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The classification of serial posets with the non-negative quadratic Tits form being principal

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ABSTRACT. Using (introduced by the first author) the method of (min, max)-equivalence, we classify all serial principal posets, i.e. the posets S satisfying the following conditions: (1) the quadratic Tits form $q_S(z) : \mathbb{Z}^{|S|+1} \to \mathbb{Z}$ of S is non-negative; (2) $\operatorname{Ker} q_S(z) := \{t \mid q_S(t) = 0\}$ is an infinite cyclic group (equivalently, the corank of the symmetric matrix of $q_S(z)$ is equal to 1); (3) for any $m \in \mathbb{N}$, there is a poset $S(m) \supset S$ such that $S(m)$ satisfies (1), (2) and $|S(m) \setminus S| = m$.

1. Introduction

In [11], for a finite quiver Q with the set of vertices Q_0 and the set of arrows Q_1, P. Gabriel introduced a quadratic form $q_Q : \mathbb{Z}^n \to \mathbb{Z}$, $n = |Q_0|$, called by him the *quadratic Tits form of the quiver* Q:

$$q_Q(z) = \sum_{i \in Q_0} z_i^2 - \sum_{i \to j} z_i z_j,$$

where $i \to j$ runs through the set Q_1. He proved that a connected quiver is of finite type over a field (i.e. has only finitely many isomorphism classes of indecomposable representations) if and only if its underlying graph is one of the (simply faced) Dynkin diagrams and that the quivers of

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finite type coincide with the quivers whose quadratic Tits form is positive. This Gabriel's work laid the foundations of a new direction in the theory of algebras dealing with the investigation of the relationships between the properties of representations of various objects and the properties of quadratic forms associated with these objects.

The above quadratic form is naturally generalized to a finite poset $S \not\ni 0$ [10]:

$$q_S(z) = z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j,i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i.$$

In [10] Yu. A. Drozd showed that a poset S is of finite type if and only if its quadratic Tits form is weakly positive, i.e. takes positive value on any non-zero vector with non-negative coordinates (representations of posets over a field were introduced by L. A. Nazarova and A. V. Roiter [16]).

In contrast to quivers, the sets of posets with weakly positive and with positive Tits form do not coincide. Therefore the investigations of posets with positive Tits form seems to be quite natural: they are analogs of the Dynkin diagrams. Posets of this type were studied by the authors in many papers (see e.g. [3] – [6]). In particular, the classification of posets with positive Tits form was obtained (see Section 2).

One has a similar situation for quivers and posets of tame type. A quiver Q is of tame type over a field if and only if its quadratic Tits form is non-negative. This follows from the facts that a connected quiver is of tame infinite type iff its underlying graph is an extended Dynkin diagram [9, 15], and that the connected quivers with non-negative, but not positive, Tits form coincide with the quivers, the underlying graphs of which are extended Dynkin diagrams [8]. A poset S is of tame type if and only if its quadratic Tits form is weakly non-negative (see [18] and [17, Theorem 15.23]). Since (in contrast to quivers) the sets of posets with weakly non-negative and with non-negative Tits forms do not coincide, the investigations of posets with non-negative Tits form also are actual.

The present paper is devoted to the study of one class of posets with non-negative quadratic Tits form.

2. Positive and principal posets

Throughout the paper, all posets are finite of order $n > 0$ without the element 0. In the case, when the elements of a poset are numbered by integer numbers, the relation of partial order is denoted by $\prec$ (and one always assumes that $i \prec j$ implies $i < j$).

2.1. Definitions on posets. A poset T is called *dual* to a poset S and is denoted by S^{op} if $T = S$ as usual sets and $x < y$ in T if and only if $x > y$ in S.

By a subposet we always mean a full one, and singletons are identified with the elements themselves. Sometime (in definitions or statements) we admit empty posets which are or may be later subposets of some posets.

