

## YURIY DROZD

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*To the 80th anniversary*



On October 15, 2024 an outstanding Ukrainian mathematician and one of the founders of the Kyiv Algebraic School Yuriy Drozd celebrated his 80th birthday.

Yu. Drozd was born in Kyiv, his father was a civil engineer and mother a physician. During the Second World War, both parents of Yu. Drozd served as army officers in the Red Army.

He enjoyed mathematics from young age at school. In particular, in 1961 Yu. Drozd became a winner of the 1st Mathematical Competition of Ukrainian Republic as well as of the 1st All-Soviet mathematical competition. After graduation from high school, Yu. Drozd became a student at the Faculty of Mathematics and Mechanics of the Taras Shevchenko Kyiv State University. Already his first scientific work “Homological algebra

in semi-abelian categories” (appeared in 1966) indicated his interest in categorical methods in representation theory.

In 1966, Yu. Drozd graduated from the university with honours. His university advisor was Andrei Roiter. The main result of his diploma work was a criterion of representation-finiteness of cubic orders. It was published in “Proceedings of the Academy of Sciences of the USSR” and provided a valuable contribution to the future development of the theory of integral representations of rings.

Continuing his collaboration with A. Roiter, Yu. Drozd started his postgraduate education at the Institute of Mathematics of Ukrainian Academy of Science in Kyiv. His scientific advisor was one of the leading Soviet mathematicians Igor Shafarevich. During his postgraduate studies, Yu. Drozd spent a significant amount of time at the Moscow State University and the Steklov Institute of Mathematics, learning, in particular, foundations of modern algebraic geometry.

In 1969, Yu. Drozd continued his academic carrier at the Kyiv State University, where he progressed from assistant professor to full professor, being the chair of Algebra and Mathematical Logic from 1980 till 1998. In 1970, Yu. Drozd defended his dissertation “Some questions of the theory of integral representations”. In particular, this thesis contained a criterion for a commutative order to have only finitely many indecomposable lattices, structure results on hereditary and Bassian orders as well as several other results, which played a central role in the theory of integral representations afterwards.

Already during his graduate studies at the Kyiv Taras Shevchenko State University, Yu. Drozd started a very fruitful collaboration with Volodymyr Kirichenko. The subject of their joint interest was the theory of integral representations, which also included a purely ring theoretic study of non-commutative orders (hereditary, Bassian, quasi-Bassian). As an outcome was their joint work “Primary orders with a finite number of indecomposable representations”. Until now, a classification of one-dimensional non-commutative orders of finite lattice type remains an open problem and the obtained result about primary orders is one of the most general ones. In 1980, a joint monograph of Yu. Drozd and V. Kirichenko “Finite dimensional algebras” appeared. In 1994, it was translated into English (and got an appendix on quasi-hereditary algebras) by Vlastimil Dlab. Up to now, this book remains one of the standard references in representation theory of finite dimensional algebras.

In the 1970s, Yu. Drozd began to study matrix problems, a new

branch of algebra and representation theory. Together with A. Roiter, M. Kleiner, S. Ovsienko and others he significantly contributed to the development of the theory of bocses (bimodules over a category with a bialgebra structure), which became one of the trademarks of the Kyiv Algebraic School. This technique allowed him to prove a famous “tame-wild theorem” stating that every finite dimensional algebra over an algebraically closed field has either finite, or tame, or wild representation type. This striking result is a milestone of the modern representation theory of finite dimensional algebras. The technique of bocses also played a key role in further works of Yu. Drozd on the representation type, most notably in his proofs of the trichotomy and semi-continuity of the representation type of bounded derived categories of finite dimensional algebras.

In 1981, he defended his Dr. Sci. dissertation “Matrix methods in representation theory and ring theory” with Leningrad State University as a leading organization.

At the beginning of the 1980s, Yu. Drozd initiated a new direction of research at the Kyiv State University devoted to the study of special classes of infinite dimensional representations of simple Lie algebras. One of the highlights of this direction is his joint work with V. Futorny and S. Ovsienko on Harish-Chandra subalgebras and Gel’fand-Zetlin modules, dedicated to the study of generalized categories of Harish-Chandra modules.

A great contribution of Yu. Drozd to the development of modern algebra was application of the technique of matrix problems to classification results in other areas of mathematics, including algebraic geometry, algebraic topology, commutative algebra, and representation theory of Lie algebras. In a joint work with V. Bondarenko, Yu. Drozd described all finite groups of tame representation type over a field of positive characteristic. In a joint work with V. Bekkert, Yu. Drozd proved tame-wild dichotomy for derived categories over finite-dimensional algebras, and in a joint work with V. Bekkert and V. Futorny, he established derived representation type of complete finitely generated local and two-point algebras over an algebraically closed field. Another his joint work with V. Bekkert and V. Futorny studied irreducible unitary representations of linear groups over Dynkinian and Euclidean algebras using the technique of bimodule categories and representations of quivers.

