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RESEARCH ARTICLE

# Quasi-valuation maps based on positive implicative ideals in BCK-algebras

**Young Bae Jun, Kyoung Ja Lee and Seok Zun Song**

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Abstract. The notion of PI-quasi-valuation maps of a BCK-algebra is introduced, and related properties are investigated. The relationship between an I-quasi-valuation map and a PI-quasi-valuation map is examined. Conditions for an I-quasi-valuation map to be a PI-quasi-valuation map are provided, and conditions for a real-valued function on a BCK-algebra to be a quasi-valuation map based on a positive implicative ideal are founded. The extension property for a PI-quasi-valuation map is established.

## 1. Introduction

Logic appears in a 'sacred' form (resp., a 'profane') which is dominant in proof theory (resp., model theory). The role of logic in mathematics and computer science is twofold; as a tool for applications in both areas, and a technique for laying the foundations. Non-classical logic including many-valued logic, fuzzy logic, etc., takes the advantage of the classical logic to handle information with various facets of uncertainty (see [11] for generalized theory of uncertainty), such as fuzziness, randomness, and so on. Non-classical logic has become a formal and useful tool for computer science to deal with fuzzy information and uncertain information. Among all kinds of uncertainties, incomparability is an important one which can

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be encountered in our life. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki (see [2–5]) and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Neggers and Kim [10] introduced the notion of $d$-algebras which is another useful generalization of BCK-algebras, and then they investigated several relations between $d$-algebras and BCK-algebras as well as some other interesting relations between $d$-algebras and oriented diagraphs. In [9], Neggers et al. discussed the ideal theory in $d$-algebras. Neggers et al. [8] introduced the concept of $d$-fuzzy function which generalizes the concept of fuzzy subalgebra to a much larger class of functions in a natural way. In addition they discussed a method of fuzzification of a wide class of algebraic systems onto $[0, 1]$ along with some consequences. In [6], Jun et al. introduced the notion of quasi-valuation maps based on a subalgebra and an ideal in BCK/BCI-algebras, and then they investigated several properties. They provided relations between a quasi-valuation map based on a subalgebra and a quasi-valuation map based on an ideal. In a BCI-algebra, they gave a condition for a quasi-valuation map based on an ideal to be a quasi-valuation map based on a subalgebra, and found conditions for a real-valued function on a BCK/BCI-algebra to be a quasi-valuation map based on an ideal. Using the notion of a quasi-valuation map based on an ideal, they constructed (pseudo) metric spaces, and showed that the binary operation $*$ in BCK-algebras is uniformly continuous. In this paper, we introduce the notion of PI-quasi-valuation maps of a BCK-algebra, and investigate related properties. We discuss the relationship between an I-quasi-valuation map and a PI-quasi-valuation map. We provide conditions for an I-quasi-valuation map to be a PI-quasi-valuation map, and find conditions for a real-valued function on a BCK-algebra to be a quasi-valuation map based on a positive implicative ideal. We finally establish an extension property for a PI-quasi-valuation map.

## 2. Preliminaries

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following axioms:

(I) $(\forall x, y, z \in X)\ (((x * y) * (x * z)) * (z * y) = 0)$,
(II) $(\forall x, y \in X)\ ((x * (x * y)) * y = 0)$,
(III) $(\forall x \in X)\ (x * x = 0)$,
(IV) $(\forall x, y \in X)\ (x * y = 0, y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra $X$ satisfies the following identity:

(V) $(\forall x \in X)\ (0 * x = 0)$,
then $X$ is called a *BCK-algebra*. Any BCK/BCI-algebra $X$ satisfies the following conditions:
(a1) $(\forall x \in X)\ (x * 0 = x)$,
(a2) $(\forall x, y, z \in X)\ (x * y = 0 \ \Rightarrow\ (x * z) * (y * z) = 0,\ (z * y) * (z * x) = 0)$,
(a3) $(\forall x, y, z \in X)\ ((x * y) * z = (x * z) * y)$,
(a4) $(\forall x, y, z \in X)\ (((x * z) * (y * z)) * (x * y) = 0)$.

We can define a partial ordering $\leqslant$ by $x \leqslant y$ if and only if $x * y = 0$. A subset $A$ of a BCK/BCI-algebra $X$ is called an *ideal* of $X$ if it satisfies the following conditions:
(b1) $0 \in A$,
(b2) $(\forall x, y \in X)\ (x * y \in A,\ y \in A \ \Rightarrow\ x \in A)$.

