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SURVEY ARTICLE

Generalized norms of groups: retrospective review and current status

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Abstract. In this survey paper the authors specify all the known findings related to the norms of a group and their generalizations (since 2016 in more details). Special attention is paid to the analysis of their own study of different generalized norms, particularly the norm of non-cyclic subgroups, the norm of Abelian non-cyclic subgroups, the norm of decomposable subgroups and relations between them.

Introduction

In group theory findings related to the study of characteristic subgroups and the impact of properties of these subgroups on the structure of a group are in the focus. Different Σ-norms of a group are characteristic subgroups of such a type.

Let Σ be a system of all subgroups of a group which have some theoretical group property. Σ-norm of a group G is the intersection of the normalizers of all subgroups of a group which belong to the system Σ. Let's denote it by $N_\Sigma(G)$. In the case $\Sigma=\varnothing$ we assume that $G = N_\Sigma(G)$.

The Σ-norm is a characteristic subgroup of a group and contains the center of a group. $N_\Sigma(G)$ is the maximal subgroup of a group that normalizes all Σ-subgroups of a group. Therefore, all subgroups of the Σ-norm, which belong to the system Σ, are normal in $N_\Sigma(G)$ (although these subgroups may not exist).

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Considering the Σ-norm, there are several problems related to the study of the group properties with the given system Σ of subgroups. There are still a number of problems in the study of groups with generalized norms: the study of groups that coincide with their Σ-norms; the study of groups that have identity Σ-norms or their Σ-norms coincide with the center; the study of groups that have non-central Dedekind Σ-norms; the study of groups that have non-Dedekind Σ-norms; the study of infinite groups that have Σ-norms of finite index. Many algebraists solved the similar problems but the choice of a system Σ (all subgroups of a group, all cyclic, all non-cyclic, all Abelian, all non-Abelian, all subnormal, all maximal, all infinite subgroups of a group) and properties of the Σ-norm varied.

Knowing the structure of Σ-norms and the nature of its attachment to a group, the properties of a group can be characterized in many cases. If the Σ-norm coincides with a group and $\Sigma \neq \varnothing$, then all subgroups of the system Σ are normal in a group. First non-Abelian groups with this property were considered in the XIX century by R. Dedekind [37], who gave a complete description of finite non-Abelian groups, all subgroups of which are normal, and called them Hamiltonian groups. Infinite Hamiltonian groups were described in 1933 by R. Baer [10]. Sets of Abelian and Hamiltonian groups combined are called the set of Dedekind groups.

However, the study of groups with other systems Σ of normal subgroups were continued only in the second part of the XX century, that slowed down the study of Σ-norms. The findings of S. M. Chernikov [32] – [35] and his disciples [63, 64, 71, 75–77, 100, 103, 106, 107, 114, 124] are from the very field of the research. Thus nowadays the structure of groups that coincide with the norm $N_\Sigma(G)$ is known for many systems of subgroup. So the question on the study of the properties of groups, in which the Σ-norm is a proper subgroup, arises naturally.

1. The norm of a group and its first generalizations

The problem of studying the properties of groups which differ from their Σ-norm, was formulated by R. Baer in the 1930s. In [3] he introduced the subgroup $N(G)$, which is the intersection of the normalizers of all subgroups of a group and called it *the norm of a group G*. The norm $N(G)$ is the Σ-norm of a group for the system Σ consisting of all subgroups of a group. It is clear that this norm is contained in all others Σ-norms, which can be considered its generalizations.

Since every subgroup of the norm $N(G)$ is normal in $N(G)$, the norm $N(G)$ is Dedekind (ie, it is an Abelian or a Hamiltonian group). Thus, the index of the norm in a group can be considered as a "measure" of Dedekindness of a group.

The norm of a group was quite actively studied by R. Baer [2]–[11], as well as a number of authors [13, 17, 22, 23, 41, 48, 54, 62, 109–111, 119, 121, 126–128, 132].

The conditions of the norm to be Abelian [11], its relations with the center of a group [9, 13, 48, 109, 111, 132], properties of groups, which norm has some index [127, 128], as well as groups that have an Abelian or cyclic quotient group $G/N(G)$ [5, 8, 126] are considered. In particular, in [126] it was proved that in a finite group G the quotient group $G/N(G)$ is cyclic if and only if a group G is nilpotent with cyclic quotient groups $P/N(P)$ for every Sylow subgroup P of a group G.

As noted above, the norm $N(G)$ is the Σ-norm of a group, in which the system of all subgroups of a group is chosen as the system Σ. Narrowing it and considering the system Σ to be other systems of subgroups of a group (containing all subgroups of a group with a certain theoretical-group property), we will obtain new Σ-norms, which can be considered as generalizations of the Baer norm $N(G)$.

The first generalizations of this kind appeared in the second half of the twentieth century and concerned mainly different systems of maximal subgroups in finite groups. Such generalizations, first of all, should be to include the notion of *the hypercenter of a group* G introduced in 1953 by R. Baer [4]. This characteristic subgroup has been defined in various ways: as the terminal member of the ascending central chain or as the smallest normal subgroup modulo which the center is E. In [4] R. Baer proved that the hypercenter of a finite group is the intersection of all normalizers of all Sylow subgroups.

Another generalization of the norm of a group was introduced by W. Kappe [57] in 1961. It is A-norm of a group, which is the intersection of the normalizers of all maximal Abelian subgroups of a group. Later W. Kappe [58]–[60] considered the E-norm of a group, which he defined as the intersection of normalizers all maximum subgroups of a group that have a certain theoretical group property E.

The subgroup $\Delta(G)$, which is related to the concept of E-norm, was studied by V. Gashuts [47] and was defined as the intersection of the normalizers of all maximum subgroups of a group. The Gashutz subgroup $\Delta(G)$ can be considered as the Σ-norm of a group for the system Σ, which consists of non-normal maximal subgroups of G. In [47] it was found that

$\Delta(G)$ is nilpotent and

$$\Delta(G)/\Phi(G) = Z(G/\Phi(G)),$$

where $\Phi(G)$ is Frattini subgroup of a group G.

The analysis of these findings were cited in the review article [40].

It should be noted that similar studies of generalized norms of Sylow subgroups of some subgroup X of a group G and related subgroups are held in our time. In particular, in [52] L. Gong, T. Ji and and B. Li considered the generalization of the norm of a group by studying on Sylow subgroups and $\mathcal{H}$-subgroups in finite groups which is denoted by $C(G)$ and $A(G)$, respectively. It is proved that the generalized norms $A(G)$ and $C(G)$ are all equal to the hypercenter of G.

