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Characterization of 3-prime near-rings via multiplicative derivations

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Abstract. In this paper, we will prove some theorems concerning the commutativity of a 3-prime near-rings which satisfy some differential identities on semigroup ideals and Jordan ideals of a near-ring $\mathcal{N}$ admitting a multiplicative derivation. After this, we discuss an example to prove that the necessity of the 3-primeness hypothesis imposed on the various theorems cannot be marginalized.

Introduction

Throughout this paper, $\mathcal{N}$ represent a right near-ring and $\mathcal{Z}(\mathcal{N})$ stands multiplicative center of $\mathcal{N}$, that is, $\mathcal{Z}(\mathcal{N}) = \{x \in \mathcal{N} \mid xy = yx \text{ for all } y \in \mathcal{N}\}$. For all $x, y \in \mathcal{N}$, we write $[x, y] = xy - yx$ and $x \circ y = xy + yx$ for the Lie product and Jordan product, respectively. A near-ring $\mathcal{N}$ is called a zero-symmetric if $x.0 = 0.x = 0$ for all $x \in \mathcal{N}$; knowing that $0.x = 0$ follows from the fact that $\mathcal{N}$ is a right near-ring. $\mathcal{N}$ is called 2-torsion free if $(\mathcal{N}, +)$ has no elements of order 2 and know as 3-prime if for $x, y \in \mathcal{N}$, $x\mathcal{N}y = \{0\}$ implies $x = 0$ or $y = 0$. In [4], the notion of Jordan ideal was defined as follows: an additive subgroup J of $\mathcal{N}$ is said to be a Jordan ideal of $\mathcal{N}$ if $j \circ n \in \mathcal{J}$ and $n \circ j \in \mathcal{J}$ for all $j \in \mathcal{J}$, $n \in \mathcal{N}$. A nonempty subset $\mathcal{I}$ of $\mathcal{N}$ is said to be a semigroup left ideal

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(resp. semigroup right ideal) if $\mathcal{N}\mathcal{I} \subseteq \mathcal{I}$ (resp. $\mathcal{I}\mathcal{N} \subseteq \mathcal{I}$); note that $\mathcal{I}$ is said to be semigroup ideal if I is both a semigroup left ideal and a semigroup right ideal of $\mathcal{N}$. An additive mapping $d : \mathcal{N} \to \mathcal{N}$ is called a derivation if $d(xy) = d(x)y + xd(y)$ hold for all $x, y \in \mathcal{N}$, or equivalently as noted in [13], that $d(xy) = xd(y) + d(x)y$ for any $x, y \in \mathcal{N}$. The concept of derivation has been generalized in different directions by various authors, for more details see for example [2], [4], [8], [10], [11], and [12], where further references can be found. In [5], the notion of multiplicative derivation on ring was introduced by Daif as follows: a mapping $d : \mathcal{N} \to \mathcal{N}$, which is not assumed to be additive, is called a multiplicative derivation if $d(xy) = xd(y)+d(x)y$ holds for all $x, y \in \mathcal{N}$. In [6], the authors have worked on these mappings thereby giving them a complete description. Clearly, any derivation is a multiplicative derivation, but the converse is not true in general. To be convinced of this difference, see for example [1, Examples 1.1 & 1.2]. Motivated by this difference, we pursue this research path and study the structure of 3-prime near-rings in which multiplicative derivations satisfy certain identities involving Jordan's ideal. The results obtained are very precise and cover others known in the literature while neglecting an important property marked in the hypotheses of various subsequent works, that is the additivity property considered in the different maps used.

1. Preliminary results

In this section, we include some well-known results which will be used for developing the proof of our main results.

Lemma 1. *Let $\mathcal{N}$ be a 3-prime near-ring and $\mathcal{I}$ be a nonzero semigroup ideal of $\mathcal{N}$.*

(*i*) [3, Lemma 1.4 (i)] *If $x, y \in \mathcal{N}$ and $x\mathcal{I}y = \{0\}$, then $x = 0$ or $y = 0$.*

(*ii*) [3, Lemma 1.3 (i)] *If $x \in \mathcal{N}$ and $x\mathcal{I} = \{0\}$ or $\mathcal{I}x = \{0\}$, then $x = 0$.*

Lemma 2 ([9, Lemma 3])**.** *Let $\mathcal{N}$ be a 3-prime near-ring admitting a nonzero multiplicative derivation d, then $\mathcal{N}$ satisfies the following partial distribution law:*

$$z(xd(y) + d(x)y) = zxd(y) + zd(x)y \text{ for all } x, y, z \in \mathcal{N}.$$

Lemma 3. *Let $\mathcal{N}$ be a 3-prime near-ring and $\mathcal{J}$ a nonzero Jordan ideal of $\mathcal{N}$.*

(i) [4, Lemma 1] *If* $\mathcal{J}x = 0$*, then* $x = 0$*.*

(ii) [4, Lemma 3] *If* $\mathcal{N}$ *is 2-torsion free and* $\mathcal{J} \subseteq \mathcal{Z}(\mathcal{N})$*, then* $\mathcal{N}$ *is a commutative ring.*

Lemma 4 ([9, Lemma 3])**.** *Let* $\mathcal{N}$ *be a 3-prime near-ring. If* $\mathcal{N}$ *admits a nonzero multiplicative derivation* d *such that* $ad(\mathcal{N}) = \{0\}$ *or* $d(\mathcal{N})a = \{0\}$*, then* $a = 0$*.*

Lemma 5 ([7, Lemma 2.1])**.** *A near-ring* $\mathcal{N}$ *admits a multiplicative derivation if and only if it is zero-symmetric.*

