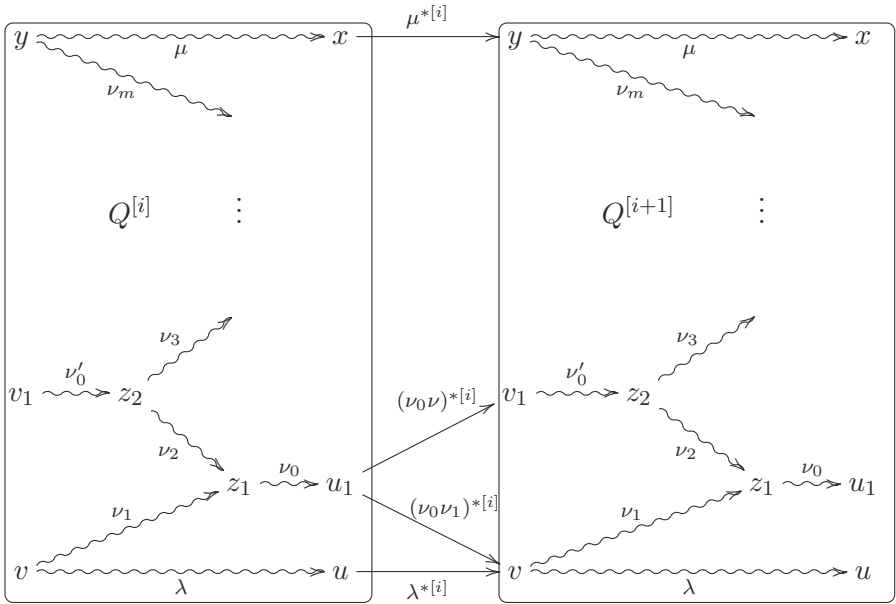


FIGURE 1.

Assume $d(y, v) = 0$. Then $y = v$ because these are sources in Q . By formulas (3.4) and (3.2) we have

$$\rho(v^{[i+1]}) = \rho(y^{[i+1]}) = \phi_{\alpha_1^{[i+1]}}^{-1} \cdots \phi_{\alpha_t^{[i+1]}}^{-1} \rho(x^{[i+1]}) = \phi_{\mu^{*[i]}} \rho(x^{[i]}).$$

If $u = x$, then $\lambda = \mu$ and hence $\phi_{\mu^{*[i]}} \rho(x^{[i]}) = \phi_{\lambda^{*[i]}} \rho(u^{[i]})$. Thus (3.5) holds.

If $u \neq x$, then $\phi_{\mu^{*[i]}} \phi_{\mu^{[i]}} = \phi_{\lambda^{*[i]}} \phi_{\lambda^{[i]}}$ by Claim 2. Since Q is an oriented tree, we have $w(u, x) = \mu\lambda^{-1}$, and $\rho(u^{[i]}) = \phi_{\lambda^{[i]}} \phi_{\mu^{[i]}}^{-1} \rho(x^{[i]})$. Therefore

$$\rho(v^{[i+1]}) = \phi_{\mu^{*[i]}} \rho(x^{[i]}) = \phi_{\lambda^{*[i]}} \phi_{\lambda^{[i]}} \phi_{\mu^{[i]}}^{-1} \rho(x^{[i]}) = \phi_{\lambda^{*[i]}} \rho(u^{[i]}),$$

and (3.5) holds.

Assume $d(y, v) \geq 1$. Then we can write $w(y, v) = \nu_1^{-1} \nu_2 \cdots \nu_{m-1}^{-1} \nu_m$ for some paths ν_1, \dots, ν_m of length at least 1 and $m \geq 2$. Set $z_1 := t(\nu_2)$, $z_2 := s(\nu_2)$. Then z_1 is a sink and z_2 is a source in $w(y, v)$. Since Q is a tree, there exists a unique maximal path of the form $\nu_0 \nu_2 \nu'_0: v_1 \rightsquigarrow u_1$ in Q for some paths ν_0, ν'_0 . We set $\nu := \nu_2 \nu'_0$. (See Figure 1, where we omitted the notations $[i], [i + 1]$ for paths in $Q^{[i]}, Q^{[i+1]}$, respectively.) Since $d(v_1, y) = d(v, y) - 1$, we have

$$\rho(v_1^{[i+1]}) = \phi_{(\nu_0 \nu)^{*[i]}} \rho(u_1^{[i]}) \tag{3.6}$$

by induction hypothesis. Since the paths $\nu^{[i+1]}(\nu_0\nu)^{*[i]}$ and $\nu_1^{[i+1]}(\nu_0\nu_1)^{*[i]}$ are parallel, we have

$$\phi_{\nu^{[i+1]}}\phi_{(\nu_0\nu)^{*[i]}} = \phi_{\nu_1^{[i+1]}}\phi_{(\nu_0\nu_1)^{*[i]}} \tag{3.7}$$

by Claim 1. Further by the result of Case 1 we have

$$\rho(v^{[i+1]}) = \phi_{\nu_1^{[i+1]}}^{-1}\phi_{\nu^{[i+1]}}\rho(v_1^{[i+1]}). \tag{3.8}$$

