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Vitaliy M. Bondarenko

To the 75th anniversary

Professor Vitaliy M. Bondarenko, a leading researcher at the Institute of Mathematics of NAS of Ukraine, was there at the beginning of the theory of matrix problems, a new branch of modern algebra created and developed to a large extent by the Kyiv algebraic school.

He was born on May 3 (according to the birth certificate, on May 13), 1948, in the village of Kapitanivka in the Cherkasy region, well-known for its sugar factory, one of the first in Ukraine (1846) and the oldest currently operating. In his school years, Vitaliy Bondarenko was most interested in mathematics and chess. He participated in district, regional and republican mathematical olympiads and became a winner of many competitions for solving chess problems (later a champion of Kyiv).

After graduation from high school and one season's work at the sugar factory, Vitaliy Bondarenko entered in 1967 the Faculty of Mechanics and Mathematics of Kyiv T. G. Shevchenko State University (now Taras Shevchenko National University). He specialized in algebra (according to him, thanks to the lectures of L. A. Nazarova at junior courses) and listened to the lectures of Yu. A. Drozd, L. A. Kaluzhnin, V. V. Kirichenko, A. V. Roiter and S. T. Zavalo. In his course work, V. Bondarenko obtained a canonical form of the mutually annihilating pairs of matrices over a field under simultaneous similarity (his proof is alternative to that of I. M. Gelfand and V. A. Ponomarev). In his diploma he proved that the problem of classifying up to similarity matrices over the ring of residue classes modulo p^m, where p is simple and $m > 1$, contains (even if the matrices satisfy the equality $x^3 = 0$ or $x^{p^2} = 1$) the classical unsolved problem of classifying pairs of matrices up to (simultaneous) similarity, i.e., in modern terms, is wild. It was the first such result for zero-divisor local rings of principal ideals. These results were obtained under the supervision of A. V. Roiter and published respectively in Zapiski Nauchnykh Seminarov LOMI (in English translation, in Journal of Soviet Mathematics) and Mathematics collection ("Naukova Dumka", Kyiv). They determined the main directions of his further scientific researches.

After graduation from the university in 1972, Vitaliy Bondarenko became an engineer at the Institute of Mathematics of AS of UkSSR (now NAS of Ukraine), where he still works as a leading researcher. In graduate school of the Institute (1972 – 1975), V. Bondarenko studied modular representations of finite p-groups over fields. After the classification of the modular representations of the Klein group (V. A. Bashev, 1961) and the proof that a similar classification problem for all other noncyclic Abelian p-groups is wild (S. A. Krugljak, 1963, for $p > 2$ and S. Brenner, 1970, for $p = 2$), the greatest interest here was to determine whether a smallest non-commutative 2-group allows a complete classification of its representations over fields of characteristic 2 or this classification problem is wild. In his Candidate of Sciences dissertation "Modular representations of dihedral groups" (defended under A. V. Roiter in 1976), Vitaliy Bondarenko classified the indecomposable representations of all dihedral groups (not just 2-groups) over any field of characteristic 2. In the process he obtained a canonical form of the pairs of matrices over a field of any characteristic when both are nilpotent of degree 2. His generalizations of ideas of the dissertation to the quasi-dihedral groups and Drozd's ideas on tame and wild problems allowed to describe all groups of tame representation type (1977). The well-known result of V. M. Bondarenko and Yu. A. Drozd is formulated as follows: a finite group is tame over a field of characteristic $p > 0$ if and only if each noncyclic Abelian p-subgroup has order at most 4 (or in more detail: a group and its p-Sylow subgroup are both simultaneously tame or not, and a noncyclic p-group is tame if and only if $p = 2$ and its factor group by the commutant is the Klein group).

Free matrix problems over fields (ones without algebraic relations) were the next step in his study of representations.

In 1972, L. A. Nazarova and A. V. Roiter introduced the representations of posets in connection with the 2nd Brauer-Trall conjecture. Vitaliy Bondarenko actively participated in the initial study of such representations. He described the minimal non-domestic posets (together with L. A. Nazarova and A. G. Zavadskij) and the faithful tame posets of non-polynomial growth; established criteria of tameness and polynomiality of growth for the posets with shurian involution (together with L. A. Nazarova and A. V. Roiter) and much more generally with equivalence relation (together with A. G. Zavadskij), etc.