A poset S is called a *sum of subposets* $A_1, A_2, \ldots, A_m$ and write $S = A_1 + A_2 + \cdots + A_m$, if $S = \cup_{i=1}^{m} A_i$ and $A_i \cap A_j = \varnothing$ for any i and $j \neq i$. If any two elements of different summands are incomparable, the sum is called *direct* and one writes also $\coprod$ instead of $+$.

A sum $S = A + B$ with $A, B \neq \varnothing$ is said to be *left* (resp. *right*) if $a < b$ (resp. $b < a$) for some $a \in A, b \in B$ and there is no $a' \in A, b' \in B$ satisfying $a' > b'$ (resp. $b' > a'$). Both left and right sums are called *one-sided.* A sum $S = A + B$ is called *two-sided* if $a < b$ and $a' > b'$ for some $a, a' \in A, b, b' \in B$. Finally, a one-sided (left or right) or two-sided sum $S = A + B$ is called *minimax* if $x < y$ with x and y belonging to different summands implies that x is minimal and y maximal in S.

2.2. The quadratic Tits form. Let S be a poset. *The quadratic form of* S is by definition the following quadratic form $q : \mathbb{Z}^{|S|+1} \to \mathbb{Z}$:

$$q = q_S(z) = z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i$$

(see [10]). Here $\mathbb{Z}$ denotes as usual the ring of integer numbers. Obviously, one can assume, without loss of generality, that $S = \{1, 2, \ldots, n\}$ $(n \geqslant 1)$; then $\mathbb{Z}^{|S|+1} = \mathbb{Z}^{n+1}$ consists of the integer vectors $(z_0, z_1, \ldots, z_n)$.

2.3. Positive posets. A poset with the quadratic Tits form being positive (i.e. $q_S(z) > 0$ for all non-zero vectors $z \in \mathbb{Z}^{|S|+1}$) is called *positive.*

In this subsection we adhere to the results of [4].

The positive posets are of two types: serial and non-serial.

A positive poset S is called *serial* if for any $m \in \mathbb{N}$, there is a positive poset $S(m) \supset S$ such that $|S(m) \setminus S| = m$, and *non-serial* otherwise. There are 108 non-serial posets up to isomorphism and duality, and 194 up to isomorphism (see Table 2 in [4]).

Now formulate two theorems on serial positive posets.

A linear ordered set with $n \geqslant 0$ elements is called a *chain of length* n. A poset with one pair of incomparable elements $a_1 < \ldots < a_p < \{b, c\} < d_1 < \ldots d_q$ $(p, q \geqslant 0)$ is called an *almost chain of length* $n = p + q + 1$ ($a_p < \{b, c\} < d_1$ means that $a_p < b < d_1$, $a_p < c < d_1$; b and c are incomparable).

Theorem 1. *A poset T is serial positive if and only if it is isomorphic to one of the following poset S:*

(1) *S is a direct sum of a chain of length $k \geqslant 0$ and a chain of length $s \geqslant 1$, where $k \leqslant s$;*

(2) *S is a left minimax sum of two chains of lengths $k \geqslant 1$ and $s \geqslant 1$, where $k + s > 3$;*

(3) *S is a direct sum of an almost chain of length $k \geqslant 1$ and a chain of length $s \geqslant 0$, where $k + s > 1$.*

Moreover, all these posets are pairwise non-isomorphic.

Theorem 2. *Any positive poset of order $n > 7$ is serial.*

2.4. Principal posets. A poset S and the quadratic Tits form $q_S(z)$ are called *principal* (see [12]) if the following conditions hold:

(1) $q_S(z)$ is non-negative (i.e. which accepts only non-negative values);

(2) $\operatorname{Ker} q_S(z) := \{t \mid q_S(t) = 0\}$ is an infinite cyclic group, i.e. $\operatorname{Ker} q_S(z) = t'\mathbb{Z}$ for some $t' \neq 0$ (equivalently, the corank of the symmetric matrix of $q_S(z)$ is equal to 1).

The principal posets form a natural class of the posets with non-negative quadratic Tits form.

By analogy with the definition of a serial positive poset, we call a principal poset S *serial* if for any $m \in \mathbb{N}$, there is a principal poset $S(m) \supset S$ such that $|S(m) \setminus S| = m$.