Recall, that a subgroup H of a group G is called an $\mathcal{H}$-subgroup if $N_G(H) \cap H^g \leqslant H$ for all $g \in G$ (see [16]). The set of $\mathcal{H}$-subgroups of a group G is denoted by $\mathcal{H}(G)$ and it is easily to see that all Sylow subgroups and all maximal subgroups of G lie in $\mathcal{H}(G)$. The authors obtained a number of properties of norms $A(G)$ and $C(G)$, and statements that summarize Baer's results on the norm of a group. In particular, it is proved that $A(G) = E$ and $C(G) = E$ if and only if $Z(G) = E$.

2. Wielandt subgroup and related generalized norms of a group

In 1958, H. Wielandt studied the properties of normalizers of subnormal subgroups [131] and introduced the subgroup $W(G)$, which was defined as the intersection of normalizers of all subnormal subgroups of a group. The Wielandt subgroup $W(G)$ is the norm of subnormal subgroups of a group. Moreover, in a nilpotent group it coincides with the Baer norm $N(G)$. In addition, in a finite group G the subgroup $W(G)$ is not identity subgroup [131].

Wielandt subgroup and its generalizations were actively studied by a number of mathematicians, in particular, F. Amin, A. Ali and M. Arif [1], J. C. Beidleman, M. R. Dixon and D. J. S. Robinson [14,15], R. Brandl, F. de Giovanni, S. Franciosi [18], J. Cossey and R. Bryce [20,25,26,36], A. Camina [28], K. Casolo [29,30], C. Franchi [44,45], O. Kegel [61], F. Mari and F. de Giovanni [101,102], E. Ormerod [104], D. Robinson [105], J. Roseblade [108], C. Wetherell [129,130], K. Zhan and K. Guo [55,135], H. Shelash and A. Ashrafi [115]. Let's note such generalizations:

– *local Wielandt subgroup* $W^p(G)$ is the intersection of normalizers of all p'-perfect subnormal subgroup of a group G. Let's regard that the

p'-perfect group is a group that has no non-identity quotient groups of order coprime with p [25];

– *m-Wielandt subgroup $U_m(G)$* of a group G that is the intersection of normalizers of all subnormal subgroups of a group G with a defect at most m for integer $m \geqslant 1$ [20, 44, 45];

– *generalized Wielandt subgroup $IW(G)$* which is the intersection of normalizers of all infinite subnormal subgroups of a group [14, 15];

– *N-Wielandt subgroup $W_N(G)$* which consists of all elements of a group G, which normalize all subnormal subgroups of N ([1]);

– *strong Wielandt subgroup $\overline{W}(G)$* is the intersection of the centralizers of nilpotent subnormal quotient groups of a group G:

$$\overline{W}(G) = \left\{ g \in G | \, [S, g] \leqslant S^R \text{ for all } S << G \right\},$$

where S^R is nilpotent residual of the subgroup S or the smallest normal subgroup N of S such that the quotient group S/N is nilpotent [30, 129, 130].

In [101] F. Mari, F. Giovanni introduced a new Σ-norm, in which the system Σ consists of all non-subnormal subgroups of a group. This *norm of non-subnormal subgroups* was denoted by $W^*(G)$. It is clear that if $W^*(G) = G$, then all subgroups are subnormal in a group G. Moreover, if a group G is a group with a finite number of normalizers of subnormal subgroups, then the quotient group $G/W^*(G)$ is finite [101].

In 2019, M. Lewis and M. Zarrin [65] considered the norm $B_n(G)$, which is the intersection of all normalizers of non n-subnormal subgroups of a group G ($B_n(G) = G$ if all subgroups of G are n-subnormal). Let's regard that subgroup K of group G is n-subnormal in G if there are a chain of subgroups in G such that

$$K = K_0 \lhd K_1 \lhd \cdots \lhd K_n = G.$$

In particular, a normal subgroup is a 1-subnormal subgroup, so $B_1(G)$ coincides with the intersection of the normalizers of all non-normal subgroups of groups G and $B_1(G) = N(G)$. In [65] it was proved that in a finite non-nilpotent group G, each non-subnormal subgroup of which is self-normalizing, $B_m(G)$ is cyclic for all integers $m = 1$. The following statement characterizes the relations between the norm $B_n(G)$ and members of the ascending central series of a group.

Theorem 1 ([65]). *If G is a finite group and n is a positive integer, then there is a positive integer m so that $B_n(G) \leqslant Z_m(G)$, where $Z_m(G)$ is the m-th term of the upper central series of G.*

3. Generalized norms of $\mathfrak{F}$-residuals of some systems of subgroups

A number of findings are devoted to the study of Σ-norms, in which the system Σ is the system of some characteristic subgroups. Sh. Li, Zh. Shen and W. Shi [67, 117] considered Σ-norm the system for derived subgroups of all subgroups of a finite group G and called it $D(G)$. They defined the D-group, i.e., $G = D(G)$, and characterized the relations between $D(G)$ and G. It was proved that D-groups have nilpotent derived subgroup. They fingered out that G is solvable with Fitting length at most 3 if all elements of G of prime order are in $D(G)$ (see [67]).

Continuing the study of Σ-norms of characteristic subgroups, L. Gong, L. Zhao and K. Guo considered the subgroup $\omega^{\mathcal{A}}(G)$, which is the norm of the derived subgroups of all subnormal subgroups, and the subgroup $\theta^{\mathcal{A}}(G)$, which is the norm of the derived subgroups of all non-subnormal subgroups in a finite group G and the relations between them [53]. It is proved that $\omega^{\mathcal{A}}(G)$ is soluble and $C_G(G') = 1$ if and only if $\omega^{\mathcal{A}}(G) = 1$.

Zh. Shen, J. Chen, and Sh. Li [86] studied the norm $CS(G)$ of all subgroups of the derived subgroup of a group G. If $G = CS(G)$, then all subgroups of the derived subgroup G' are normal in a group G. The groups with such a property were studied by I. Ya. Subbotin [124], so the authors called them CS-groups (or Chernikov-Subbotin groups).

In [69], the so-called D^*-norm, close to the norm $D(G)$, was considered. It was denoted by $D^*(G)$ and defined as the intersection of the normalizers of the derived subgroups of all subgroups H of G such that H is generated by two elements of G and H' is nilpotent. In this paper authors show that $D^*(G) = D(G)$ if G is a finite group. Moreover it was proved that $G = D^*(G)$ in a finite soluble group.