At the International Mathematical Congress in Nice (1970), I. M. Gelfand posed a linear algebra problem in connection with the classification of Harish-Chandra modules over $\mathrm{SL}(2, \mathbb{R})$ (an unsolved problem at that time). In modern terms it is the problem of classifying the indecomposable representations of the quiver with vertices $1, 2, 3$, arrows $\alpha_+ : 1 \to 2$, $\beta_+ : 3 \to 2$, $\alpha_- : 2 \to 1$, $\beta_- : 2 \to 3$ and relations $\alpha_+\alpha_- = \beta_+\beta_- = \gamma$, γ is nilpotent.

The first step in solving the Gelfand problem was made by L. A. Nazarova and A. V. Roiter in 1973, for any field and without the nilpotency condition. They reduced it to a class of free matrix problems which are closed under a special reduction, called by them self-reproducing systems. Their algorithmic answer essentially meant that these matrix problems and therefore the Gelfand problem are tame (at that time there was no strict definition of such problems). Later, in 1981, S. M. Khoroshkin indirectly solved the problem on representations of the Gelfand quiver over the field $\mathbb{R}$ by constructing indecomposable Harish-Chandra modules over $\mathrm{SL}(2, \mathbb{R})$ in a different way. In 1989, W. Crawley-Boevey classified indecomposable representations over a field with at least 3 elements of a quiver with relations Morita equivalent to the Gelfand one. After many years of study, Vitaliy Bondarenko came to a solution of the Nazarova-Roiter matrix problems in explicit and invariant (reduction-independent) form, which was first published in 1988. The received by this way explicit solution of the Gelfand problem over any field was improved by him in 1991, namely, the main invariants of the indecomposable representations of the Gelfand quiver were indicated in terms of the quiver itself.

In the process V. Bondarenko also received an explicit solution of some class of free matrix problems generalizing the Nazarova-Roiter self-reproducing systems, which were called by him the representations of bundles of semichains. As a consequence he generalized his results for the Gelfand quiver to an infinite series of quivers with relations (later named the Gelfand graphs) which arises in the problem of description of Harish-Chandra modules over some other simple Lie algebras.

From the 1990s until now, Bondarenko's classification theorem on representations of bundles of semichains has been used more than 70 times by various authors in their investigations of different problems of representations, algebraic geometry, algebraic topology, etc.

Vitaliy Bondarenko himself continued to use this method in many of his studies. He described all tame cases of the classification problem of linear operators in finite-dimensional graded (in the most general sence) vector spaces, satisfying a fixed polynomial relation, and tame "symmetric" cases of the classification problem of linear maps between two such vector spaces (1997 – 2003). In 2004 he obtained a classification of the representations of the generalized quaternion groups over fields of characteristic 2, the last series of tame p-groups whose modular representations at that time remained unclassified. In all these problems, the main role was played by representations of bundles of semichains.

The final point in the development of the concept of free matrix problems which arose under solving the Gelfand problem was defined by Vitaliy M. Bondarenko in 2002. He introduced the notion of dispersing representations of quivers, which contains representations of marked quivers and bundles of semichains, vector space categories (the most general case of linear matrix problems), etc. The natural extension of this notion to quiver with relations leads, in particular, to dispersing representations of groups, algebras, and so on.

At the same tame (1999 – 2004), V. Bondarenko received a number of results on representations of classical objects, which consist in the singling out of wild cases. Namely, they provided

– sufficient conditions of wildness of the problem of classifying up to equivalence matrix representations of a finite p-group over the ring of formal power series of one variable with coefficients in a complete discrete valuation ring (together with P. M. Gudyvok);

– a wildness of an algebra over a field of characteristic zero, generated by n idempotents with their sum equal a natural number m as the single defining relation, if $m \geq 2$ and $n \geq m + 2$;

– a wildness of idempotent generated algebras associated to extended Dynkin diagrams and their imaginary roots.

