

Domination polynomial of clique cover product of graphs*

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ABSTRACT. Let G be a simple graph of order n . We prove that the domination polynomial of the clique cover product $G^{\mathcal{C}} \star H^{V(H)}$ is

$$D(G^{\mathcal{C}} \star H, x) = \prod_{i=1}^k \left[((1+x)^{n_i} - 1)(1+x)^{|V(H)|} + D(H, x) \right],$$

where each clique $C_i \in \mathcal{C}$ has n_i vertices. As an application, we study the \mathcal{D} -equivalence classes of some families of graphs and, in particular, describe completely the \mathcal{D} -equivalence classes of friendship graphs constructed by coalescing n copies of a cycle graph of length 3 with a common vertex.

1. Introduction

All graphs in this paper are simple of finite orders, i.e., graphs are undirected with no loops or parallel edges and with finite number of vertices. Graph polynomials are a well-developed area useful for analyzing properties of graphs. Li and Gutman [18] introduced a general graph polynomial. Let f be a complex-valued function defined on the set of graphs G such that $G_1 \sim G_2$ implies $f(G_1) = f(G_2)$. Let G be a graph on n vertices and $S(G)$ be the set of all subgraphs of G . Define $S_k(G) = \{H : H \in$

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$S(G)$ and $|V(H)| = k$, $p(G, k) = \sum_{H \in S_k(G)} f(H)$. Then, the general graph polynomial of G is defined as $P(G, x) = \sum_{k=0}^n p(G, k)x^k$. Non-isomorphic graphs may have the same graph polynomial. Two graphs G and H are said to be P -equivalent, written as $G \sim^P H$, if $P(G, x) = P(H, x)$. The P -equivalence class of G is $[G] = \{H : H \sim^P G\}$. A graph G is said to be P -unique if $[G] = \{G\}$. There are two interesting problems on the equivalence classes:

- (i) Determine the P -equivalence classes for some families of graphs.
- (ii) Which graphs are P -unique?

These problems have been widely studied for the chromatic polynomial (see for example, [9]). The domination polynomial of graph G is the generating function for the number of dominating sets of G , i.e., $D(G, x) = \sum_{i=1}^{|V(G)|} d(G, i)x^i$ (see [2, 5]). Note that the domination polynomial of G is a special case of the general graph polynomial where $f(H) = f_G(H)$ is the indicator function for when $V(H)$ is a dominating set of G . Calculating the domination polynomial of a graph G is difficult in general, as the smallest power of a non-zero term is the domination number $\gamma(G)$ of the graph, and determining whether $\gamma(G) \leq k$ is known to be NP-complete [12]. But for certain classes of graphs, we can find a closed form expression for the domination polynomial.

The equivalence classes of the domination polynomial are called \mathcal{D} -equivalence classes. It is known that cycles [2] and cubic graphs of order 10 [3] (particularly, the Petersen graph) are \mathcal{D} -unique, while if $n \equiv 0 \pmod{3}$, the paths of order n are not [2]. In [7], a necessary and sufficient condition for the complete r -partite graphs to be \mathcal{D} -unique was given. This result in the bipartite case, settles in an affirmative conjecture in [1]. In [15] the \mathcal{D} -equivalence class of barbell graph (and its generalization) was described and it was showed that there are many families of connected graphs in the \mathcal{D} -equivalence class of nK_r , where nK_r is disjoint union of n complete graph K_r .

The join $G + H$ of two graph G and H with disjoint vertex sets $V(G)$ and $V(H)$ and edge sets $E(G)$ and $E(H)$ is the graph union $G \cup H$ together with all the edges joining $V(G)$ and $V(H)$. For two graphs $G = (V, E)$ and $H = (W, F)$, the corona $G \circ H$ is the graph arising from the disjoint union of G with $|V|$ copies of H , by adding edges between the i th vertex of G and all vertices of i th copy of H [11]. It is easy to see that the corona operation of two graphs does not have the commutative property. An induced subgraph $\langle U \rangle$ of G is a graph with the vertex set U and the edge set consists of edges in G which connect vertices in U , if $U \subseteq V(G)$. A

clique is a subset of vertices of an undirected graph such that its induced subgraph is complete; that is, every two distinct vertices in the clique are adjacent.