Another class of subgroups that have a significant impact on a group structure is the class of centralizers. In [133] the subgroup $C(G)$, which is the intersection of the normalizers of the centralizers of all elements of a group G

$$C(G) = \bigcap_{a \in G} N_G(C_G(a)),$$

and its impact on group properties were studied.

A large number of findings of recent years concern to the norms of of different systems of residuals. Recall that $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ of a group G is the smallest normal subgroup N of G such that $G/N \in \mathfrak{F}$, where $\mathfrak{F}$ is some class of groups. For example, if $\mathfrak{F} = \mathcal{A}$ is a class of Abelian groups, then the $\mathcal{A}$-residual $G^{\mathcal{A}}$ of a group G coincides with the derived subgroup

G'. Accordingly, if $\mathfrak{F} = \mathcal{N}$ is a class of nilpotent groups, then the nilpotent residual $G^{\mathcal{N}}$ of a group G is the intersection of all normal subgroups of the group, the quotient groups of which are nilpotent.

In [123] N. Su and Ja. Wang considered generalized norm $N_{\mathfrak{F}}(G)$ of $\mathfrak{F}$-residuals of all subgroups of a group G. It is the intersection of normalizes of $\mathfrak{F}$-residuals of all subgroups of a group G:

$$N_{\mathfrak{F}}(G) = \bigcap_{H \leqslant G} N_G(H^{\mathfrak{F}}),$$

where $\mathfrak{F}$ is some formation, that is, a class of groups closed with respect to the quotient groups by normal subgroups and their intersections.

In this paper authors release several deep relations between $G = N_{\mathfrak{F}}^{\infty}(G)$ and $G \in \mathfrak{F}$, where $\mathfrak{F}$ is the class of all groups which $\mathfrak{F}$-residuals are nilpotent. The terminal term of ascending series:

$$N_{\mathfrak{F}}^{0}(G) = 1 \leqslant N_{\mathfrak{F}}^{1}(G) \leqslant \cdots \leqslant N_{\mathfrak{F}}^{i}(G) \leqslant N_{\mathfrak{F}}^{i+1}(G) \leqslant \ldots,$$

where $N_{\mathfrak{F}}^{i+1}(G)/N_{\mathfrak{F}}^{i}(G) = N_{\mathfrak{F}}(G/N_{\mathfrak{F}}^{i}(G))$, is denoted by $N_{\mathfrak{F}}^{\infty}(G)$.

Obviously, the norm $N_{\mathfrak{F}}(G)$ is a characteristic subgroup and every element of $N_{\mathfrak{F}}(G)$ normalizes the $\mathfrak{F}$-residual of each subgroup of G. The norm of $\mathfrak{F}$-residual has many interesting properties and also closely related to the global properties of a given group. At the same time, a large number of findings are devoted to the study of the relations between the norm $N_{\mathfrak{F}}(G)$ and the p-length, Fitting length, solvability, p-nilpotency for some formations $\mathfrak{F}$ (see [31, 49–51, 66, 118]).

For example, if $\mathfrak{F} = \mathcal{A}$, we have $N_{\mathcal{A}}(G) = D(G)$, where $D(G)$ is the norm of derived subgroups of all subgroups of G [67, 117].

In [66] authors studied a local version of the norm $D(G)$ of derived subgroups of all subgroups of G, which is the intersection of the normalizers of the residuals of all subgroups of G with respect to the class of all Abelian p-groups, where p is a prime. The norm $D_p(G)$ of Abelian p-group-residuals is the subgroup

$$D_p(G) = \bigcap_{H \leqslant G} N_G(H'(p)),$$

where $H'(p)$ is the Abelian p-residual of the subgroup H (i.e. the smallest normal subgroup of G for which the corresponding quotient group is an Abelian p-group). It was proved, that $D_p(G) = 1$ if and only if $C_G(G'(p)) = 1$.

Theorem 2 ([66])**.** *Let G be a finite group. Then $D_p(G) = A \leftthreetimes P$, where P is the Sylow p-subgroup and A is a Hall p'-subgroup of $D_p(G)$. Moreover, $P = D(P)$ and $A = N(A)$. In particular, $\bigcap_{p||G|} D_p(G) = N(G)$.*

Zh. Shen, W. Shi and G. Qian [118] studied the nilpotent residuals of subgroups. They considered the case when $\mathfrak{F} = \mathcal{N}$ is a class of nilpotent groups. The authors introduced the notion of the *norm $N_{\mathcal{N}}(G)$ of nilpotent residuals of all subgroups of G* and denoted it by $S(G)$. It is the intersection of normalizes of nilpotent residuals of all subgroups of a group:

$$S(G) = \bigcap_{H \leqslant G} N_G(H^{\mathcal{N}}).$$

It is clear that this norm contains the Baer norm and the norm $D(G)$ of the derived subgroups of all subgroups:

$$N(G) \subseteq D(G) \subseteq S(G).$$

It was proved some properties of the norm of nilpotent residuals in finite groups: $G^{\mathcal{N}} \cap S(G) = 1$ if and only if $Z(G^{\mathcal{N}}) = 1$. Moreover, in a finite group G, which is a direct product of two subgroups of coprime order $G = A \times B$, $(|A|, |B|) = 1$, the norm $S(G)$ is the direct product of norms of such subgroups: $S(G) = S(A) \times S(B)$.

In [118], the author studied groups which coincide with the norm $S(G)$ (they were called S-groups). It was proved that the nilpotent residual $G^{\mathcal{N}}$ of a finite supersoluble S-group G is Abelian. Moreover, if all elements of prime order of G are in $S(G)$, then G is soluble.

The norm of nilpotent residuals were also studied by L. Gong and K. Guo in [49]. The authors, in fact, obtained the same results, as Zh. Shen, W. Shi and G. Qian [118], although others approaches to their proof have been used.