Lemma 6 ([3, Lemma 1.2 (iii)])**.** *Let* $\mathcal{N}$ *be a 3-prime near-ring. If* $z \in \mathcal{Z}(\mathcal{N}) \setminus \{0\}$ *and* $xz \in \mathcal{Z}(\mathcal{N})$*, then* $x \in \mathcal{Z}(\mathcal{N})$*.*

2. Main results

This section is devoted to the study of the behavior of a right near-ring $\mathcal{N}$ under the action of certain algebraic identities by taking into account only the two particular subsets Jordan ideal and semigroup ideal of $\mathcal{N}$ instead of integer $\mathcal{N}$. The results obtained vary between the commutativity of $\mathcal{N}$ and the non-existence of such a nonzero multiplicative derivation d which gives important meaning to the different hypotheses considered. Indeed, we have obtained the following results.

Theorem 1. *Let* $\mathcal{N}$ *be a 2-torsion free 3-prime near-ring,* $\mathcal{I}$ *be a nonzero semigroup ideal of* $\mathcal{N}$ *and* $\mathcal{J}$ *be a nonzero Jordan ideal of* $\mathcal{N}$*. If* $\mathcal{N}$ *admits a multiplicative derivation* d *such that* $d(\mathcal{J})$ *is nonzero, then the following assertions are equivalent:*

(i) $d([x,j]) = [x,j]$ *for all* $x \in \mathcal{I}, j \in \mathcal{J}$*;*

(ii) $d([x,j]) = -[x,j]$ *for all* $x \in \mathcal{I}, j \in \mathcal{J}$*;*

(iii) $d([x,j]) = [x,d(j)]$ *for all* $x \in \mathcal{I}, j \in \mathcal{J}$*;*

(iv) $\mathcal{N}$ *is a commutative ring.*

Proof. It is easy to verify that condition (iv) implies both properties (i), (ii), and (iii).
Let's show that $(i) \Rightarrow (iv)$, for this suppose that

$$d([x,j]) = [x,j] \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{1}$$

Substituting xj for x in (1), because of $[xj, j] = [x, j]j$, we infer that

$$xjd(j) = jxd(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{2}$$

Taking tx instead of x in (2), where $t \in \mathcal{N}$, we arrive at

$$tjxd(j) = jtxd(j) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J},$$

which leads to

$$[t, j]\mathcal{I}d(j) = \{0\} \text{ for all } t \in \mathcal{N}, j \in \mathcal{J}. \tag{3}$$

In view of Lemma 1 (i), equation (3) shows that

$$d(j) = 0 \text{ or } j \in \mathcal{Z}(\mathcal{N}) \text{ for all } j \in \mathcal{J}. \tag{4}$$

By hypotheses we have $d(\mathcal{J}) \neq \{0\}$, so there exists an element $j_0 \in \mathcal{J}$ such that $d(j_0) \neq 0$, and hence (4) assures that $j_0 \in \mathcal{Z}(\mathcal{N})$. Now, substituting xj_0 for x in (1), we get

$$xjd(j_0) = jxd(j_0) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{5}$$

Replacing x by tx, where $t \in \mathcal{N}$, in (5) and using it again, we obtain

$$tjxd(j_0) = jtxd(j_0) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}.$$

So that,

$$[t, j]\mathcal{I}d(j_0) = \{0\} \text{ for all } t \in \mathcal{N}, j \in \mathcal{J}. \tag{6}$$

In virtue of Lemma 1 (i) and $d(j_0) \neq 0$, (6) gives $\mathcal{J} \subseteq \mathcal{Z}(\mathcal{N})$ which means that $\mathcal{N}$ is a commutative ring by Lemma 3 (ii).

$(ii) \Rightarrow (iv)$, using arguments similar to those used above with some suitable modifications, we obtain the required result.

$(iii) \Rightarrow (iv)$ Assume that

$$d([x, j]) = [x, d(j)] \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{7}$$

Putting xj instead of x in (7) and using it again, we arrive at

$$-jxd(j) + xd(j)j = 0 \text{ for all } x \in \mathcal{I}, j \in \mathcal{J},$$

that is

$$xd(j)j = jxd(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{8}$$

Taking tx instead of x in (8), where $t \in \mathcal{N}$, we find that

$$tjxd(j) = jtxd(j) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J},$$

which can be written as

$$[t, j]\mathcal{I}d(j) = \{0\} \text{ for all } t \in \mathcal{N}, j \in \mathcal{J}.$$

The rest of the proof is similar to the steps used after equation (3) in the proof of $(i) \Rightarrow (iv)$. □

Theorem 2. *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring, $\mathcal{I}$ be a nonzero semigroup ideal of $\mathcal{N}$ and $\mathcal{J}$ be a nonzero Jordan ideal of $\mathcal{N}$. There is no multiplicative derivation d of $\mathcal{N}$ such that $d(\mathcal{J}) \neq \{0\}$ and d satisfies one of the following conditions:*

(i) $d([x, j]) = x \circ j$ for all $x \in \mathcal{I}, j \in \mathcal{J}$;

(ii) $d([x, j]) = -(x \circ j)$ for all $x \in \mathcal{I}, j \in \mathcal{J}$;

(iii) $d(x \circ j) = [x, j]$ for all $x \in \mathcal{I}, j \in \mathcal{J}$;

(iv) $d(x \circ j) = -[x, j]$ for all $x \in \mathcal{I}, j \in \mathcal{J}$.