Some class of principal posets of order $n = 6, 7, 8$ (which in our terminology means the non-serial ones) were written by G. Marczak, D. Simson and K. Zając with the help of programming in Maple and Python (see the paper [13] for $n = 6, 7$ and the preprint [14] for $n = 8$).

In this paper, using the method of (min, max)-equivalence, we classify all serial principal posets.

3. Main results

We adhere to the definitions of Section 2 and give some new definitions.

For subposets A, B of a poset S we write $A < B$ if $a < b$ for any $a \in A, b \in B$. A poset P is called a *semichain of length s* if it has the form $P = \cup_{i=1}^{s} P_i$ with $P_1 < P_2 < \cdots < P_s$, where every P_i consists of one or two incomparable elements. The number of two-element P_i is said to be 2-*length* of the semichain P. Note that chains and almost chains (see subsection 2.3) are semichains of 2-length 0 and 1, respectively.

We can now formulate the main theorems of this paper.

Theorem 3. *A poset T is serial principal if and only if it is isomorphic to one of the following poset S:*

(I) *S is a direct sum of a chain of length $k \geqslant 0$, and a semichain of length $s \geqslant 2$ and 2-length 2;*

(II) *S is a direct sum of a semichain of length $k \geqslant 1$ and 2-length 1, and a semichain of length $s \geqslant 1$ and 2-length 1, where $k \leqslant s$;*

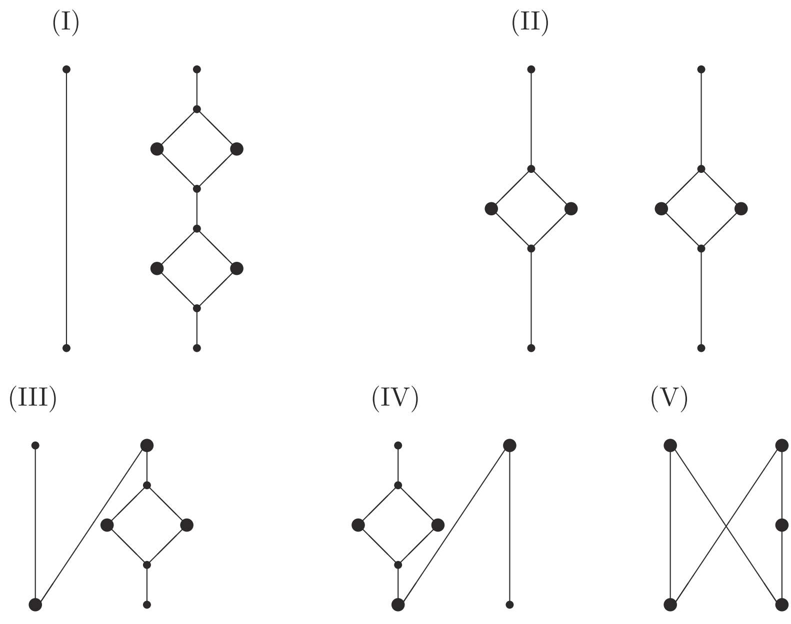
(III) *S is a left minimax sum of a chain of length $k \geqslant 1$, and a semichain of length $s \geqslant 2$ and 2-length 1 with the only maximal element;*

(IV) *S is a left minimax sum of a semichain of length $k \geqslant 2$ and 2-length 1 with the only minimal element, and a chain of length $s \geqslant 1$;*

(V) *S is a two-sided minimax sum of a chain of length $k \geqslant 2$ and a chain of length $s \geqslant 3$, where $k \leqslant s$.*

Moreover, all these posets are pairwise non-isomorphic.

In the language of Hasse diagrams the posets indicated in the theorem have the following form



Here vertical lines are chains, and inclined segments do not contain intermediate points. The large points indicated in the figures (unlike small and intermediate ones) must always be present.

The second main theorem is the following.

Theorem 4. *Any principal poset of order $n > 8$ is serial.*

4. Minimax equivalence of posets

In this section we recall notation and results from the papers [1], [4] and formulate some corollaries which will be used in the proof of the main theorems.