N. Su and Ja. Wang [122] continued to study the norms of $\mathfrak{F}$-residuals ($\mathfrak{F}$ is some formation) of all subgroups of a group G (the norm was denoted by $D_{\mathfrak{F}}(G)$), the properties of such norms and their impact on some characteristics of a group. They studied the norm $D^p_{\mathfrak{F}}(G)$, which is the intersection of normalizers of subgroups $H^{\mathfrak{F}} O_{p'}(G)$ for all subgroups H of a finite group G (see [122]). The authors summarized the results of the findings [49, 118, 123] regarding the norm of residuals of all subgroups of a group and studied the properties of the terminal members $D^{\infty}_{\mathfrak{F}}(G)$ and $D^{\infty}_{\mathfrak{F},p}(G)$ of the ascending series of $D^i_{\mathfrak{F}}(G)$ and $D^i_{\mathfrak{F},p}(G)$ respectively:

$$D^0_{\mathfrak{F}}(G) = D^0_{\mathfrak{F},p}(G) = 1, \ldots, D^{i+1}_{\mathfrak{F}}(G)/D^i_{\mathfrak{F}}(G) = D_{\mathfrak{F}}(G/D^i_{\mathfrak{F}}(G)),$$
$$D^{i+1}_{\mathfrak{F},p}(G)/D^i_{\mathfrak{F},p}(G) = D_{\mathfrak{F},p}(G/D^i_{\mathfrak{F},p}(G)).$$

It was proved that under certain hypotheses, the $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ is nilpotent (respectively, p-nilpotent) if and only if $G = D^{\infty}_{\mathfrak{F}}(G)$ (respectively, $G = D^{\infty}_{\mathfrak{F},p}(G)$). Further more, if the formation $\mathfrak{F}$ is either the class of all nilpotent groups or the class of all Abelian groups, then $G^{\mathfrak{F}}$ is p-nilpotent if and only if every cyclic subgroup of order p and 4 of G (if $p = 2$) is contained in $D^{\infty}_{\mathfrak{F},p}(G)$.

In the case $\mathfrak{F} = \mathcal{N}_p$ is the class of all p-nilpotent groups, X. Guo and X. Li [68] introduced the norm of $\mathcal{N}_p$-residual of a group G (this norm is denoted by $N^{\mathcal{N}_p}(G)$). As a local version of L. Gong results, X. Li and X. Guo characterized the relations between $C_G(G^{\mathcal{N}_p})$ and $N^{\mathcal{N}_p}(G)$. In particular, they investigated the relations between the norm $N^{\mathcal{N}}(G)$ of the nilpotent residuals of all subgroups of G and $N^{\mathcal{N}_p}(G)$.

K. Chen and W. Guo [31] studied $\mathfrak{h}\mathfrak{F}$-norm $N_{\mathfrak{h}\mathfrak{F}}(G)$ of a group G, that is the intersection of normalizes of products of $\mathfrak{F}$-residual of all subgroups of a group G and $\mathfrak{h}$-radical of a group G

$$N_{\mathfrak{h},\mathfrak{F}}(G) = \bigcap_{H \leqslant G} N_G(H^{\mathfrak{F}} G_{\mathfrak{h}}),$$

where $\mathfrak{h}$ is Fitting class, $\mathfrak{F}$ is formation. Let's regard that $\mathfrak{h}$-radical $G_{\mathfrak{h}}$ of a group G is maximal normal $\mathfrak{h}$-subgroup of a group G.

Ch. Fu, Zh. Shen and Q. Yan in [46] considered the norm $N^{\mathfrak{D}_p}(G)$, which is the intersection of the normalizers of the p-decomposable residuals of all subgroups of G for a prime p. As in the findings mentioned above, ascending series are constructed from the conditions

$$N_0^{\mathfrak{D}_p}(G) = 1, N_{i+1}^{\mathfrak{D}_p}(G)/N_i^{\mathfrak{D}_p}(G) = N^{\mathfrak{D}_p}(G/N_i^{\mathfrak{D}_p}(G))$$

for $i = 0, 1, 2, \ldots$. The terminal term of the ascending series is denoted by $N_{\infty}^{\mathfrak{D}_p}(G)$. The authors proved that $G = N_{\infty}^{\mathfrak{D}_p}(G)$ if and only if the p-decomposable residual $G^{\mathfrak{D}_p}$ is nilpotent. Furthermore, if the index of $N_{\infty}^{\mathfrak{D}_p}(G)$ is coprime with p, then G is p-solvable with $l_p(G) = 1$.

In [112, 113] V. Selkin studied π-decomposable norm $N_\pi(G)$ of a finite group G. π-*decomposable norm* $N_\pi(G)$ is the intersection of normalizers of π-decomposable residuals of all subgroups of a finite group G. Let's regard that a finite group G is π-decomposable, if it can be presented as a direct product

$$G = O_\pi(G) \times O_{\pi'}(G),$$

where $O_\pi(G)$ is the maximal normal π-subgroup of a group G. A group G is meta-π-decomposable if G is an extension of a π-decomposable group by a π-decomposable group. It was proved that a finite group G meta-π-decomposable if and only if the quotient group $G/N_\pi(G)$ is π-decomposable.

In [113] π-special norm $N_{\pi sp}(G)$ of a finite group G, which is the intersection of normalizes of π-special residuals of all subgroups of a group G, was considered. Let's regard that a finite group G is called *π-special,* if

$$G = O_{p_1}(G) \times O_{p_2}(G) \times \cdots \times O_{p_n}(G) \times O_{\pi'}(G), \pi = \{p_1, \dots, p_n\} \subseteq P.$$

It was proved that a finite group G is meta-π-special if and only if the quotient group $G/N_{\pi sp}(G)$ is meta-π-special. Moreover, a number of statements summarized the results of [49, 118] for the norm of nilpotent residuals of all subgroups of a group G.

In 2020 B. Hu, J. Huang and A. Skiba [56, 134] considered the *σ-nilpotent norm* $N_\sigma(G)$ of a group G, which is the intersection of the normalizers of the σ-nilpotent residuals of all subgroups of G.

A group G is said to be *σ-nilpotent* if $G = G_1 \times \cdots \times G_t$ for some σ-primary groups $G_1, \dots, G_t$, where $\sigma = \{\sigma_i | i \in I\}$ is some partition of the set of all primes. The group is called *σ-soluble* if every chief factor of G is σ-primary (i.e. is a σ_i-group for some i). If G is σ-soluble, then the *σ-nilpotent length* $l_\sigma(G)$) of G is the length of the shortest normal chain of G with σ-nilpotent factors.

The authors studied the relations of the σ-nilpotent length with the σ-nilpotent norm of G and proved that the σ-nilpotent length of a σ-soluble group G is at most r $(r > 1)$ if and only if $l_\sigma(G/N_\sigma(G)) \leqslant r$. Moreover, the following result takes place.