Zhu in [21] defined an operation of graphs called the clique cover product. Given two graphs G and H , assume that $\mathcal{C} = \{C_1, C_2, \dots, C_k\}$ is a clique cover of G and U is a subset of $V(H)$. Construct a new graph from G , as follows: for each clique $C_i \in \mathcal{C}$, add a copy of the graph H and join every vertex of C_i to every vertex of U . Let $G^{\mathcal{C}} \star H^U$ denote the new graph. In fact, the clique cover product of graphs is a common generalization of some known operations of graphs. For instance: If each clique C_i of the clique cover \mathcal{C} is a vertex, then $G^{V(G)} \star H^{V(H)}$ is the corona of G and H . If we take $H = 2K_1$ and $U = V(2K_1)$, then $G^{\mathcal{C}} \star H^U$ is the graph $\mathcal{C}\{G\}$ obtained by Stevanović [19] using the clique cover construction. It has been proven that the independence polynomial of $G^{\mathcal{C}} \star H^U$ is

$$I(G^{\mathcal{C}} \star H^U) = (I(H, x))^k I\left(G, \frac{xI(H - U, x)}{I(H, x)}\right),$$

and shown that $I(G^{\mathcal{C}} \star H^U)$ is unimodal [21]. We prove that the domination polynomial of the clique cover product $G^{\mathcal{C}} \star H^{V(H)}$ or simply $G^{\mathcal{C}} \star H$ is

$$D(G^{\mathcal{C}} \star H, x) = \prod_{i=1}^k D(K_{n_i} + H, x),$$

where $K_{n_i} = \langle C_i \rangle$.

In the next section, we consider graphs which are constructed from the path P_n by the clique cover construction and study their domination polynomials and \mathcal{D} -equivalence classes. In Section 3, We extend the results of Section 2 and obtain the domination polynomial of clique cover product of graphs, and as some consequences, we determine graphs in the class of some specific k -trees. We completely describe graphs in the class $[F_n]$, where $F_n = K_1 + nK_2$ is a friendship graph, in Section 4.

2. \mathcal{D} -equivalence class of a family of graphs

In this section, we investigate the \mathcal{D} -equivalence classes of a family of graphs. We state the following definition (see [17, 19]).

Definition 2.1. A clique cover of a graph G is a spanning subgraph of G , each component of which is a clique. If $\mathcal{C} = \{C_1, C_2, \dots, C_q\}$ is a clique cover of G , construct a new graph H from G , which is denoted by

$H = \mathcal{C}\{G\}$, as follows: for each clique $C \in \mathcal{C}$, add two new non-adjacent vertices and join them to all the vertices of C . Note that all old edges of G are kept in $\mathcal{C}\{G\}$.

In Figure 1, the set $\mathcal{C} = \{\{v_1, v_2, v_3\}, \{v_4, v_5\}, \{v_6\}\}$ is a clique cover of G that has a clique consisting of one vertex.



FIGURE 1. Graphs G and $H = \mathcal{C}\{G\}$, respectively.

Now, we consider graphs of the form $H_n = \mathcal{C}\{P_n\}$, which are constructed from the path P_n by the clique cover construction. Note that in $H_n = \mathcal{C}\{P_n\}$ (Figure 2), for even n , we take $\mathcal{C} = \{\{1, 2\}, \{3, 4\}, \dots, \{n - 1, n\}\}$, and for odd n , we take $\mathcal{C} = \{\{1\}, \{2, 3\}, \dots, \{n - 3, n - 2\}, \{n - 1, n\}\}$. By H_0 we mean the null graph. We shall study the \mathcal{D} -equivalence class of H_n . To compute the domination polynomial of H_n , we need some preliminaries and well known results.

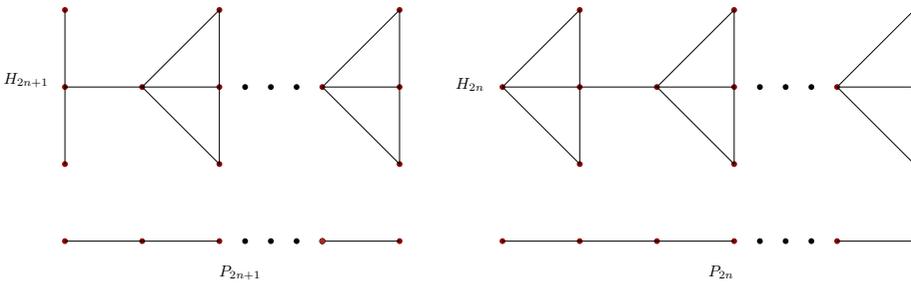


FIGURE 2. Graphs H_{2n+1} and H_{2n} , respectively.