Since the beginning of this century, after receiving the Doctor of

Sciences degree in 2000, Vitaliy Bondarenko has been actively cooperating with the Faculty of Mechanics and Mathematics of the Kyiv Taras Shevchenko University, for 20 years he was working part-time as a guest lecturer; 6 out of 10 of his PhD students at the faculty graduated from it. He also cooperates with the Faculty of Mathematics and Digital Technologies of the Uzhgorod University; 4 of his dissertation students, including one Doctor of Sciences, graduated from it.

In total, from 2007 to 2020, 12 PhD and one Doctor of Sciences dissertations were defended under the supervision of Vitaliy M. Bondarenko (and two correspondence graduate students, successfully working on their dissertations, preferred other career opportunities). In joint works with his PhD students, he had investigated a wide range of problems in the representation theory of groups and semigroups, the theory of matrices over fields and local rings, the theory of posets and their Tits quadratic forms, etc; many of his ideas related to new topics. In particular, joint papers with PhD students provided

– a representation type of the problem of classification up to similarity of pairs of annihilating n-potent and m-potent matrices over a good field, and a classification of triples of idempotent cyclically annihilating matrices over any field;

– necessary and sufficient conditions (separately and together) of the indecomposability and irreducibility of monomial matrices over commutative local rings;

– sufficient conditions of wildness of the problem of classifying up to equivalence (rectangular) matrices over an integral domain in a generalized case when equivalence and wildness are considered modulo ideals;

– the following theorem (on six twos): a finite (non-trivial) 2-group is wild over a local factorial ring of characteristic not 2 with residue field of characteristic 2 if the ring contains 2 non-associated primes and the number of non-associated prime factors of 2 is greater than 2;

– a criterion of tame type for elementary Abelian groups with respect to modular representations of constant Jordan type;

– classifications of various classes of representations over fields of a direct product of symmetric groups and semigroups of degree 2 (which is as a rule wild);

– representation type of finite semigroups generated by idempotents with partial null multiplication;

– the notion of Σ-function for a finite system of representations of an algebra as a discrete characteristic of the category of representations, which in the case of a compete system of non-equivalent indecomposable representations can be interpreted as a “spectral dimension” of the corresponding Auslander algebra;

– a proof that all 35 monoids of fourth order are tame and classification of their indecomposable representations;

– a description of infinite posets with positive Tits quadratic form, proof that there does not exist an infinite poset critical with respect to positivity of Tits quadratic form and finite complete system of non-isomorphic critical posets;

– the concept of minimax isomorphism of finite posets which preserves up to $\mathbb{Z}$-equivalence Tits quadratic forms and a proof that a poset is a critical with respect to positivity of Tits quadratic form if and only if it is minimax isomorphic to that with respect to weakly positivity;

– description of finite posets with positive Tits quadratic form and the minimal ones with non-positive Tits quadratic form as analogues of Dynkin and extended Dynkin diagrams, respectively;

– the concept of local deformations of real quadratic forms, description of their parameters for different classes of integer positive ones, calculations of deformational diameters of Dynkin diagrams.

The current research of Vitaliy Bondarenko is devoted to categories of matrix representations of classical and modern objects, and classification, combinatorial and probabilistic problems of the theory of Tits quadratic forms.

Vitaliy M. Bondarenko has published about 150 scientific papers and is the author of two single-author monographs: “Representations of Gelfand graphs" (Kyiv, 2005) and "Linear operators on vector spaces graded by posets with involution: tame and wild cases" (Kyiv, 2006).

In 2007, Vitaliy M. Bondarenko was awarded the State Prize of Ukraine.

For more than 10 years, Professor Bondarenko has been a member of both Specialized Academic Council for the defence of dissertations that awards the degree of Doctor of Sciences in algebra (at Kyiv Taras Shevchenko National University and Institute of Mathematics of NAS of Ukraine). For many years he has been on the editorial board of Scientific Bulletin of Uzhgorod University, in recent years a reviewer of international scientific projects of the Ministry of Education and Science of Ukraine.

We warmly congratulate Vitaliy Bondarenko on the occasion of his 75th birthday and wish him strong health and many successful years of research and teaching.

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