Proof. (i) Assume that $\mathcal{N}$ admits a multiplicative derivation d such that $d(\mathcal{J}) \neq \{0\}$ and d satisfies

$$d([x, j]) = x \circ j \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \quad (9)$$

Substituting xj for x in (9), we get

$$[x, j]d(j) = 0 \text{ for all } x \in \mathcal{I}, j \in \mathcal{J},$$

and thus

$$xjd(j) = jxd(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \quad (10)$$

Replacing x by tx, where $t \in \mathcal{N}$, in (10) and using it again, we obtain

$$tjxd(j) = jtxd(j) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J},$$

which implies that

$$[t, j]\mathcal{I}d(j) = \{0\} \text{ for all } t \in \mathcal{N}, j \in \mathcal{J}.$$

Invoking Lemma 1 (i), we obtain

$$d(j) = 0 \text{ or } j \in \mathcal{Z}(\mathcal{N}) \text{ for all } j \in \mathcal{J}. \quad (11)$$

Suppose there is an element $j_0 \in \mathcal{J}$, such that $d(j_0) = 0$. As $d(\mathcal{J}) \neq \{0\}$, there exists an element $i_0 \in \mathcal{J}$ such that $d(i_0) \neq 0$, then (11) assures that $i_0 \in \mathcal{Z}(\mathcal{N})$. Now, returning to (9) and replacing respectively x and j by xi_0 and j_0, we find that

$$xj_0d(i_0) = j_0xd(i_0) \text{ for all } x \in \mathcal{I}. \tag{12}$$

Taking tx instead of x, where $t \in \mathcal{N}$, in (12) and using it again, we infer that

$$tj_0xd(i_0) = j_0txd(i_0) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I},$$

that is $[t, j_0]\mathcal{I}d(i_0) = \{0\}$ for all $t \in \mathcal{N}$. In view of Lemma 1 (i) and the fact that $d(i_0) \neq 0$, the last result gives $j_0 \in \mathcal{Z}(\mathcal{N})$ and therefore (11) implies that $j \in \mathcal{Z}(\mathcal{N})$ for all $j \in \mathcal{J}$. So, $\mathcal{N}$ is a commutative ring by Lemma 3 (ii). In this case, by 2-torsion freeness, equation (9) reduces to $xj = 0$ for all $x \in \mathcal{I}, j \in \mathcal{J}$. Thus $\mathcal{I}j = \{0\}$ for all $j \in \mathcal{J}$, it follows that $j = 0$ for all $j \in \mathcal{J}$ by Lemma 1 (ii). Accordingly, $\mathcal{J} = \{0\}$ which contradicts our original assumption that $\mathcal{J} \neq \{0\}$.

(ii) Using similar arguments, we get the required result.

(iii) Assume that $\mathcal{N}$ admits a multiplicative derivation d satisfies $d(\mathcal{J}) \neq \{0\}$ and

$$d(x \circ j) = [x, j] \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{13}$$

Substituting xj for x in (13), we get

$$(x \circ j)d(j) = 0 \text{ for all } x \in \mathcal{I}, j \in \mathcal{J},$$

and thus

$$xjd(j) = (-j)xd(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{14}$$

Replacing x by tx, where $t \in \mathcal{N}$, in (14) and using (14), we obtain

$$t(-j)xd(j) = (-j)txd(j) \text{ for all } x \in \mathcal{I}, t \in \mathcal{N}, j \in \mathcal{J},$$

which implies that

$$[t, -j]\mathcal{I}d(j) = \{0\} \text{ for all } t \in \mathcal{N}, j \in \mathcal{J}.$$

Invoking Lemma 1 (i), the last equation shows that

$$d(j) = 0 \text{ or } -j \in \mathcal{Z}(\mathcal{N}) \text{ for all } j \in \mathcal{J}. \tag{15}$$

Suppose there is an element $j_0 \in \mathcal{J}$ such that $d(j_0) = 0$. As $d(\mathcal{J}) \neq \{0\}$, there exists an element $i_0 \in \mathcal{J}$ and $d(i_0) \neq 0$, then (15) assures that

$-i_0 \in \mathcal{Z}(\mathcal{N})$. Now, we prove that $d(-i_0) \neq 0$. Indeed suppose that $d(-i_0) = 0$, in this case, replacing j by $-i_0$ in (13), we arrive at

$$d(x+x)(-i_0) = 0 \text{ for all } x \in \mathcal{I}.$$

Right multiplying the previous relation by t, where $t \in \mathcal{N}$, and using the fact $-i_0 \in \mathcal{Z}(\mathcal{N})$, we find that

$$d(x+x)t(-i_0) = 0 \text{ for all } x \in \mathcal{I}, t \in \mathcal{N},$$

and hence

$$d(x+x)\mathcal{N}(-i_0) = \{0\} \text{ for all } x \in \mathcal{I}.$$

By the 3-primeness of $\mathcal{N}$, we find that

$$d(x+x) = 0 \text{ or } -i_0 = 0 \text{ for all } x \in \mathcal{I}. \tag{16}$$

If $-i_0 = 0$, then $i_0 = 0$ which implies that $d(i_0) = d(0.0) = 0.d(0) + d(0).0 = 0$ by Lemma 5; a contradiction. Now, we treat the firste case of (16) that is, we have that $d(x+x) = 0$ for all $x \in \mathcal{I}$. Writing xi_0 instead of x in previous expression and using it again, we arrive at

$$2xd(i_0) = 0 \text{ for all } x \in \mathcal{I}.$$

In view of the 2-torsion freeness of $\mathcal{N}$, the last result gives $\mathcal{I}d(i_0) = \{0\}$ which implies, because of Lemma 1 (ii), that $d(i_0) = 0$, a contradiction again. Consequently, $d(-i_0) \neq 0$. On the other hand, replacing x and j by $x(-i_0)$ and j_0, respectively, in (13), we arrive at

$$xj_0d(-i_0) = (-j_0)xd(-i_0) \text{ for all } x \in \mathcal{I}. \tag{17}$$

Substituting tx for x, where $t \in \mathcal{N}$, in (17) and using it again, we get

$$[t, -j_0]\mathcal{I}d(-i_0) = \{0\} \text{ for all } t \in \mathcal{N}. \tag{18}$$