Theorem 3 ([56])**.** *If G is σ-soluble and every element of G of odd prime order is in $N_\sigma(G)$, then $l_{\sigma_i}(G) \leqslant 1$ and $G/O_{\sigma_i',\sigma_i}(G)$ is σ-nilpotent for all i such that $2 \in \sigma_i'$.*

Yu. Lv and Ya. Li [74] considered the norm closed to the residuals of some classes of subgroups. Let $\mathfrak{F}$ be a formation and G be a finite group. The *weak norm of H in G with respect to $\mathfrak{F}$* is defined by

$$N_{\mathfrak{F}}(G, H) = \bigcap_{T \leqslant H} N_G(T^{\mathfrak{F}}).$$

In particular, $N_{\mathfrak{F}}(G, G) = N_{\mathfrak{F}}(G)$. Obviously, that

$$N_{\mathfrak{F}}(G) \leqslant N_{\mathfrak{F}}(G, G) \leqslant N_{\mathfrak{F}}(G, H) \leqslant G.$$

In this paper, for the case $\mathfrak{F} \in \{\mathfrak{U}_p, \mathfrak{U}\}$, where $\mathfrak{U}_p$ ($\mathfrak{U}$, respectively) is the class of all finite p-supersolvable groups (supersolvable groups), the authors characterize the structure of some finite groups by the properties of the weak norm of some subgroups in G with respect to $\mathfrak{F}$. It was proved that for a p-soluble group G and it's normal subgroup H the condition $N_{\mathfrak{U}_p}(G, H) = 1$ is equal to the condition $C_G(H^{\mathfrak{U}_p}) = 1$.

The following statements hold in the class of solvable groups (see [74], Theorem 3.13 and Theorem 3.14 respectively).

Theorem 4. *Let G be a soluble group and H be a normal subgroup of G, then $\bigcap_{p\in\pi(H)} N_{\mathfrak{U}_p}(G, H) = G$ if and only if $N_{\mathfrak{U}}(G, H) = G$.*

Theorem 5. *Let G be a soluble group and H be a normal subgroup of G, then $\bigcap_{p\in\pi(H)} N_{\mathfrak{U}_p}(G, H) = 1$ if and only if $N_{\mathfrak{U}}(G, H) = 1$.*

Moreover, in [74] the properties of groups which coincide with the norm $N_{\mathfrak{U}_p}(G, H)$ were studied. They were called $\mathcal{N}_1$-group with respect to H and $\mathfrak{U}_p$. Obviously, G is an $\mathcal{N}_1$-group with respect to H and $\mathfrak{U}_p$ if and only if $K^{\mathfrak{U}_p} \trianglelefteq G$ for every $K \leqslant H$. Let's point the following statement among the properties of $\mathcal{N}_1$-group (see [74], Theorem 4.4 and Theorem 4.5).

Theorem 6. *Let G be an $\mathcal{N}_1$-group with respect to itself and $\mathfrak{U}$. Then G is soluble. In particular, $N_{\mathfrak{U}}(G)$ and $N_{\mathfrak{F}}^{\infty}(G)$ are soluble.*

In this theorem $N_{\mathfrak{F}}^{\infty}(G)$ is the terminal term of an upper series $N_{\mathfrak{F}}^{i}(G), i \geqslant 1$ of G: $N_{\mathfrak{F}}^{0}(G) = 1, N_{\mathfrak{F}}^{i+1}(G)/N_{\mathfrak{F}}^{i}(G) = N_{\mathfrak{F}}(G/N_{\mathfrak{F}}^{i}(G))$.

Theorem 7. *Let G be a group and H be a subgroup of G. If G is an $\mathcal{N}_1$-group with respect to H and $\mathfrak{U}_p$, where p is the smallest prime in $\pi(H)$, then H is p-solvable.*

Let's point out that the idea of considering the intersection of normalizers of all subgroups of some subgroup H in a group G belongs I. Ya. Subbotin [125], and such a norm was called the invariator of the subgroup H in a group G.

4. Generalized norms of system Σ of some natural types of subgroups

Quite a large number of findings are devoted to the study of the properties of Σ-norms and the impact of these properties on the characteristics of the whole group for systems consisting of subgroups of natural types (for example, all Abelian, all non-Abelian, all cyclic, all non-cyclic subgroups, etc.). Let's note that if a system Σ is the system of all Abelian and even all cyclic subgroups of G, it does not lead to an extension of the norm $N(G)$. However, choosing others natural systems of subgroups (for example, all non-cyclic or all non-Abelian subgroups), we get Σ-norms, which differ from the Baer norm $N(G)$ in the general case.

Among such Σ-norms let's note the non-cyclic norm N_G, which is the intersection of the normalizers of all non-cyclic subgroups of G (provided that a group contains such subgroups). This norm was introduced by F. M. Lyman in [70], where it was proved that in classes of infinite locally finite, as well as torsion locally soluble by finite groups the norm N_G has a finite index if and only if a group G is central-by-finite. Moreover, in this case the norm N_G is Dedekind (Abelian in torsion groups) if $N_G \neq G$.

Conditions under which the norm of non-cyclic subgroups of a torsion group is Dedekind was considered in [99].

Theorem 8. *The non-cyclic norm N_G of a torsion locally soluble by finite group G is Dedekind, if the one of the following conditions takes place:*

1) *a group G contains a non-cyclic subgroup A such that $A \bigcap N_G = E$;*
2) *the non-cyclic norm N_G of a group G is torsion;*
3) *a group G contains a non-identity cyclic N_G-admissible subgroup $\langle g \rangle$, such that $\langle g \rangle \bigcap N_G = E$;*
4) *a group G contains a free Abelian subgroup of rank 2;*
5) *a group G has the torsion center $Z(G)$;*
6) *a group G contains a finite Abelian non-cyclic subgroup;*
7) *G is a torsion-free group.*

Impact of properties of the non-cyclic norm on properties and structure of quite wide classes of groups under the condition of non-Dedekindness of the norm N_G have been studied in [79, 90, 91, 94, 95, 98]. Under these restrictions, the authors not only got some characteristics of groups, but also described their structure in classes of locally finite and torsion locally soluble by finite groups. In particular, in [98] the authors got that an torsion locally soluble by finite group with the non-Dedekind non-cyclic norm is a group in which all non-cyclic subgroups are normal. Such groups were studied by F. M. Lyman in [75].

Let's note that the properties of the norm of non-cyclic subgroups in the class of finite groups and its impact on a group were also studied by Zh. Shen, W. Shi, J. Zhan [119, 120].