A subset $A$ of a BCK-algebra $X$ is called a *positive implicative ideal* of $X$ if it satisfies (b1) and
(b3) $(\forall x, y, z \in X)\ ((x * y) * z \in A,\ y * z \in A \ \Rightarrow\ x * z \in A)$.

**Proposition 2.1.** [7] *For a subset $A$ of a BCK-algebra $X$, the following are equivalent:*
(1) *$A$ is a positive implicative ideal of $X$.*
(2) *$A$ is an ideal, and for any $x, y \in X$, $(x * y) * y \in A$ implies $x * y \in A$.*

We refer the reader to the books [1,7] for further information regarding BCK/BCI-algebras.

## 3. Quasi-valuation maps based on a positive implicative ideal

**Definition 3.1** ([6]). Let $X$ be a BCK/BCI-algebra. By a *quasi-valuation map* of $X$ based on a subalgebra (briefly *$S$-quasi-valuation map* of $X$), we mean a mapping $f : X \to \mathbb{R}$ which satisfies the following condition:

$$(\forall x, y \in X)\ (f(x * y) \geqslant f(x) + f(y)). \tag{3.1}$$

**Proposition 3.2** ([6]). *For any $S$-quasi-valuation map $f$ of a BCK-algebra $X$, we have*
(c1) $(\forall x \in X)\ (f(x) \leqslant 0)$.

For any real-valued function $f$ on a BCK/BCI-algebra $X$, we consider the following conditions:
(c2) $f(0) = 0$.
(c3) $f(x) \geqslant f(x * y) + f(y)$ for all $x, y \in X$.

(c4) $f(x*y) \geqslant f(((x*y)*y)*z) + f(z)$ for all $x, y, z \in X$.
(c5) $f(x*z) \geqslant f((x*y)*z) + f(y*z)$ for all $x, y, z \in X$.
(c6) $f(x*y) \geqslant f((x*y)*y)$ for all $x, y \in X$.
(c7) $f((x*z)*(y*z) \geqslant f((x*y)*z)$ for all $x, y, z \in X$.

**Definition 3.3** ([6])**.** Let $X$ be a BCK/BCI-algebra. By a *quasi-valuation map* of $X$ based on an ideal (briefly *I-quasi-valuation map* of $X$), we mean a mapping $f : X \to \mathbb{R}$ which satisfies the conditions (c2) and (c3).

**Definition 3.4.** Let $X$ be a BCK-algebra. By a *quasi-valuation map* on $X$ based on a positive implicative ideal (briefly *PI-quasi-valuation map* of $X$), we mean a mapping $f : X \to \mathbb{R}$ which satisfies the conditions (c2) and (c5).

**Example 3.5.** Let $X = \{0, a, b\}$ be a BCK-algebra with the $*$-operation given by Table 1.

TABLE 1. $*$-operation.

| $*$ | 0 | $a$ | $b$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| $a$ | $a$ | 0 | 0 |
| $b$ | $b$ | $b$ | 0 |

Let $f$ be a real-valued function on $X$ defined by

$$f = \begin{pmatrix} 0 & a & b \\ 0 & 0 & -2 \end{pmatrix}.$$

Then $f$ is a PI-quasi-valuation map of $X$.

**Example 3.6.** Let $X = \{0, a, b, c\}$ be a BCK-algebra with the $*$-operation given by Table 2.

TABLE 2. $*$-operation.

| $*$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| $a$ | $a$ | 0 | 0 | $a$ |
| $b$ | $b$ | $a$ | 0 | $b$ |
| $c$ | $c$ | $c$ | $c$ | 0 |

Let $f$ be a real-valued function on $X$ defined by

$$f = \begin{pmatrix} 0 & a & b & c \\ 0 & 0 & 0 & -7 \end{pmatrix}.$$

Then $f$ is a PI-quasi-valuation map of $X$.

**Theorem 3.7.** *Let $X$ be a BCK-algebra. Every PI-quasi-valuation map of $X$ is an I-quasi-valuation map of $X$.*

*Proof.* Let $f : X \to \mathbb{R}$ be a PI-quasi-valuation map on a BCK-algebra $X$. If we take $z = 0$ in (c5) and use (a1), then we have the condition (c3). Hence $f$ is an I-quasi-valuation map of $X$. □

The converse of Theorem 3.7 may not be true as shown by the following example.

**Example 3.8.** Let $X = \{0, a, b, c\}$ be a BCK-algebra with the $*$-operation given by Table 2 and let $g$ be a real-valued function on $X$ defined by

$$g = \begin{pmatrix} 0 & a & b & c \\ 0 & -2 & -3 & 0 \end{pmatrix}.$$

Then $g$ is an I-quasi-valuation map of $X$, but not a PI-quasi-valuation map of $X$ since $g(b * a) = -2 < 0 = g((b * a) * a) + g(a * a)$.