Since $d(-i_0) \neq 0$, equation (18) implies that $-j_0 \in \mathcal{Z}(\mathcal{N})$ and therefore (15) reduces to $-j \in \mathcal{Z}(\mathcal{N})$ for all $j \in \mathcal{J}$ which, in virtue of $(\mathcal{J}, +)$ is an additive subgroup, means that $\mathcal{J} \subseteq \mathcal{Z}(\mathcal{N})$. Accordingly, $\mathcal{N}$ is a commutative ring by Lemma 3 (ii). Now, returning to (13), we can see that

$$d(xj+xj) = 0 = (x+x)d(j) + d(x+x)j \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

In particular, putting $x = xj$ in the latter term of the previous relation and taking account that $\mathcal{N}$ is 2-torsion free, we infer that $jxd(j) = 0$ for all $x \in \mathcal{I}, j \in \mathcal{J}$ which can be rewritten as $j\mathcal{I}d(j) = \{0\}$ for all $j \in \mathcal{J}$. Now, applying Lemma 1 (i), we conclude that $j = 0$ or $d(j) = 0$ for all $j \in \mathcal{J}$. But in the both cases, we have $d(j) = 0$ and hence $d(\mathcal{J})$ is zero which is contrary to our hypothesis that $d(\mathcal{J}) \neq \{0\}$.

(*iv*) Using a similar arguments, we get the required result. This completes the proof of our Theorem. □

Theorem 3. *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring, $\mathcal{I}$ be a nonzero semigroup ideal of $\mathcal{N}$ and $\mathcal{J}$ be a nonzero Jordan ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a multiplicative derivation d such that $d(\mathcal{Z}(\mathcal{N})) \neq \{0\}$, and d satisfies one of the following conditions:*

(*i*) $d([x, j]) \in \mathcal{Z}(\mathcal{N})$ *for all* $x \in \mathcal{I},\ j \in \mathcal{J}$*;*

(*ii*) $d(x \circ j) \in \mathcal{Z}(\mathcal{N})$ *for all* $x \in \mathcal{I},\ j \in \mathcal{J}$*;*

(*iii*) $d([x, j]) \circ t \in \mathcal{Z}(\mathcal{N})$ *for all* $x \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{N}$*;*

(*iv*) $d(x \circ j) \circ t \in \mathcal{Z}(\mathcal{N})$ *for all* $x \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{N}$*;*

then $\mathcal{N}$ is a commutative ring.

Proof. (*i*) Let $z \in \mathcal{Z}(\mathcal{N})$ such that $d(z) \neq 0$. By hypotheses given, we have $d([xz, j])t = td([xz, j])$ for all $t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}$ which implies that

$$d(z[x, j])t = td(z[x, j]) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}. \tag{19}$$

In the light of Lemma 2, equation (19) gives

$$d(z)[x, j]t = td(z)[x, j] \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}. \tag{20}$$

Substituting xj for x in (20) and using it, we get

$$d(z)[x, j]jt = d(z)[x, j]tj \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}.$$

Once again, because of (20), the previous relation can be rewritten as

$$[j, t]d(z)[x, j] = 0 \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}. \tag{21}$$

Now, right multiplying equation (21) by m, where $m \in \mathcal{N}$, and applying (20), we find that

$$[j, t]md(z)[x, j] = 0 \text{ for all } t, m \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}$$

which implies that

$$[j,t]\mathcal{N}d(z)[x,j]=\{0\} \text{ for all } t\in\mathcal{N}, x\in\mathcal{I}, j\in\mathcal{J}.$$

Using the 3-primeness of $\mathcal{N}$, we get

$$j\in\mathcal{Z}(\mathcal{N}) \text{ or } d(z)[x,j]=0 \text{ for all } x\in\mathcal{I}, j\in\mathcal{J}. \tag{22}$$

Suppose there is an element $j_0\in\mathcal{J}$ such that $j_0\notin\mathcal{Z}(\mathcal{N})$, then

$$d(z)[x,j_0]=0 \text{ for all } x\in\mathcal{I}. \tag{23}$$

Returning to (19) and replacing j by j_0, and using the fact that $z\in\mathcal{Z}(\mathcal{N})$, we find that $d([x,j_0]z)t=td([x,j_0]z)$ for all $t\in\mathcal{N}, x\in\mathcal{I}$. Applying Lemma 2 and after simplifying, we obtain $[x,j_0]d(z)t=t[x,j_0]d(z)$ for all $t\in\mathcal{N}, x\in\mathcal{I}$ which means that $[x,j_0]d(z)\in\mathcal{Z}(\mathcal{N})$. Right multiplying equation (23) by $d(z)m$, where $m\in\mathcal{N}$, we find that $d(z)[x,j_0]d(z)m=0$ for all $m\in\mathcal{N}, x\in\mathcal{I}$. It follows that

$$d(z)\mathcal{N}[x,j_0]d(z)=\{0\} \text{ for all } x\in\mathcal{I}.$$

Using the 3-primeness of $\mathcal{N}$ and taking account that $d(z)\neq 0$, we conclude that $[x,j_0]d(z)=0$ for all $x\in\mathcal{I}$ which yields to

$$xj_0d(z)=j_0xd(z_0) \text{ for all } x\in\mathcal{I}. \tag{24}$$

Taking tx instead of x, where $t\in\mathcal{N}$, in (24) and using it again, we arrive at

$$[t,j_0]\mathcal{I}d(z)=\{0\} \text{ for all } t\in\mathcal{N}.$$

In view of Lemma 1 (i) together $d(z)\neq 0$, the previous expression assures that $j_0\in\mathcal{Z}(\mathcal{N})$. Consequently, (22) reduces to $\mathcal{J}\subseteq\mathcal{Z}(\mathcal{N})$ and Lemma 3 (ii) shows that $\mathcal{N}$ is a commutative ring.