The studies of the non-Abelian norm $N^*(G)$ of a group G introduced by F. Mari, F. de Giovanni [101] was quite fruitful. The norm $N^*(G)$ is the intersection of the normalizers of all non-Abelian subgroups of a group G. It is clear that in groups with the condition $N^*(G) = G$ all non-Abelian subgroups are normal. Such groups were studied by G. M. Romalis and M. F. Sesekin [106,107,114], V. T. Nagrebetsky [103], O. A. Makhnev [100], M. M. Semko and M. F. Kuzenny [63,64] and were called metahamiltonian. In [101] it was proved that a locally graded group G with the finite quotient group $G/N^*(G)$ has the finite derived subgroup G'. This result generalizes well-known Schur's theorem on the finiteness of the derived subgroup in central-by-finite groups.

The study of the norm of non-Abelian subgroups was continued by M. Falko, F. de Giovanni, L. Kurdachenko and C. Musela in [42,43]. The authors called it the *metanorm of a group* and denoted by $M(G)$. They studied properties of locally finite groups with Abelian metanorm. It was proved that locally finite groups with non-nilpotent metanorm $M(G)$ and $|(M(G))'| = p$ (p is a prime), are metahamiltonian (Theorem 4.8 [42]). Moreover, for groups with non-Abelian metanorm the following statement takes place [42].

Theorem 9. *Let G be a locally finite group which metanorm $M(G)$ is not metabelian, and p be a characteristic of $M(G)$. Then every Sylow p-subgroup of G is nilpotent of class 2.*

In [43] the authors continued the study embedding properties of the metanorm. In particular, it will be shown that the subgroup $[M(G), G]$ is torsion for any locally graded group G, and the behavior of the metanorm in locally nilpotent groups will be studied in detail. Moreover, it was proved that if G is any locally graded group with a non-nilpotent metanorm, then the only obstruction to a strong hypercentral embedding of $M(G)$ in G is represented by the section $(M(G))'/(M(G))''$, which is a small eccentric chief factor. In fact, in this situation it turns out that the subgroup $(M(G))''$ is contained in the centre $Z(G)$ of G and $M(G)/(M(G))'$ lies in the center of $G/(M(G))'$.

They are quite effective to study infinite groups with the restrictions on different systems Σ of infinite subgroups of a group. Findings of such a type are [79,83,84,99] for systems: all infinite, all infinite Abelian, all infinite cyclic and all cyclic subgroups of non-prime order of a group, provided that given systems of subgroups are not empty. The corresponding Σ-norms were denoted by:

– $N_G(\infty)$ is the norm of infinite subgroups;
– $N_G(A_\infty)$ is the norm of infinite Abelian subgroups;
– $N_G(C_\infty)$ is the norm of infinite cyclic subgroups;

– $N_G(C_{\bar{p}})$ is the norm of cyclic subgroups of non-prime order of a group G.

In the class of torsion groups we get the following relation

$$Z(G) \subseteq N(G) \subseteq N_G(\infty) \subseteq N_G(A_\infty) \subseteq N_G(C_\infty).$$

In [79] it was proved that in a torsion group G these norms coinside with the center of a group and

$$N(G) = N_G(\infty) = N_G(A_\infty) = N_G(C_\infty) = Z(G),$$

if $Z(G)$ contains elements of infinite order. The last statement is a generalization of Baer's results [11] on the relations between the norm $N(G)$ of group and its center.

The following statement [99] characterizes sufficient conditions for the Abelianity of these norms in torsion groups.

Theorem 10. *In a non-periodic group G the norms $N_G(C_\infty)$, $N_G(A_\infty)$, $N_G(C_{\overline{p}})$ are Abelian in each of the following cases:*

1) *the center $Z(G)$ contains elements of infinite order;*
2) *every of the given Σ-norms is torsion;*
3) *a group G contains an infinite cyclic subgroup $\langle x\rangle$ which has an identity intersection with the Σ-norm ($N_G(C_\infty)$, $N_G(A_\infty)$, $N_G(C_{\overline{p}})$);*
4) *a group G contains a Σ-subgroup A which has an identity intersection with Σ-norm;*
5) *G is an involution-free group.*

A number of findings [79, 83, 84] related to these norms of infinite subgroups, concerned the study of the impact of the properties of these norms on the properties of a group under the condition of their non-Dedekindness or the finiteness of their index in a group. In particular, in [84] it was described the structure of the norm under the condition of its non-Dedekindness and it was proved that in torsion free groups such a norm is Abelian.

In [79] the structure of the norm $N_G(A_\infty)$ of infinite Abelian subgroups in the class of torsion groups and its impact on properties of a group with non-Dedekind norm $N_G(A_\infty)$ is described.

Groups with the above restrictions on the norm $N_G(\infty)$ of infinite subgroups were studied in [79, 83]. It was proved that torsion non-Abelian groups with the norm $N_G(\infty)$ of finite index are mixed and finite extensions of their centers.

Infinite locally finite groups, which are finite by the norm $N_G(\infty)$, or are finite extensions of their centers, or have the finite center and are finite extensions of the direct product of a finite number of quasicyclic p-subgroups (p is the same prime) were considered. The detailed description of main properties of the norms of infinite subgroups mentioned above are given in [40].

M. Brescia and A. Russo [19] studied the cyclic norm $C(G)$ of a group G, which is the intersection of the normalizers of every maximal locally cyclic subgroup of G. $C(G)$ coincides with the set of all the elements of G including cyclic automorphisms on G. The quotient group $C(G)/Z(G)$ is isomorphic with $Inn(G)/CAut(G)$, where $CAut(G)$ is the group of cyclic automorphisms of a group G. The authors proved that if $C(G)$ has finite index in G, then G is central-by-finite.

In [96] F. M. Lyman and T. D. Lukashova introduced one more generalized norm – the norm N_G^A of Abelian non-cyclic subgroups of a group G, which is the intersection of normalizers of all Abelian non-cyclic subgroups of a group. This norm is considered only for groups, where the system of Abelian non-cyclic subgroups is non-empty.

The properties of the norm N_G^A and its impact on properties of a group under some restrictions were studied in [38,39,78,80,82,85–87,93,96,97,99] for classes of locally finite and torsion groups. Some results are given in the survey article [40].

A number of general properties and conditions of the Dedekind norm of Abelian non-cyclic subgroups are described in [38,93,96,99]. In [93,96] the properties of locally finite p-groups (p is a prime) with the non-Dedekind norm N_G^A were studied. It was proved that a locally finite p-group with the non-Dedekind norm N_G^A does not contain elementary Abelian subgroups of order p^3. This statement summarizes the corresponding F. M. Lyman's results [76, 77] for $\overline{HA}_p$-groups, which are the norms of their Abelian non-cyclic subgroups.