An approach to computing the domination polynomial of a graph is in term of those of its subgraphs. For instance, one can deduce that

$$D(G_1 \cup G_2, x) = D(G_1, x)D(G_2, x),$$

and the following theorem which gives the domination polynomial of join of two graphs.

Theorem 2.2. [2] Let G_1 and G_2 be graphs of orders n_1 and n_2 , respectively. Then

$$D(G_1 + G_2, x) = \left((1 + x)^{n_1} - 1 \right) \left((1 + x)^{n_2} - 1 \right) + D(G_1, x) + D(G_2, x).$$

The following theorem which is for computation of domination polynomial of corona products of two graphs.

Theorem 2.3. [4, 16] Let $G = (V, E)$ and $H = (W, F)$ be nonempty graphs of order n and m , respectively. Then

$$D(G \circ H, x) = (x(1+x)^m + D(H, x))^n.$$

We also denote by $G - e$ the subgraph of G , obtained by deleting an edge e of $E(G)$. An irrelevant edge is an edge $e \in E(G)$, such that $D(G, x) = D(G - e, x)$, and a vertex $v \in V(G)$ is domination-covered, if every dominating set of $G - v$ includes at least one vertex adjacent to v in G [16]. The following theorem gives a necessary and sufficient condition for a vertex to be a domination-covered vertex.

Theorem 2.4. [16] Let $G = (V, E)$ be a graph. A vertex $v \in V$ is domination-covered if and only if there is a vertex $u \in N[v]$ such that $N[u] \subseteq N[v]$.

Using Theorem 2.4 we are able to determine an irrelevant edge.

Theorem 2.5. [16] Let $G = (V, E)$ be a graph. An edge $e = \{u, v\} \in E$ is an irrelevant edge in G , if and only if u and v are domination-covered in $G - e$.

Now, we are ready to use Theorem 2.5 to obtain the domination polynomials of H_n :

Theorem 2.6. Let H_n be the graphs in the Figure 2.

- (i) For every $n \in \mathbb{N}$, $D(H_{2n}, x) = (x^4 + 4x^3 + 6x^2 + 2x)^n$.
- (ii) For every $n \in \mathbb{N}$, $D(H_{2n+1}, x) = (x^3 + 3x^2 + x)(x^4 + 4x^3 + 6x^2 + 2x)^n$.

Proof. (i) Let $G = K_1 + P_3$ be a graph of order 4 and e_1, \dots, e_n be the edges with end-vertices of degree 4, whose connect each two G in H_{2n} . By Theorem 2.4 two end-vertices of every edge e_i are domination-covered in H_{2n} , and so by Theorem 2.5 every edge e_i is an irrelevant edge of H_{2n} . Since $D(G, x) = x^4 + 4x^3 + 6x^2 + 2x$, using induction we have

$$D(H_{2n}, x) = (x^4 + 4x^3 + 6x^2 + 2x)^n.$$

(ii) Let e be an edge joining H_{2n} and P_3 in H_{2n+1} . By Theorem 2.4 two end-vertices of edge e are domination-covered in H_{2n+1} . So, by Theorem 2.5 the edge e is an irrelevant edge of H_{2n+1} . Therefore $D(H_{2n+1}, x) = D(P_3 \cup H_{2n}, x)$ and by Part (i) we have the result. \square

As an immediate consequence of Theorem 2.6, we see that for each natural number n , the graphs H_{2n} and H_{2n+1} are not \mathcal{D} -unique. We discuss more in Theorem 2.8. The following lemma present many graphs in the class $[H_{2n}]$:

Lemma 2.7. Let G be a graph of order n . The graphs of the form $G \circ P_3$ and H_{2n} have the same domination polynomial.

Proof. By Theorem 2.3 we can deduce that for each arbitrary graph G ,

$$\begin{aligned} D(G \circ P_3, x) &= \left(x(1+x)^3 + D(P_3, x)\right)^{|V(G)|} \\ &= (x(1+x)^3 + x^3 + 3x^2 + x)^n \\ &= \left(x(x^3 + 4x^2 + 6x + 2)\right)^n \\ &= D(H_{2n}, x). \end{aligned} \quad \square$$