(ii) Let $z\in\mathcal{Z}(\mathcal{N})$ such that $d(z)\neq 0$. By hypothesis we have

$$d(xz\circ j)t=td(xz\circ j) \text{ for all } t\in\mathcal{N}, x\in\mathcal{I}, j\in\mathcal{J}.$$

So that,

$$d((x\circ j)z)t=td((x\circ j)z) \text{ for all } t\in\mathcal{N}, x\in\mathcal{I}, j\in\mathcal{J}. \tag{25}$$

Applying Lemma 2 and after simplifying, equation (25) yields

$$(x\circ j)d(z)t=t(x\circ j)d(z) \text{ for all } t\in\mathcal{N}, x\in\mathcal{I}, j\in\mathcal{J}$$

it follows that

$$(x \circ j)d(z) \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{26}$$

Returning to (25) and using the fact that $z \in \mathcal{Z}(\mathcal{N})$, we find that

$$d(z(x \circ j))t = td(z(x \circ j)) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}.$$

Once again, in view of Lemma 2, we obtain

$$d(z)(x \circ j)t = td(z)(x \circ j) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J},$$

which means that

$$d(z)(x \circ j) \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{27}$$

Replacing x by xj in (27), we get

$$d(z)(x \circ j)j \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

Hence, in the light of Lemma 6, the previous result shows that

$$d(z)(x \circ j) = 0 \text{ or } j \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{28}$$

Assume that

$$d(z)(x \circ j) = 0 \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{29}$$

Right multiplying (29) by $d(z)m$, where $m \in \mathcal{N}$, and invoking (26) we find that

$$d(z)m(x \circ j)d(z) = 0 \text{ for all } m \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}$$

that is,

$$d(z)\mathcal{N}(x \circ j)d(z) = \{0\} \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

Using the 3-primeness of $\mathcal{N}$ together $d(z) \neq 0$, we infer that

$$xjd(z) = -jxd(z) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{30}$$

Substituting tx for x in (30), where $t \in \mathcal{N}$, we have

$$t(-j)xd(z) = (-j)txd(z) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}.$$

So that

$$[t, -j]\mathcal{I}d(z) = \{0\} \text{ for all } t \in \mathcal{N}, j \in \mathcal{J}.$$

In view of Lemma 1 (i) and $d(z) \neq 0$, the last equation assures that $j \in \mathcal{Z}(\mathcal{N})$ for all $j \in \mathcal{J}$, and hence $\mathcal{N}$ is a commutative ring by Lemma 3 (ii). Accordingly, from (29) and 2-torsion freeness of $\mathcal{N}$ we get $d(z)xj = 0$ for all $x \in \mathcal{I}, j \in \mathcal{J}$, so that $d(z)\mathcal{I}j = \{0\}$ for all $j \in \mathcal{J}$. In view of Lemma 1 (i) and $d(z) \neq 0$, we conclude that $J = \{0\}$, a contradiction. Consequently, there is $x_0 \in \mathcal{I}$, $j_0 \in \mathcal{J}$ such that $d(z)(x_0 \circ j_0) \neq 0$. From (28), it follows that $j_0 \in \mathcal{Z}(\mathcal{N})$. On the other hand, as $\mathcal{N}$ is a zero symmetric near-ring and $d(z)(x_0 + x_0)j_0 \neq 0$, then necessarily $d(z) \neq 0$, $x_0 + x_0 \neq 0$ and $j_0 \neq 0$. Now, replacing j by j_0 in (26), we have

$$(x + x)d(z)j_0 \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}.$$

As $0 \neq j_0 \in \mathcal{Z}(\mathcal{N})$, applying Lemma 6 to the previous equation, we obtain

$$(x + x)d(z) \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}. \tag{31}$$

Taking $r(x_0 \circ j_0)$ instead of x in (31), where $r \in \mathcal{N}$, we find that

$$(r + r)(x_0 \circ j_0)d(z) \in \mathcal{Z}(\mathcal{N}) \text{ for all } r \in \mathcal{N}.$$

In view of (26) and Lemma 6, it follows that

$$(x_0 \circ j_0)d(z) = 0 \text{ or } r + r \in \mathcal{Z}(\mathcal{N}) \text{ for all } r \in \mathcal{N}. \tag{32}$$