It should be noted that the non-Dedekindness condition of the norm N_G^A is a rather strong restriction and significantly impacts on the properties of a group. This is confirmed by the results [82,96], where the comprehensive description of infinite locally finite p-groups (p is a prime) with these restrictions on N_G^A was given.

It was proved in [96] that the class of infinite locally finite p-groups ($p \neq 2$) with the non-Dedekind norm N_G^A coincides with the class of infinite $\overline{HA}_p$-groups (i.e., non-Abelian p-groups in which all Abelian non-cyclic subgroups are normal). The class of infinite locally finite 2-groups with the non-Dedekind norm N_G^A is richer. Exept $\overline{HA}_2$-groups it contains groups with non-normal Abelian non-cyclic subgroups [82]. All such groups satisfy the condition of minimality for Abelian subgroups and are finite extensions of the quasicyclic subgroups. Similar results were obtained for infinite locally finite groups with the non-Dedekind locally nilpotent norm N_G^A [80].

A number of findings [38, 85–87, 97] were devoted to the study of finite p-groups (p is a prime) with the non-Dedekind norm N_G^A. The authors obtained the complete description of the groups structure, which finally puts an end to their study.

Among the latest studies, let's note the main result of [97], which characterizes the finite 2-group with cyclic center and the metacyclic non-Dedekind norm N_G^A of Abelian non-cyclic subgroups.

Theorem 11. *An arbitrary finite* 2*-group with the cyclic center and the non-Dedekind metacyclic norm N_G^A of Abelian non-cyclic subgroups is a group of one of the following types:*

1) $G = \langle a\rangle\langle b\rangle, |a| = 2^n, n > 2, |b| = 8, b^4 = a^{2^{n-1}}, b^{-1}ab = a^{-1}$, $N_G^A = G$;
2) $G = \langle a\rangle\lambda\langle b\rangle, |a| = 2^n, |b| = 2^m, n \geqslant 2, m \geqslant 1, [a, b] = a^{2^{n-1}}$, $N_G^A = G$;
3) $G = \langle y\rangle \lambda \langle b\rangle, |y| = 8, |b| = 2, [y, b] = y^3; N_G^A = \langle y^2\rangle \lambda \langle b\rangle$;
4) $G = (H \times \langle b\rangle) \langle a\rangle, H = \langle h_1, h_2\rangle, |h_1| = 2^k > 4, h_1^{2^{k-1}} = h_2^2, a^2 = h_1^{2^{k-2}}, h_2^{-1}h_1h_2 = h_1^{-1}, |b| = 2, [a, h_1] = a^4, [a, h_2] = b, [a, b] = a^4$; $N_G^A = \langle a\rangle \lambda \langle b\rangle$;
5) $G = \langle y\rangle \langle b\rangle, \langle y\rangle \cap \langle b\rangle = E, |y| = 2^k, k \geqslant 4, |b| = 2^m, m \geqslant 2$, $[y, b] = y^{2^{k-m}s}b^{2^{m-1}t}, (s, 2) = 1, t \in \{0, 1\}; N_G^A = \langle y^{2^{m-1}}\rangle\lambda \langle b\rangle$.

Study of the properties of the non-Dedekind norm N_G^A and its impact on the properties of a group in torsion groups were conducted by F. M. Lyman and M. G. Drushlyak in [39, 78]. It was proved that a torsion group G, in which the norm N_G^A is non-Hamiltonian $\overline{HA}$-group, does not contain free Abelian subgroups of rank 2 if and only if the norm N_G^A does not contain such subgroups.

The above results summarize the results of [76, 77], which relate to different classes of non-Hamiltonian groups with the condition $G = N_G^A$, ie, groups in which all Abelian non-cyclic subgroups are normal.

The impact of one more generalized norm of a group – *the norm N_G^d of decomposable subgroups* – on different characteristics of a group was considered in [72]. This is Σ-norm of a group G, where Σ consists of all decomposable subgroups of a group (if there is no decomposable subgroup in a group, we consider $G = N_G^d$).

Since the structure of rather broad classes of groups in which this norm coincides with a group (ie, groups that do not contain decomposable subgroups and groups in which every decomposable subgroup is normal) is known (see, for example, [71]), it is natural to study the properties of groups, which the norm N_G^d satisfies some restrictions and is a proper subgroup of a group. In [72, 73, 81, 89] as such a restriction the non-Dedekindness of this norm was chosen.

Due to the rather close relations between the classes of decomposable Abelian and Abelian non-cyclic subgroups the relations between the norms of decomposable and Abelian non-cyclic subgroups were studied in [72, 73, 89, 92]. In [72], such relations were considered for the class of locally finite groups. It was proved that in a locally finite group G, which contains an Abelian non-cyclic subgroup, there is one of inclusions: $N_G^A = N_G^d$, or $N_G^A \supset N_G^d$, or $N_G^A \subset N_G^d$. In particular, relations between these norms in the class of locally finite p-groups are described in the following theorem (see [72]).

Theorem 12. *In a locally finite p-group G which contains an Abelian non-cyclic subgroup, the norms of Abelian non-cyclic subgroups and decomposable subgroups coincide: $N_G^A = N_G^d$.*

It follows from the above Theorem that in a locally finite p-group, which contains at least one Abelian non-cyclic subgroup, the norm N_G^d of decomposable subgroups is non-Dedekind if and only if the norm N_G^A of Abelian non-cyclic subgroups is non-Dedekind. So, an infinite p-group with the non-Dedekind norm N_G^d is a finite extension of a quasicyclic subgroup.

On the other hand, in the class of finite non-primary and infinite periodic locally nilpotent non-primary groups these norms are connected by the inclusion of $N_G^A \supseteq N_G^d$ (this inclusion can not be strict). Under the additional condition of non-Dedekindness of the norm N_G^d the following statment takes place for torsion locally nilpotent groups. It reduces the study of such groups to locally finite p-groups with the non-Dedekind norm of Abelian non-cyclic subgroups.