**Example 3.9.** Let $X = \{0, a, b, c\}$ be a BCK-algebra with the $*$-operation given by Table 3.

TABLE 3. $*$-operation

| $*$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $0$ |
| $a$ | $a$ | $0$ | $0$ | $0$ |
| $b$ | $b$ | $b$ | $0$ | $0$ |
| $c$ | $c$ | $c$ | $b$ | $0$ |

Let $f$ be a real-valued function on $X$ defined by

$$f = \begin{pmatrix} 0 & a & b & c \\ 0 & 0 & -3 & -4 \end{pmatrix}.$$

Then $f$ is an I-quasi-valuation map of $X$, but not a PI-quasi-valuation map of $X$ since $f(c * b) = -3 < 0 = f((c * b) * b) + f(b * b)$.

We give conditions for an I-quasi-valuation map to be a PI-quasi-valuation map. We first consider the following lemma.

**Lemma 3.10.** [6] *For any I-quasi-valuation map $f$ of $X$, we have the following assertions:*

(1) *$f$ is order reversing.*
(2) *$f(x * y) + f(y * x) \leqslant 0$ for all $x, y \in X$.*
(3) *$f(x * y) \geqslant f(x * z) + f(z * y)$ for all $x, y, z \in X$.*

**Theorem 3.11.** *Let $f$ be an I-quasi-valuation map of a BCK-algebra $X$. If $f$ satisfies the condition* (c6)*, then $f$ is a PI-quasi-valuation map of $X$.*

*Proof.* Let $f$ be an I-quasi-valuation map of $X$ which satisfies the condition (c6). Notice that $((x * z) * z) * (y * z) \leqslant (x * z) * y = (x * y) * z$ for all $x, y, z \in X$. Since $f$ is order reversing, it follows that

$$f(((x * z) * z) * (y * z)) \geqslant f((x * y) * z)$$

so from (c6) and (c3) that

$$\begin{aligned} f(x * z) \geqslant f((x * z) * z) &\geqslant f(((x * z) * z) * (y * z)) + f(y * z) \\ &\geqslant f((x * y) * z) + f(y * z). \end{aligned}$$

Therefore $f$ is a PI-quasi-valuation map of $X$. □

For any function $f : X \to \mathbb{R}$, consider the following set:

$$I_f := \{x \in X \mid f(x) = 0\}.$$

**Lemma 3.12.** [6] *Let $X$ be a BCK-algebra. If $f$ is an I-quasi-valuation map of $X$, then the set $I_f$ is an ideal of $X$.*

**Lemma 3.13.** [6] *In a BCK-algebra, every I-quasi-valuation map is an S-quasi-valuation map.*

**Lemma 3.14.** *Every PI-quasi-valuation map $f$ of a BCK-algebra $X$ satisfies the condition* (c6)*.*

*Proof.* Let $f$ be a PI-quasi-valuation map of $X$. Then $f$ is an I-quasi-valuation map of $X$ by Theorem 3.7. If we take $z = y$ in (c5), then $f(x * y) \geqslant f((x * y) * y) + f(y * y) = f((x * y) * y) + f(0) = f((x * y) * y)$ for all $x, y \in X$. Thus the condition (c6) is valid. □

**Theorem 3.15.** *Let $X$ be a BCK-algebra. If $f$ is a PI-quasi-valuation map of $X$, then the set $I_f$ is a positive implicative ideal of $X$.*

*Proof.* Suppose $f$ is a PI-quasi-valuation map of $X$. Then $f$ is an I-quasi-valuation map of $X$ by Theorem 3.7, and so $I_f$ is an ideal of $X$ by Lemma 3.12. Let $x, y \in X$ be such that $(x * y) * y \in I_f$. Then $f((x * y) * y) = 0$ and so $f(x * y) \geqslant f((x * y) * y) = 0$ by Lemma 3.14. Using Lemma 3.13 and Proposition 3.2, we get $f(x) \leqslant 0$ for all $x \in X$. Thus $f(x * y) = 0$ which means that $x * y \in I_f$. Thus, by Proposition 2.1, we conclude that $I_f$ is a positive implicative ideal of $X$. □

The following examples show that the converse of Theorem 3.15 may not be true, that is, there exist a BCK-algebra $X$ and a function $f : X \to \mathbb{R}$ such that

(1) $f$ is not a PI-quasi-valuation map of $X$,
(2) $I_f$ is a positive implicative ideal of $X$.