We prove that $(x_0 \circ j_0)d(z) \neq 0$. In fact, suppose that $(x_0 \circ j_0)d(z) = 0$. In this case, left multiply both sides of the equation by $md(z)$, where $m \in \mathcal{N}$, we get $md(z)(x_0 \circ j_0)d(z) = 0$ for all $m \in \mathcal{N}$. Since, because of (27), $d(z)(x_0 \circ j_0) \in \mathcal{Z}(\mathcal{N})$, we can see that

$$d(z)(x_0 \circ j_0)\mathcal{N}d(z) = \{0\}.$$

By the 3-primeness of $\mathcal{N}$ and $d(z) \neq 0$, the previous relation gives $d(z)(x_0 \circ j_0) = 0$ which contradicts our hypothesis. Consequently, $(x_0 \circ j_0)$ $d(z) \neq 0$ and therefore (32) shows that $r + r \in \mathcal{Z}(\mathcal{N})$ for all $r \in \mathcal{N}$. Now, replacing r by r^2 in the last relation and invoking Lemma 6, we arrive at $2r = 0$ or $r \in \mathcal{Z}(\mathcal{N})$ for all $r \in \mathcal{N}$. Since $\mathcal{N}$ is 2-torsion free, the first condition yields $r = 0 \in \mathcal{Z}(\mathcal{N})$. So, from the both cases, we conclude that $\mathcal{N} \subseteq \mathcal{Z}(\mathcal{N})$ and thus $\mathcal{N}$ is a commutative ring.

(iii) By hypothesis, we have

$$d([x, j]) \circ t \in \mathcal{Z}(\mathcal{N}) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}. \tag{33}$$

Substituting $td([x,j])$ for t in (33), we get

$$(d([x,j]) \circ t)d([x,j]) \in \mathcal{Z}(\mathcal{N}) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}.$$

By application of Lemma 6, we obtain

$$d([x,j]) \circ t = 0 \text{ or } d([x,j]) \in \mathcal{Z}(\mathcal{N}) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}. \quad (34)$$

Let $(x,j) \in \mathcal{I} \times \mathcal{J}$. If there exists an element $t \in \mathcal{N}$ such that $d([x,j]) \circ t \neq 0$, then from (34) we conclude that $d([x,j]) \in \mathcal{Z}(\mathcal{N})$. Else, we will have $d([x,j]) \circ t = 0$ for all $t \in \mathcal{N}$, then

$$(-d([x,j]))t = td([x,j]) \text{ for all } t \in \mathcal{N}. \quad (35)$$

Taking tr instead of t, where $r \in \mathcal{I}$, in (35) and using it again, we find that

$$(-d([x,j]))tr = t(-d([x,j]))r \text{ for all } t \in \mathcal{N}, r \in \mathcal{I}$$

and hence $[-d([x,j]), t]\mathcal{I} = \{0\}$ for all $t \in \mathcal{N}$. In virtue of Lemma 1(ii), the preceding result shows that $-d([x,j]) \in \mathcal{Z}(\mathcal{N})$. Consequently, for all $(x,j) \in \mathcal{I} \times \mathcal{J}$, we have either

$$d([x,j]) \in \mathcal{Z}(\mathcal{N}) \text{ or } -d([x,j]) \in \mathcal{Z}(\mathcal{N}). \quad (36)$$

As $d(\mathcal{Z}(\mathcal{N})) \neq \{0\}$, there exists an element $z \in \mathcal{Z}(\mathcal{N})$ such that $z \neq 0$. In particular, replacing t by z in (33), we obtain

$$(d([x,j]) + d([x,j]))z \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

Since $z \neq 0$ and in view of Lemma 6, we infer that

$$(d([x,j]) + d([x,j])) \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

Once again, taking $t = d([x,j])$ in (33), we find that

$$(d([x,j]) + d([x,j]))d([x,j]) \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J},$$

which, because of Lemma 6, implies that

$$d([x,j]) \in \mathcal{Z}(\mathcal{N}) \text{ or } d([x,j]) + d([x,j]) = 0 \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

In another way,

$$d([x,j]) \in \mathcal{Z}(\mathcal{N}) \text{ or } d([x,j]) = -d([x,j]) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \quad (37)$$

Accordingly, from (36) and (37) we conclude that $d([x,j]) \in \mathcal{Z}(\mathcal{N})$ for all $x \in \mathcal{I}, j \in \mathcal{J}$, so that $\mathcal{N}$ is a commutative ring by (i).

(iv) By hypotheses, we have

$$d(x \circ j) \circ t \in \mathcal{Z}(\mathcal{N}) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}. \tag{38}$$

Substituting $td(x \circ j)$ for t in (38), we get

$$(d(x \circ j) \circ t)d(x \circ j) \in \mathcal{Z}(\mathcal{N}) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}.$$

By application of Lemma 6, the preceding expression indicates that

$$d(x \circ j) \circ t = 0 \text{ or } d(x \circ j) \in \mathcal{Z}(\mathcal{N}) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J}. \tag{39}$$

Now, consider $x \in \mathcal{I}$, $j \in \mathcal{J}$ and suppose that $d(x \circ j) \circ t = 0$ for all $t \in \mathcal{N}$. It follows that

$$(-d(x \circ j))t = td(x \circ j) \text{ for all } t \in \mathcal{N}. \tag{40}$$

Taking tr instead of t, where $r \in \mathcal{I}$, in (40) and using it again, we find that $(-d(x \circ j))tr = t(-d(x \circ j))r$ for all $t \in \mathcal{N}, r \in \mathcal{I}$ which can be written as $[-d(x \circ j), t]\mathcal{I} = \{0\}$ for all $t \in \mathcal{N}$. Applying Lemma 1 (ii), we get $-d(x \circ j) \in \mathcal{Z}(\mathcal{N})$. Hence, (39) reduces to

$$d(x \circ j) \in \mathcal{Z}(\mathcal{N}) \text{ or } -d(x \circ j) \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{41}$$

In (38), replace t with a nonzero element z of $\mathcal{Z}(\mathcal{N})$ and using the same arguments as those used between the two relations (36) and (37), we arrive at

$$d(x \circ j) \in \mathcal{Z}(\mathcal{N}) \text{ or } d(x \circ j) = -d(x \circ j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{42}$$

Combining (41) and (42) we conclude that $d(x \circ j) \in \mathcal{Z}(\mathcal{N})$ for all $x \in \mathcal{I}$, $j \in \mathcal{J}$ and hence $\mathcal{N}$ is a commutative ring by (ii). □

As an application of the previous theorems, we can get the following corollary if d acts as a derivation of $\mathcal{N}$.