One can see that graphs constructed by clique cover of a graph G , may not be isomorphic. As instance, two non-isomorphic graphs G_1 and G_2 , depicted in Figure 3, are obtained by different clique covers of P_5 , namely $\mathcal{C}_1 = \{\{1, 2\}, \{3\}, \{4, 5\}\}$ and $\mathcal{C}_2 = \{\{1\}, \{2, 3\}, \{4, 5\}\}$. We notice that the domination polynomials of these two graphs are the same, i.e.,

$$D(G_1, x) = D(G_2, x) = (x^4 + 4x^3 + 6x^2 + 2x)^2(x^3 + 3x^2 + x).$$

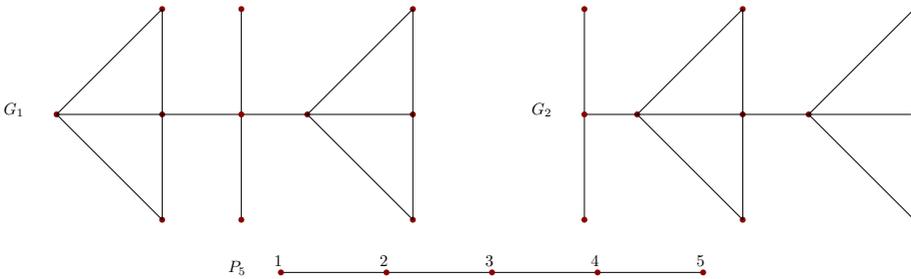


FIGURE 3. The graphs $G_1 = \mathcal{C}_1\{P_5\}$ and $G_2 = \mathcal{C}_2\{P_5\}$, respectively.

Finally, we present some other families of graphs whose are in the \mathcal{D} -equivalence classes of H_n graphs in Figure 2.

Theorem 2.8. (i) Let e_1, \dots, e_n be the edges with end-vertices of degree four in the graph H_{2n} . For every $1 \leq i \leq n$, the disconnected graphs obtained from deletion of any number of the edges e_i and the graph H_{2n}

have the same domination polynomial. As well as all graphs obtained by adding each number of the edges between every two vertices of degree four, and adding the edges between two vertices of degree three and four in the graph H_{2n} .

(ii) Let \mathcal{C} be a clique cover of P_{2n+1} that has a clique consisting of one vertex. All non-isomorphic graphs $\mathcal{C}\{P_{2n+1}\}$ and the graph H_{2n+1} have the same domination polynomial. As well as the union of the graph P_3 and every graph in $[H_{2n}]$.

Proof. (i) Similar to the proof of Theorem 2.6, for every $1 \leq i \leq n$, the edge e_i is an irrelevant edge of H_{2n} , that is for every $1 \leq i \leq n$, $D(H_{2n}, x) = D(H_{2n} - e_i, x)$, and so we have the first results. Remains to show that each added edge with the mentioned conditions in this graph is an irrelevant edge. This can be achieved by using Theorems 2.4 and 2.5.

(ii) Similar to the proof of Part (ii) of Theorem 2.6 we have the result. □

3. Domination polynomial of clique cover product

The domination polynomials of binary graph operations, such as, join and corona has been computed [4]. Also, recently, recurrence formulae and properties of the domination polynomials of families of graphs obtained by various products, has been investigated [7]. In this section, we generalize the results in Section 2 and consider the clique cover product and formulate the domination polynomial for the clique cover product $G^{\mathcal{C}} \star H^{V(H)}$ or simply $G^{\mathcal{C}} \star H$. The following theorem gives the domination polynomial of $G^{\mathcal{C}} \star H^U$.

Theorem 3.1. For two graphs G and H , let $\mathcal{C} = \{C_1, C_2, \dots, C_k\}$ be a clique cover of G and $U \subseteq V(H)$. Then

$$D(G^{\mathcal{C}} \star H^U, x) = \prod_{i=1}^k D(H^*, x),$$

where H^* is the subgraph of order $|V(H)| + |C_i|$ in $G^{\mathcal{C}} \star H^U$ obtained by adding a copy of the graph H and joining every vertex of C_i to every vertex of U . Moreover,

$$D(G^{\mathcal{C}} \star H, x) = \prod_{i=1}^k \left[((1+x)^{n_i} - 1)(1+x)^{|V(H)|} + D(H, x) \right],$$

where n_i is the order of C_i .