**Example 3.16.** Let $X = \{0, a, b, c, d\}$ be a BCK-algebra with the $*$-operation given by Table 4.

TABLE 4. $*$-operation.

| $*$ | $0$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $a$ | $a$ | $0$ | $a$ | $0$ | $a$ |
| $b$ | $b$ | $b$ | $0$ | $b$ | $0$ |
| $c$ | $c$ | $a$ | $c$ | $0$ | $c$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $0$ |

Let $g$ be a real-valued function on $X$ defined by

$$g = \begin{pmatrix} 0 & a & b & c & d \\ 0 & 0 & -8 & 0 & -6 \end{pmatrix}.$$

Then $I_g = \{0, a, c\}$ is a positive implicative ideal of $X$. But $g$ is not a PI-quasi-valuation map of $X$ since $g(b * c) = g(b) = -8 \ngeqslant -6 = g((b * d) * c) + g(d * c)$.

**Proposition 3.17.** *Let $X$ be a BCK-algebra. Then every PI-quasi-valuation map $f$ of $X$ satisfies the condition* (c7).

*Proof.* Let $f$ be a PI-quasi-valuation map of $X$. Then $f$ satisfies the condition (c6) (see Lemma 3.14) and $f$ is an I-quasi-valuation map $f$ of $X$ (see Theorem 3.7). It follows from [6, Proposition 3.13] that $f$ satisfies the condition (c7). □

Notice that an I-quasi-valuation map $f$ of a BCK-algebra $X$ does not satisfy the condition (c7). In fact, consider a BCK-algebra $X = \{0, a, b, c\}$ in which the $*$-operation is given by the Table 5.

TABLE 5. $*$-operation.

| $*$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $0$ |
| $a$ | $a$ | $0$ | $0$ | $a$ |
| $b$ | $b$ | $a$ | $0$ | $b$ |
| $c$ | $c$ | $c$ | $c$ | $0$ |

Let $f$ be a real-valued function on $X$ defined by

$$f = \begin{pmatrix} 0 & a & b & c \\ 0 & -3 & -3 & -8 \end{pmatrix}.$$

Then $f$ is an I-quasi-valuation map of $X$. Since

$$f((b * a) * (a * a)) = f(a * 0) = f(a) = -3 < 0 = f((b * a) * a),$$

$f$ does not satisfy the condition (c7).

**Theorem 3.18.** *Let $X$ be a BCK-algebra. If an I-quasi-valuation map $f$ of $X$ satisfies the condition* (c7)*, then it is a PI-quasi-valuation map of $X$.*

*Proof.* Let $f$ be an I-quasi-valuation map of $X$ which satisfies the condition (c7). For any $x, y, z \in X$, we have

$$f(x * z) \geqslant f((x * z) * (y * z)) + f(y * z) \geqslant f((x * y) * z) + f(y * z)$$

by (c3) and (c7). Therefore $f$ is a PI-quasi-valuation map of $X$. □

**Theorem 3.19.** *Let $f$ be a real-valued function on a BCK-algebra $X$. If $f$ satisfies conditions* (c2) *and* (c4)*, then $f$ is a PI-quasi-valuation map of $X$.*

*Proof.* Assume that $f$ satisfies conditions (c2) and (c4). Then

$$f(x) = f(x * 0) \geqslant f(((x * 0) * 0) * z) + f(z) = f(x * z) + f(z)$$

for all $x, z \in X$. Hence $f$ is an I-quasi-valuation map of $X$. Taking $z = 0$ in (c4) and using (a1) and (c2), we have

$$f(x * y) \geqslant f(((x * y) * y) * 0) + f(0) = f((x * y) * y)$$

for all $x, y \in X$. It follows from Theorem 3.11 that $f$ is a PI-quasi-valuation map of $X$. □

**Proposition 3.20.** *Every PI-quasi-valuation map $f$ of a BCK-algebra $X$ satisfies the following implication for all $x, y, a, b \in X$:*

$$(((x * y) * y) * a) * b = 0 \;\Rightarrow\; f(x * y) \geqslant f(a) + f(b). \tag{3.2}$$

*Proof.* Note that $f$ is an I-quasi-valuation map of $X$ by Theorem 3.7. Assume that $(((x * y) * y) * a) * b = 0$ for all $x, y, a, b \in X$. Using [6, Proposition 3.14], we have $f((x * y) * y) \geqslant f(a) + f(b)$. It follows from (III), (a1) and (c7) that

$$f(x*y) = f((x*y)*0) = f((x*y)*(y*y)) \geqslant f((x*y)*y) \geqslant f(a)+f(b).$$