Corollary 1. *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring, $\mathcal{I}$ be a nonzero semigroup ideal of $\mathcal{N}$ and $\mathcal{J}$ be a nonzero Jordan ideal of $\mathcal{N}$. If $\mathcal{N}$ admitting a derivation d such that $d(\mathcal{Z}(\mathcal{N})) \neq \{0\}$, then the following assertions are equivalent:*

(i) $d([x,j]) \in \mathcal{Z}(\mathcal{N})$ *for all* $x \in \mathcal{I}, j \in \mathcal{J}$*;*

(ii) $d(x \circ j) \in \mathcal{Z}(\mathcal{N})$ *for all* $x \in \mathcal{I}, j \in \mathcal{J}$*;*

(iii) $\mathcal{N}$ *is a commutative ring.*

Theorem 4. *Let* $\mathcal{N}$ *be a 2-torsion free 3-prime near-ring,* $\mathcal{I}$ *be a nonzero semigroup ideal of* $\mathcal{N}$ *and* $\mathcal{J}$ *be a nonzero Jordan ideal of* $\mathcal{N}$*. There is no nonzero multiplicative derivation* d *of* $\mathcal{N}$ *such that* $d(\mathcal{J}) \neq \{0\}$ *and satisfies one of the following conditions:*

(i) $d([x,j]) = x \circ d(j)$ *for all* $x \in \mathcal{I},\ j \in \mathcal{J}$*;*

(ii) $d(x \circ j) = [x, d(j)]$ *for all* $x \in \mathcal{I},\ j \in \mathcal{J}$*.*

Proof. (i) Assume that $\mathcal{N}$ admits a multiplicative derivation d such that $d(\mathcal{J}) \neq \{0\}$ and d satisfies

$$d([x,j]) = x \circ d(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{43}$$

Substituting xj for x in (43) and using it again, we get

$$[x,j]d(j) + (x \circ d(j))j = xj \circ d(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

After solving this expression, we find that

$$xd(j)j = jxd(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{44}$$

Replacing x by tx, where $t \in \mathcal{N}$, in (44) and invoking (44), we obtain

$$tjxd(j) = jtxd(j) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J},$$

which implies that $[t,j]\mathcal{I}d(j) = \{0\}$ for all $t \in \mathcal{N}, j \in \mathcal{J}$. In view of Lemma 1 (i), the last equation assures that

$$d(j) = 0 \text{ or } j \in \mathcal{Z}(\mathcal{N}) \text{ for all } j \in \mathcal{J}. \tag{45}$$

Suppose there is an element $j_0 \in \mathcal{J}$, such that $d(j_0) = 0$. Since $d(\mathcal{J}) \neq \{0\}$, there exists an element $i_0 \in \mathcal{J}$ such that $d(i_0) \neq 0$, then (45) assures that $i_0 \in \mathcal{Z}(\mathcal{N})$. Now, returning to (43) and replacing respectively x and j by xi_0 and j_0, we find that

$$xj_0d(i_0) = j_0xd(i_0) \text{ for all } x \in \mathcal{I}. \tag{46}$$

Taking tx instead of x, where $t \in \mathcal{N}$, in (46), we infer that

$$[t, j_0]\mathcal{I}d(i_0) = \{0\} \text{ for all } t \in \mathcal{N}.$$

In view of Lemma 1 (i) and the fact that $d(i_0) \neq 0$, we conclude that $j_0 \in \mathcal{Z}(\mathcal{N})$ and hence (45) shows that $j \in \mathcal{Z}(\mathcal{N})$ for all $j \in \mathcal{J}$. So, $\mathcal{N}$ is a commutative ring by Lemma 3 (ii). In this case, our hypothesis becomes $2xd(j) = 0$ for all $x \in \mathcal{I}$, $j \in \mathcal{J}$. By 2-torsion freeness of $\mathcal{N}$, we can see that $\mathcal{I}d(j) = \{0\}$ for all $j \in \mathcal{J}$ and therefore $d(J) = \{0\}$ by Lemma 1 (ii), but this contradicts our initial assumption that $d(\mathcal{J}) \neq \{0\}$.

(ii) By hypotheses given, we have

$$d(x \circ j) = [x, d(j)] \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{47}$$

Substituting xj for x in (47) and after simplifying, we arrive at

$$xd(j)j = (-j)xd(j) \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}. \tag{48}$$

Putting tx instead of x in (48), where $t \in \mathcal{N}$, and invoking (48), we obtain

$$t(-j)xd(j) = (-j)txd(j) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I}, j \in \mathcal{J},$$

implying that

$$[t, -j]\mathcal{I}d(j) = \{0\} \text{ for all } t \in \mathcal{N}, j \in \mathcal{J}.$$

In virtue of Lemma 1(i), the last equation yields

$$d(j) = 0 \text{ or } -j \in \mathcal{Z}(\mathcal{N}) \text{ for all } j \in \mathcal{J}. \tag{49}$$