Proof. Since every vertex of C_i is dominated by every vertex in U of H thus by Theorem 2.4 two end-vertices of all edges which connect each two C_i in $G^{\mathcal{C}} \star H^U$ are domination-covered in new graph, and so by Theorem 2.5 every edge e_i is an irrelevant edge of $G^{\mathcal{C}} \star H^U$. Therefore by definition of an irrelevant edge and induction, we have the result. Now, suppose that $U = V(H)$. Thus we can deduce that $D(G^{\mathcal{C}} \star H, x) = \prod_{i=1}^k D(K_{n_i} + H, x)$. Note that in the complete graph K_n , any nonempty set of vertices is a dominating set, so it follows that $D(K_n, x) = (1 + x)^n - 1$. Therefore by Theorem 2.2 have the result. \square

Remark 3.2. If each clique C_i of the clique cover \mathcal{C} is a vertex, then $G^{V(G)} \star H$ is the corona of G and H . So the clique cover product of graphs is a generalization of corona product and hence by Theorem 3.1 we have

$$\begin{aligned} D(G \circ H, x) &= D(G^{V(G)} \star H) \\ &= \prod_1^n [((1 + x)^1 - 1)(1 + x)^m + D(H, x)] \\ &= \left(x(1 + x)^m + D(H, x)\right)^n, \end{aligned}$$

which is another approach for proof of Theorem 2.3.

Here we shall apply Theorem 3.1 to get the following results on \mathcal{D} -equivalence class of some graphs.

Corollary 3.3. (i) Let G and H be two graphs and \mathcal{C} and \mathcal{C}' be two clique covers of G . If $|\mathcal{C}| = |\mathcal{C}'| = k$, and $|\mathcal{C}_i| = |\mathcal{C}'_i|$ for $1 \leq i \leq k$, then the graphs $G^{\mathcal{C}} \star H$ and $G^{\mathcal{C}'} \star H$ have the same domination polynomial, i.e.,

$$D(G^{\mathcal{C}} \star H, x) = D(G^{\mathcal{C}'} \star H, x) = \prod_{i=1}^k D(K_{n_i} + H, x),$$

where $K_{n_i} = \langle C_i \rangle$.

(ii) Given two graphs G and H , assume that \mathcal{C} is a clique cover of G . The graph $G^{\mathcal{C}} \star H$ and all graphs obtained from deletion of any number of the edges between two cliques in the graph $G^{\mathcal{C}} \star H$ have the same domination polynomial. As well as all graphs obtained by adding each number of the edges between every two cliques of \mathcal{C} in the graph $G^{\mathcal{C}} \star H$.

As an example of application of clique cover for determining the \mathcal{D} -equivalence classes of some graphs, we determine some graphs in the class of k -stars. Let us to recall some preliminaries. The class of k -trees is a very

important subclass of triangulated graphs. Harary and Palmer [13] first introduced 2-trees in 1968. Beineke and Pippert [8] gave the definition of a k -tree in 1969.

Definition 3.4. For a positive integer k , a k -tree, denoted by T_n^k , is defined recursively as follows: The smallest k -tree is the k -clique K_k . If G is a k -tree with $n \geq k$ vertices and a new vertex v of degree k is added and joined to the vertices of a k -clique in G , then the larger graph is a k -tree with $n + 1$ vertices.

A k -star, $S_{k,n-k}$, has the vertex set $\{v_1, \dots, v_n\}$, where $\langle \{v_1, \dots, v_k\} \rangle \cong K_k$ and $N(v_i) = \{v_1, \dots, v_k\}$ for $k + 1 \leq i \leq n$. The authors in [14] calculated the domination polynomials for some k -tree related graphs, specially k -star graph and investigated domination roots of this graph.

Theorem 3.5. [14] For every $k \in \mathbb{N}$ and $n > k$,

$$D(S_{k,n-k}, x) = (1 + x)^{n-k}((1 + x)^k - 1) + x^{n-k}.$$

Here, we present graphs whose domination polynomials are

$$\prod_{i=1}^m D(S_{k_i, n_i - k_i}, x).$$

Theorem 3.6. Let G be a graph and $\mathcal{C} = \{C_1, C_2, \dots, C_m : |C_i| = k_i\}$ is a clique cover of G . If H is an empty graph, then the graphs $G^{\mathcal{C}} \star H$ and disjoint union of m , k_i - star have the same domination polynomial.

Proof. Since $S_{k,n-k} = K_k + S$, where S is an empty graph, by applying Theorem 3.1, we have the result. \square

The following lemma present many graphs in $[mS_{k,n-k}]$ whose domination polynomials are $D(S_{k,n-k}, x)^m$.

Lemma 3.7. (i) Let G be a graph of order m , then the graphs $G^{V(G)} \star S_{k-1, n-k+1} = G \circ S_{k-1, n-k+1}$ and $mS_{k,n-k}$ have the same domination polynomial.