Theorem 13 ([72])**.** *A periodic locally nilpotent group G with a non-cyclic Abelian subgroup has the non-Dedekind norm N_G^d of decomposable subgroups if and only if G is a locally finite p-group with non-Dedekind norm N_G^A of Abelian non-cyclic subgroups.*

In [89], the properties of locally finite non-primary (respectively, not locally nilpotent) groups in which the norm of decomposable subgroups is a non-Dedekind locally nilpotent subgroup were considered. It was proved that under this condition in the class of infinite locally finite groups with an Abelian non-cyclic subgroup the norm N_G^d also coincides with the norm N_G^A of Abelian non-cyclic subgroups.

Exhaustive characterization of infinite locally finite not locally nilpotent groups in which the norm N_G^d of decomposable subgroups satisfies the given restrictions is in the following theorem.

Theorem 14 ([89]). *An infinite locally finite non-locally nilpotent group G has the non-Dedekind locally nilpotent norm N_G^d of decomposable subgroups if and only if*

$$G = (A \times \langle b \rangle) \leftthreetimes \langle c \rangle \leftthreetimes \langle h \rangle \,,$$

where A is a quasicyclic p-group (p is an odd prime, $p \neq 2^k \cdot 3^l + 1$ for any non-negative integers k and l), $|b| = |c| = p$, $[A, \langle c \rangle] = 1$, $[b, c] = a \in A$, $|a| = p$, $|h| = q^n$ for a prime $q > 3$ and $n \geqslant 1$, q^n divides $(p-1)$, $h^{-1}bh = b^r, h^{-1}ch = c^s$ for integers r and s with $1 < r < p$, $1 < s < p$ such that $r \neq s$ and $rs \not\equiv 1 \,(mod\ p)$, $C_G(y) = \langle h \rangle$ for each non-identity element $y \in \langle h \rangle$.

Moreover, $N_G^d = (A \times \langle b \rangle) \leftthreetimes \langle c \rangle$.

The study of the impact of the norm N_G^d of decomposable subgroups on the properties of torsion locally soluble groups was continued in [73, 81, 92].

It was found that if the norm N_G^d of decomposable subgroups is non-Dedekind, then a group G contains some systems of decomposable subgroups only when such subgroups exist in its norm N_G^d (this applies to systems of decomposable, non-primary Abelian and free Abelian subgroups of rank greater than 1). Moreover, a decomposable Abelian subgroup of a group G is mixed if and only if every decomposable Abelian subgroup of the norm N_G^d is mixed.

In [73] it was proved that a torsion locally soluble group G with the torsion non-Dedekind norm N_G^d which does not contain decomposable subgroups, coincides with this norm. Its structure was described.

In [81] torsion locally soluble groups with locally nilpotent non-Dedekind norm of decomposable subgroups were considered. It was found that under this condition the norm N_G^d cannot be a torsion group. It was also proved that in the class of torsion locally nilpotent groups the condition of non-Dedekindness of the norm of decomposable subgroups is equivalent to the condition of normality of all decomposable subgroups in a group. The characterization of such groups is given in the following theorem.

Theorem 15 ([81]). *In a torsion locally nilpotent group G the norm N_G^d of decomposable subgroups is non-Dedekind if and only if $G = N_G^d$ and G is a group of one of the following types:*

1) *$G = Q \times B$, where Q is the quaternion group of order 8, B is an Abelian torsion free group of rank 1;*
2) *$G = \langle a \rangle \leftthreetimes B$, where $|a| = p^n$, p is a prime, $n > 1$, B is not p-divisible Abelian torsion free group of rank 1 and $[\langle a \rangle, B] = \langle a^{p^{n-1}} \rangle$.*

Moreover, the complete description of torsion locally soluble and non-locally nilpotent groups with non-Dedekind locally nilpotent norm N_G^d is given in [81]. The relations between norms of decomposable and Abelian non-cyclic subgroups in the class of locally soluble torsion groups are got.

Theorem 16 ([81]). *If at least one of the norms N_G^A or N_G^d is non-Dedekind and the norm N_G^d is infinite in a torsion locally soluble group G, then one of the inclusions $N_G^A \subseteq N_G^d$ or $N_G^d \subseteq N_G^A$ takes place.*

The study of relations between the norm N_G^d of decomposable subgroups and the norm N_G^A of Abelian non-cyclic subgroups in torsion groups were continued in [92]. In particular, the properties of torsion groups with identity intersection of these norms were considered. If Abelian non-cyclic subgroups exist in a group and $N_G^d \neq E$, $N_G^d \cap N_G^A = E$, then the norm N_G^d of decomposable subgroups is a finite cyclic group of an odd order (and of a prime odd order if the norm N_G^A is non-Dedekind), the norm N_G^A is an Abelian torsion free group of rank 1 or a finite cyclic extension of such a group.

Theorem 17 ([92]). *If a non-periodic locally soluble group G has an Abelian non-cyclic subgroup, the norm N_G^A of Abelian non-cyclic subgroups is non-Dedekind, the norm N_G^d of decomposable subgroups is nonidentity and $N_G^d \cap N_G^A = E$, then the following conditions take place:*

1) *$Z(G) = N(G) = E$, where $N(G)$ is the norm of G;*
2) *the norm of decomposable subgroups $N_G^d = \langle c \rangle$ is a cyclic group of a prime odd order;*
3) *the norm N_G^A of Abelian non-cyclic subgroups is a group of the type $N_G^A = A \leftthreetimes \langle b \rangle$, where A is a group isomorphic to an additive group of p-adic fractions (p is prime, $(p, 2) = 1$), $|b| = 2$ and $b^{-1}ab = a^{-1}$ for any element $a \in A$;*
4) *any infinite cyclic subgroup has a nonidentity intersection with the norm N_G^A;*
5) *a group G does not contain free Abelian subgroups of rank 2;*

6) *a group G does not contain finite non-cyclic Abelian subgroups;*
7) *a group G does not contain torsion non-cyclic locally cyclic subgroups;*
8) *the factor-group G/N_G^A is torsion.*

Conclusion

Thus, the study of generalized norms of groups is a very effective and popular research direction in group theory. Taking additional restrictions on generalized norms of a group, it is possible to obtain various extensions of already known classes of groups. For example, in Dedekind groups the norm $N(G)$ coincides with a whole group. Considering the situation when such a norm has a finite index in an infinite group, we obtain an extension of the class of infinite Dedekind groups. In this case, we can study the relations between this class of groups and the class of groups in which every subgroup is normal by finite.

Similarly, we can get the problem of studying the groups in which Σ-norm is non-Dedekind and a proper subgroup. How show the above results, the choice as a determinant restriction the non-Dedekindness of Σ-norm is quite successful and allows to characterize the structure of a group in many cases.

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