Yu. Drozd made a fundamental contribution to the representation theoretic study of Cohen-Macaulay modules over curve and surface sin-

ularities. Together with A. Roiter, he proved that simple curve singularities have only finitely many indecomposable Cohen-Macaulay modules. The already mentioned joint work with V. Kirichenko on representation finite orders belong to the same research domain. In a series of joint works with G.-M. Greuel, he proved semi-continuity/finite-tame-wild-trichotomy of the representation type of reduced curve singularities, explicitly characterizing singularities of tame representation type. In a subsequent joint work with I. Kashuba and G.-M. Greuel, Yu. Drozd established tameness of surface cusp singularities. Jointly with V. Gavran, this approach was generalized on the setting of non-commutative surface singularities, allowing to established tameness of some non-commutative analogues of minimally elliptic surface singularities. Together with I. Burban, Yu. Drozd discovered a new class of tame matrix problems called decorated bunches of chains and proved using this technique the tameness of degenerate cusp surface singularities.

Investigation of Cohen-Macaulay modules is closely related to the study of vector bundles and torsion free sheaves on singular curves. In 1957, Atiyah classified indecomposable vector bundles on a smooth elliptic curve. Despite a steady interest, for several decades afterwards there were no efficient tools to study vector bundles on degenerate elliptic curves. Together with G.-M. Greuel, Yu. Drozd proved tame-wild dichotomy for the categories of vector bundles and torsion free sheaves on arbitrary projective curves. In a subsequent joint works with I. Burban, the latter results were generalized for coherent sheaves as well as for the corresponding derived categories. In particular, it was shown that the derived category of coherent sheaves on a Kodaira cycle of projective lines has tame representation type. In a joint work with L. Bodnarchuk and G.-M. Greuel, Yu. Drozd applied again the technique of matrix problems to classify simple vector bundles on degenerate elliptic curves of wild representation type. All together, it led to a comprehensive theory of vector bundles, coherent sheaves and their derived categories on projective curves of arithmetic genus one.

In the 1990s, Yu. Drozd began a fruitful collaboration with H.-G. Baues from Max-Planck Institute for Mathematics in Bonn in the field of algebraic topology. In several joint papers, they in particular used the technique of matrix problems to achieve a crucial progress in stable homotopic classification of polyhedra.

One of the current research areas of Yu. Drozd is devoted to non-commutative algebraic geometry. The origins of this direction go back

to methods of integral representation theory and the theory of orders. In the 1990s, Yu. Drozd introduced the notion of a nodal order and proved that these are the only pure noetherian algebras of discrete or tame representation type. In a later joint work with I. Burban it was proven that nodal orders are even derived-tame. The global analogues of nodal orders, the so-called tame non-commutative nodal projective curves, appeared recently in a quite essential way in the framework of the homological mirror symmetry for compact surfaces with non-empty boundary. In a recent work with I. Burban, a version of Morita theorem for non-commutative schemes was established.

These and other results of Yu. Drozd were published in top international journals. He had more than 30 Ph.D. students, four of whom later defended Dr. Sci. dissertations. From 2006 till 2021 Yu. Drozd served as the Head of the Department of Algebra and Topology at the Institute of Mathematics of National Academy of Science of Ukraine, the main mathematical organization in Ukraine. Since 2021 he is a Chief researcher of the Department of Algebra and Topology at the Institute of Mathematics of NAS of Ukraine.

Yu. Drozd wrote several university textbooks. Many students use his “Galois Theory”, “Theory of Algebraic Numbers”, “Introduction to Algebraic Geometry”, “Basic Mathematical Logic”, “Discrete Mathematics” and others. The book “Introduction to Algebraic Geometry” (based on the lecture course given by Yu. Drozd at the University of Kaiserslautern in the winter term 1998/99) is the first and so far the only textbook in algebraic geometry written in Ukrainian language.

We warmly congratulate Yuriy Anatolijovych Drozd on the occasion of his 80th birthday and wish him strong health and many successful years of research and teaching.

*V. Bavula, V. Bekkert, O. Bezushchak,  
V. Bondarenko, Ie. Bondarenko, I. Burban,  
V. Futorny, R. Grigorchuk, L. Kurdachenko,  
V. Lyubashenko, V. Mazorchuk, V. Nekrashevych,  
A. Oliynyk, B. Oliynyk, A. Petravchuk, Ya. Sysak,  
E. Zelmanov, A. Zhuchok, Yu. Zhuchok*