Let j_0 an arbitrary element of $\mathcal{J}$ such that $d(j_0) = 0$. As $d(\mathcal{J}) \neq \{0\}$, there exists an element $i_0 \in \mathcal{J}$ such that $d(i_0) \neq 0$, then (49) assures that $-i_0 \in \mathcal{Z}(\mathcal{N})$. Now, in (47), replacing x and j by $x(-i_0)$ and j_0, respectively, we find that

$$xj_0d(-i_0) = (-j_0)xd(-i_0) \text{ for all } x \in \mathcal{I}. \tag{50}$$

Taking tx instead of x, where $t \in \mathcal{N}$, in (50) and using it again, we infer that

$$t(-j_0)xd(-i_0) = (-j_0)txd(-i_0) \text{ for all } t \in \mathcal{N}, x \in \mathcal{I},$$

which can be rewritter as

$$[t, -j_0]\mathcal{I}d(-i_0) = \{0\} \text{ for all } t \in \mathcal{N}.$$

In view of Lemma 1 (i), we get

$$d(-i_0) = 0 \text{ or } -j_0 \in \mathcal{Z}(\mathcal{N}).$$

Since $d(-i_0) \neq 0$, then $-j_0 \in \mathcal{Z}(\mathcal{N})$ and therefore (49) implies that $\mathcal{J} \subseteq \mathcal{Z}(\mathcal{N})$. So, $\mathcal{N}$ is a commutative ring by Lemma 3 (i) and thus (47) becomes $d(xj + xj) = d((x + x)j) = (x + x)d(j) + d(x + x)j = 0$ for all $x \in \mathcal{I}$, $j \in \mathcal{J}$. Substituting xj for x in the last equation and by defining property of d, we get $(j + j)xd(j) = 0$ for all $x \in \mathcal{I}$, $j \in \mathcal{J}$. Using the 2-torsion freeness of $\mathcal{N}$, we obtain $j\mathcal{I}d(j) = \{0\}$ for all $j \in \mathcal{J}$. Thereby, in view of Lemma 1 (i) we conclude that $d(\mathcal{J}) = \{0\}$, leading to a contradiction. □

Theorem 5. *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring, $\mathcal{I}$ be a nonzero semigroup ideal of $\mathcal{N}$ and $\mathcal{J}$ be a nonzero Jordan ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a nonzero multiplicative derivation d which satisfies $d(x) \circ j = 0$ for all $x \in \mathcal{I}$, $j \in \mathcal{J}$, then $\mathcal{N}$ cannot be a commutative ring.*

Proof. In the following we assume that the multiplicative law of $\mathcal{N}$ is commutative. By hypotheses given, we have $d(x) \circ j = 0$ for all $x \in \mathcal{I}$, $j \in \mathcal{J}$, so that

$$2jd(x) = 0 \text{ for all } x \in \mathcal{I}, j \in \mathcal{J}.$$

By the 2-torsion freeness of $\mathcal{N}$, the preceding equation gives $jd(x) = 0$ for all $x \in \mathcal{I}$. Taking $x = xt$, where $t \in \mathcal{N}$, by defining d and Lemma 2, we find that $jxd(t) = 0$ for all $j \in \mathcal{J}, x \in \mathcal{I}, t \in \mathcal{N}$. Which, because of Lemma 3 (ii), forces that $\mathcal{J} = \{0\}$ or $d = 0$, but in both cases we will have a contradiction. Thus our proof is complete. □

The following example demonstrates that the 3-primeness condition is essential in the assumptions of our theorems.

Example 1. Let $\mathcal{S}$ be a 2-torsion free zero-symmetric unitary right near-ring. Let us defined $\mathcal{N}$, $\mathcal{J}$, $\mathcal{I}$ and $d : \mathcal{N} \to \mathcal{N}$ by

$$\mathcal{N} = \left\{ \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix} \mid 0, x, y \in \mathcal{S} \right\}, \quad \mathcal{J} = \left\{ \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mid 0, x \in \mathcal{S} \right\},$$

$$\mathcal{I} = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & 0 & 0 \end{pmatrix} \mid 0, x \in \mathcal{S} \right\} \quad \text{and} \quad d\begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & x^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It is easy to verify that $\mathcal{N}$ is a 2-torsion free right near-ring which is not a 3-prime, $\mathcal{I}$ is a nonzero semigroup ideal of $\mathcal{N}$, $\mathcal{J}$ is a nonzero Jordan ideal of $\mathcal{N}$ and d is a nonzero multiplicative derivation of $\mathcal{N}$. Further, it can also be verified that $d(\mathcal{J}) \neq \{0\}$, $d(\mathcal{Z}(\mathcal{N})) \neq \{0\}$ and d satisfies the following properties:

1. $d([A, J]) = [A, J]$,
2. $d([A, J]) = -[A, J]$,
3. $d([A, J]) = [A, d(J)]$,
4. $d([A, J]) = A \circ J$,
5. $d([A, J]) = -(A \circ J)$,
6. $d(A \circ J) = [A, J]$,
7. $d(A \circ J) = -[A, J]$,
8. $d([A, J]) \in \mathcal{Z}(\mathcal{N})$,
9. $d(A \circ J) \in \mathcal{Z}(\mathcal{N})$,
10. $d([A, J]) \circ B \in \mathcal{Z}(\mathcal{N})$,
11. $d(A \circ J) \circ B \in \mathcal{Z}(\mathcal{N})$,
12. $d([A, J]) = A \circ d(J)$,
13. $d(A \circ J) = [A, d(J)]$,
14. $d(A \circ J) = 0$

for all $A \in \mathcal{I}, J \in \mathcal{J}, B \in \mathcal{N}$. However, $\mathcal{N}$ is not a commutative ring.

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