

80th birthday of Günter Pilz

Erhard Aichinger*



On 19 March 2025, Professor Günter Pilz, one of the editors of “Algebra and Discrete Mathematics”, celebrated his 80th birthday. To his honour, the research seminar of the Institute of Algebra at the Johannes Kepler University Linz, which he headed until his retirement in 2013, organized a lecture on his main scientific topic “Near-rings”. A *near-ring* is an algebraic structure in which only one distributive law holds. While the endomorphisms of an abelian group with the composition of functions as multiplication form a ring, the set of all functions on a group forms a near-ring. Hence near-rings model non-linear functions, or, using

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the words of Günter Pilz, “if you try to non-linearize, you will find the near-rings nice”. The structure of many near-rings, like that of rings, can be determined with the help of Density Theorems and Radicals. Günter Pilz’s monograph on these structures, “Near-rings”, published by North Holland in 1977, is perhaps his most influential scientific work.

Günter Pilz completed his doctorate at the University of Vienna in 1967 under Professor Wilfried Nöbauer with a dissertation on order theory in composition rings. Many of Nöbauer’s doctoral students at the time went on to pursue academic careers, and it shows a sense of community that they still meet regularly in Vienna at the “Heuriger”, a typical Austrian kind of wine tavern. Günter Pilz’s academic career took him via Vienna to Tucson in Arizona, until the then newly established Faculty of Engineering and Natural Sciences at the Johannes Kepler University Linz invited him to set up the field of algebra in Linz for research and teaching. Several textbooks published by the Linz university publisher Rudolf Trauner date from this founding period: an “Introduction to Mathematics” (1981) for economics students, the “travel guide” “Algebra – Ein Reiseführer durch die schönsten Gebiete”¹ for the major lectures on abstract algebra and the first version of “Applied Abstract Algebra”, written jointly with Rudolf Lidl and published by Springer. The “travel guide” is a didactic masterpiece: in a concise, reader-friendly style, it offers an introduction to groups, rings and fields, but also to lattices, universal algebras and semigroups, organised in portions suitable for lectures. The work concludes with an appendix entitled “Life assistance”, in which the author suggests, for example, that the phrase “without loss of generality” should sometimes simply be read as “we only treat an easy case”. My personal copy of this book is now completely tattered and serves me to this day as a model of how to fit a lot of content into little lecture time. In addition to the well thought-out structure, Pilz’s lectures have also been entertaining due to many varied ideas: axes and pitchforks were used in the lectures at appropriate occasions, and I particularly remember the (albeit rarely used) breakdown triangle that Pilz set up when a proof had to be repaired.

Pilz also endeavoured very successfully to bring international visiting professors to the university. For example, he brought Abdollah Alhevaz, Gerhard Betsch, Gary Birkenmeier, James Clay, Yuen Fong, Kalle Kaarli, Valdis Laan, Rudolf Lidl, Dragan Mašulović, Carl Maxson, Gary

¹In English, “Algebra – a travel guide through the most beautiful areas”.

Peterson, Boris Schein, Stuart Scott and Richard Wiegandt to the JKU for longer guest stays. The international collaboration by no means ended with Pilz's retirement in 2013: in recent years, an intensive collaboration has developed with Anatolii and Yurii Zhuchok on free semigroup-like structures. However, most of Günter Pilz's scientific work deals with near-rings and their applications, initially to algebraic, later also to information-theoretic and statistical questions. Together with Yong Sian So, Pilz determined the affine complete residue class rings of the integers. Together with Peter Fuchs and Carl Maxson, Pilz showed that an infinite (not necessarily commutative) ring in which every polynomial $p = \sum_{i=0}^n a_i x^i \neq 0$ has only a finite number of zeros is zero-divisor-free, and that a division ring with this property is a field. A common aspect of many of Günter Pilz's works is that they pose natural, often seemingly simple questions that have not yet been answered. For example, we know that the polynomial functions of a direct product of two finite rings with unit can be studied component by component: a function is a polynomial function if it is a polynomial function modulo both projection kernels. In 1980, Pilz formulated the conjecture that finite expanded groups have this property if and only if they have no skew congruences. In 2015, Kalle Kaarli, Peter Mayr and the author of this article were able to confirm this conjecture for the wider class of algebras with a Mal'cev term. In the 1990s, Pilz turned his attention to planar near-rings: the Italian mathematician Giovanni Ferrero had obtained block designs from these near-rings, as required for statistical experimental design. Pilz used these designs to test the efficiency of fertilisers or to detect defective ion channels in human cells. A particularly interesting mathematical problem arose from the study of near-ring-generated codes: Is there $k \in \mathbb{N}$ and a non-empty finite subset I of \mathbb{N} such that the symmetric difference of all sets $\{i, 2i, 3i, \dots, ki\}$ with $i \in I$ contains less than k elements? Together with Po-Yi Huang and Wen-Fong Ke, Pilz was able to prove in 2010 that there is no such k if I is an initial segment of the natural numbers; the general case is still open.

Thanks to his friendly manner, his sound judgement, his sense of justice, his composure and his unwavering sense of humour, Pilz was also appointed to many leading positions at the Johannes Kepler University Linz: he was department head, Dean of Studies, Vice-Rector for Research and the Austrian Commissioner for Scientific Integrity and in these functions he always pursued the goal of orienting the university towards high

international standards. Pilz still actively participates in the Institute's research seminars and publishes scientific papers. We wish him and his wife Gerti, to whom he has been married for 56 years, good health and continued enthusiasm for mathematics.

CONTACT INFORMATION

E. Aichinger

Institute for Algebra, Johannes Kepler
University Linz, Altenbergerstraße 69,
4040 Linz, Austria
E-Mail: erhard.aichinger@jku.at