It follows from (3.6), (3.7) and (3.8) that

$$\rho(v^{[i+1]}) = \phi_{(\nu_0\nu_1)^{*[i]}}\rho(u_1^{[i]}).$$

(If $u_1 = u$, then $\nu_0\nu_1 = \lambda$ and this already gives (3.5).) Again by the result of Case 1 we have

$$\rho(u_1^{[i]}) = \phi_{(\nu_0\nu_1)^{[i]}}\phi_{\lambda^{[i]}}^{-1}\rho(u^{[i]}).$$

Since the paths $\lambda^{*[i]}\lambda^{[i]}$ and $(\nu_0\nu_1)^{*[i]}(\nu_0\nu_1)^{[i]}$ are parallel, we have

$$\phi_{\lambda^{*[i]}}\phi_{\lambda^{[i]}} = \phi_{(\nu_0\nu_1)^{*[i]}}\phi_{(\nu_0\nu_1)^{[i]}}$$

by Claim 1. The last three equalities give (3.5). □

4. Main result

Theorem 4.1. *Let A be a piecewise hereditary algebra of tree type and ϕ an automorphism of \hat{A} with jump n . Then $\hat{A}/\langle\phi\rangle$ and $T_{\phi_0}^n(A)$ are derived equivalent, where we set $\phi_0 := (\mathbb{1}^{[0]})^{-1}\nu^{-n}\phi\mathbb{1}^{[0]}$.*

Proof. Let T be the tree type of A . Then by Lemma 2.1 there exists an admissibly oriented tree Q with $\bar{Q} = T$. We set $\phi' := \nu_A^n\hat{\phi}_0 (= \hat{\phi}_0\nu_A^n)$. Then $T_{\phi_0}^n(A) = \hat{A}/\langle\phi'\rangle$. By Proposition 2.3(2) there exist some $\psi, \psi' \in \text{Aut}(\widehat{\mathbb{k}Q})$ both with jump n such that $\hat{A}/\langle\phi\rangle$ (resp. $\hat{A}/\langle\phi'\rangle$) is derived equivalent to $\widehat{\mathbb{k}Q}/\langle\psi\rangle$ (resp. $\widehat{\mathbb{k}Q}/\langle\psi'\rangle$), and the actions of ψ and ψ' coincide on the objects of $\widehat{\mathbb{k}Q}$. Then by Proposition 3.4 we have $\widehat{\mathbb{k}Q}/\langle\psi\rangle \cong \widehat{\mathbb{k}Q}/\langle\psi'\rangle$. Hence $\hat{A}/\langle\phi\rangle$ and $T_{\phi_0}^n(A)$ are derived equivalent. □

Definition 4.2. Let Λ be a generalized n -fold extension of a piecewise hereditary algebra A of tree type T , say $\Lambda = \hat{A}/\langle\phi\rangle$ for some $\phi \in \text{Aut}(A)$ with jump n . Further let Q be an admissibly oriented tree with $\bar{Q} = T$.

Then by Proposition 2.3 there exists $\psi \in \text{Aut}(\widehat{\mathbb{k}Q})$ with jump n such that $\hat{A}/\langle\phi\rangle$ is derived equivalent to $\widehat{\mathbb{k}Q}/\langle\psi\rangle$. We define the (*derived equivalence*) *type* $\text{type}(\Lambda)$ of Λ to be the triple $(T, n, \bar{\pi}(\psi_0))$, where $\psi_0 := (\mathbb{1}^{[0]})^{-1} \nu_{\mathbb{k}Q}^{-n} \psi \mathbb{1}^{[0]}$ and $\bar{\pi}(\psi_0)$ is the conjugacy class of $\pi(\psi_0)$ in $\text{Aut}(T)$. $\text{type}(\Lambda)$ is uniquely determined by Λ .

By Theorem 4.1, we can extend the main theorem in [3] as follows.

Theorem 4.3. *Let Λ, Λ' be generalized multifold extensions of piecewise hereditary algebras of tree type. Then the following are equivalent:*

- (i) Λ and Λ' are derived equivalent.
- (ii) Λ and Λ' are stably equivalent.
- (iii) $\text{type}(\Lambda) = \text{type}(\Lambda')$.

Finally we pose a question concerning a refinement of Theorem 4.1.

Question. Under the setting of Theorem 4.1, when are the algebras $\hat{A}/\langle\phi\rangle$ and $T_{\phi_0}^n(A)$ isomorphic?

By Proposition 3.4 this is affirmative if A is hereditary.

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