(ii) Let G be a graph and $\mathcal{C} = \{C_1, C_2, \dots, C_m : |C_i| = k\}$ is a clique cover of G . If H is an empty graph of order $n - k$, then the graphs $G^{\mathcal{C}} \star H$ and all graphs obtained from deletion of any number of the edges joining C_i and C_j of \mathcal{C} in the graph $G^{\mathcal{C}} \star H$ have the same domination polynomial. As well as all graphs obtained by adding each number of the edges between C_i and C_j of \mathcal{C} in the graph $G^{\mathcal{C}} \star H$.

4. \mathcal{D} -equivalence classes of friendship graphs

The friendship (or Dutch-Windmill) graph F_n is a graph that can be constructed by the coalescence of n copies of the cycle graph C_3 of length 3 with a common vertex. The Friendship Theorem of Paul Erdős, Alfred Rényi and Vera T. Sós [10], states that graphs with the property that every two vertices have exactly one neighbour in common are exactly the friendship graphs. Figure 5 shows some examples of friendship graphs. The nature and location of domination roots of friendship graphs have been studied in [6] and shown that for every $n \geq 3$, F_n is not \mathcal{D} -unique. The authors considered the n -book graph B_n which can be constructed by bonding n copies of the cycle graph C_4 along a common edge $\{u, v\}$, see Figure 4. The vertex contraction G/u of a graph G by a vertex u is the

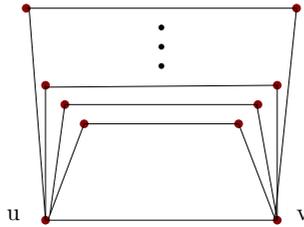


FIGURE 4. The book graph B_n .

operation under which all vertices in $N(u)$ are joined to each other and then u is deleted (see[20]). The following result proves that the friendship graph F_n is not \mathcal{D} -unique.

Theorem 4.1. [6] For each natural number $n \geq 3$, the friendship graph F_n is not \mathcal{D} -unique, as F_n and B_n/v have the same domination polynomial.

In this section, we describe $[F_n]$ completely. Since $F_n = K_1^{V(K_1)} \star (nK_2) = K_1 + nK_2$, we have the following theorem.

Theorem 4.2. [6] For every $n \in \mathbb{N}$, $D(F_n, x) = (2x + x^2)^n + x(1 + x)^{2n}$.

We shall extend Theorem 4.1 and present all families of graphs whose are in the $[F_n]$. The following theorem gives us the domination polynomial of graphs of the form $H \circ K_1$ which is the first result for domination polynomial of specific corona of two graphs and we need it to obtain our result.

Theorem 4.3. [2] $D(G, x) = x^n(x + 2)^n$ if and only if $G = H \circ K_1$ for some graph H of order n .

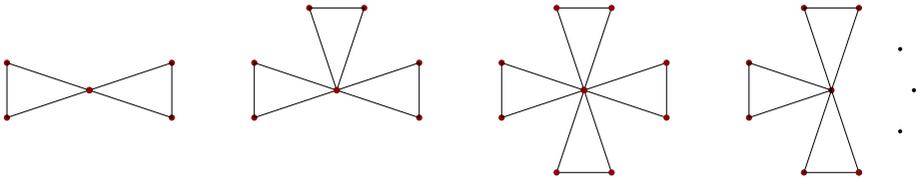


FIGURE 5. Friendship graphs F_2, F_3, F_4 and F_n , respectively.

As shown in [7], the following corollary is a consequence of Theorem 2.2.

Corollary 4.4. For graphs G_1, G_2 , and H , $D(G_1 + H, x) = D(G_2 + H, x)$ if and only if $D(G_1, x) = D(G_2, x)$.

The following theorem gives the \mathcal{D} -equivalence classes of $[F_n]$:

Theorem 4.5. Let G be a graph of order n . Then

$$[F_n] = \{(G \circ K_1) + K_1 : |G| = n\}.$$

Proof. Since $F_n = nK_2 + K_1$, by Corollary 4.4 to obtain $[F_n]$, it suffices to find $[nK_2]$. Using Theorem 4.3 we have

$$[nK_2] = \{(G \circ K_1) : |G| = n\}.$$

So we have the result. □

The graph B_n/v which has found in Theorem 4.1 is in the form $(K_n \circ K_1) + K_1$. And F_n is one of the graphs in form $(G \circ K_1) + K_1$ where G is empty graph of order n . If $n = 1$ then $F_1 = K_3$ and by [2, Corollary 2] the complete graphs K_n are \mathcal{D} -unique for every natural number n .

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