This completes the proof. □

**Lemma 3.21.** [6, Theorem 3.16] *If a real-valued function $f$ on $X$ satisfies the conditions* (c2) *and*

$$(\forall x, y, z \in X)\ ((x * y) * z = 0 \;\Rightarrow\; f(x) \geqslant f(y) + f(z)), \tag{3.3}$$

*then $f$ is an I-quasi-valuation map of $X$.*

**Theorem 3.22.** *Let $f$ be a real-valued function on a BCK-algebra $X$. If $f$ satisfies conditions* (c2) *and* (3.2), *then $f$ is a PI-quasi-valuation map of $X$.*

*Proof.* Let $x, y, z \in X$ be such that $(x * y) * z = 0$. Then

$$(((x * 0) * 0) * y) * z = 0.$$

It follows from (a1) and (3.2) that $f(x) = f(x * 0) \geqslant f(y) + f(z)$. Thus $f$ is an I-quasi-valuation map of $X$ by Lemma 3.21. Since

$$(((x * y) * y) * ((x * y) * y)) * 0 = 0$$

for all $x, y \in X$, we have $f(x * y) \geqslant f((x * y) * y) + f(0) = f((x * y) * y)$ by (3.2) and (c2). Therefore, by Theorem 3.11, $f$ is a PI-quasi-valuation map of $X$. □

**Proposition 3.23.** *Every PI-quasi-valuation map of a BCK-algebra $X$ satisfies the following implication for all $x, y, z, a, b \in X$:*

$$(((x * y) * z) * a) * b = 0 \;\Rightarrow\; f((x * z) * (y * z)) \geqslant f(a) + f(b). \tag{3.4}$$

*Proof.* Let $x, y, z, a, b \in X$ be such that $(((x * y) * z) * a) * b = 0$. Using Propositions 3.17, Theorem 3.7 and [6, Proposition 3.14], we have

$$f((x * z) * (y * z)) \geqslant f((x * y) * z) \geqslant f(a) + f(b)$$

which is the desired result. □

**Theorem 3.24.** *Let $X$ be a BCK-algebra. If a real-valued function $f$ on $X$ satisfies two conditions* (c2) *and* (3.4), *then $f$ is a PI-quasi-valuation map of $X$.*

*Proof.* Let $x, y, a, b \in X$ be such that $(((x * y) * y) * a) * b = 0$. Using (a1), (III) and (3.4), we have

$$f(x * y) = f((x * y) * 0) = f((x * y) * (y * y)) \geqslant f(a) + f(b).$$

It follows from Theorem 3.22 that $f$ is a PI-quasi-valuation map of $X$. □

**Theorem 3.25.** (Extension Property) *Let $f$ and $g$ be $I$-quasi-valuation maps of a BCK-algebra $X$ such that $f(x) \geqslant g(x)$ for all $x \in X$. If $g$ is a PI-quasi-valuation map of $X$, then so is $f$.*

*Proof.* Let $x, y, z \in X$. Using (a3), Proposition 3.17, (III) and (c2), we have

$$\begin{aligned}
&f(((x * z) * (y * z)) * ((x * y) * z)) \\
&\quad = f(((x * z) * ((x * y) * z)) * (y * z)) \\
&\quad = f(((x * ((x * y) * z)) * z) * (y * z)) \\
&\quad \geqslant g(((x * ((x * y) * z)) * z) * (y * z)) \\
&\quad \geqslant g(((x * ((x * y) * z)) * y) * z) \\
&\quad = g(((x * y) * ((x * y) * z)) * z) \\
&\quad = g(((x * y) * z) * ((x * y) * z)) \\
&\quad = g(0) = 0.
\end{aligned}$$

It follows from (c3) that

$$\begin{aligned}
f((x * z) * (y * z)) &\geqslant f(((x * z) * (y * z)) * ((x * y) * z)) + f((x * y) * z) \\
&= f((x * y) * z).
\end{aligned}$$

So from Theorem 3.18 we have that $f$ is a PI-quasi-valuation map of $X$. □

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### Contact information

**Young Bae Jun** Department of Mathematics Education,
Gyeongsang National University, Jinju 52828,
Korea
*E-Mail(s)*: `skywine@gmail.com`

**Kyoung Ja Lee** Department of Mathematics Education,
Hannam University, Daejeon 34430, Korea
*E-Mail(s)*: `lsj1109@hotmail.com`

**Seok Zun Song** Department of Mathematics,
Jeju National University, Jeju 63243, Korea
*E-Mail(s)*: `szsong@cheju.ac.kr`

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