Let S be a poset. For a minimal (resp. maximal) element a of S, denote by $T = S_a^{\uparrow}$ (resp. $T = S_a^{\downarrow}$) the following poset: $T = S$ as usual sets, $T \setminus a = S \setminus a$ as posets, the element a is maximal (resp. minimal) in T, and a is comparable with x in T if and only if they are incomparable in S. A poset T is called *minimax equivalent* or (min, max)-*equivalent* to a poset S, if there are posets $S_1, \dots, S_p$ ($p \geqslant 0$) such that, if one puts $S = S_0$ and $T = S_p$, then, for every $i = 0, 1, \dots, p$, either $S_{i+1} = (S_i)_{x_i}^{\uparrow}$ or $S_{i+1} = (S_i)_{y_i}^{\downarrow}$ (the case $p = 0$ means that S is minimax equivalent to S).

The notion of minimax equivalence can be naturally continued to the notion of *minimax isomorphism*: posets S and S' are minimax isomorphic if there exists a poset T, which is minimax equivalent to S and isomorphic to S'.

The definition of posets of the form $T = S_a^{\uparrow}$ (resp. $T = S_a^{\downarrow}$) can be extended to subposets. Namely, let S be a poset and A its lower (resp. upper) subposet, i.e. $x \in A$ whenever $x < y$ (resp. $x > y$) and $y \in A$. By $T = S_A^{\uparrow}$ (resp. $T = S_A^{\downarrow}$) we denote the following poset: $T = S$ as usual sets, partial orders on A and $S \setminus A$ are the same as before, but comparability and incomparability between elements of $x \in A$ and $y \in S \setminus A$ are interchanged and the new comparability can only be of the form $x > y$ (resp. $x < y$). Note that S and $S_A^{\uparrow}$ (resp. S and $S_A^{\downarrow}$) are minimax equivalent.

We write $S_{AB}^{\uparrow\uparrow}$ instead of $(S_A^{\uparrow})_B^{\uparrow}$, $S_{AB}^{\uparrow\downarrow}$ instead of $(S_A^{\uparrow})_B^{\downarrow}$, etc. Obviously, $S_{AA}^{\uparrow\downarrow} = S$, $S_{AA}^{\downarrow\uparrow} = S$, $S_A^{\uparrow} = S_{S\setminus A}^{\downarrow}$, $S_A^{\downarrow} = S_{S\setminus A}^{\uparrow}$.

From the definitions we have the following corollary.

Corollary 1. $(S_A^{\downarrow})^{\mathrm{op}} = (S^{\mathrm{op}})_{A^{\mathrm{op}}}^{\uparrow}$.

The main motivation for introducing the notion of minimax equivalence is the fact that the Tits forms of minimax equivalent posets are $\mathbb{Z}$-equivalent. This follows from the next proposition.

Proposition 1. *Let S be a poset and let $T = S_A^{\uparrow}$ or $T = S_A^{\downarrow}$. Then $q_S(z) = q_T(z')$, where $z_0' = z_0 - \sum_{a\in A} z_a$, $z_x' = -z_x$ for $x \in A$ and $z_x' = z_x$ for $x \notin A$.*

Corollary 2. *Let S and T be the same as in Proposition 2, and let $x \notin A$. Then $T \setminus x$ is positive if so is $S \setminus x$.*

Corollary 3. *A poset minimax equivalent to a principal one is also principal.*

5. Proofs of Theorems 3 and 4

We first prove that all posets of the form (I)–(V) are principal; then their seriality is obvious.

It is easy to see that

(a) if S is of the form (I) and L denotes the chain of length $k \geqslant 0$, then $S_L^{\uparrow}$ is also a poset of the form (I) (with the empty chain);

(b) if S is of the form (II) and L denotes the first semichain, then $S_L^{\uparrow}$ is a poset of the form (I) (with the empty chain);

(c) if S is of the form (III) and p denotes the minimal element of the chain of length $k \geqslant 1$, then $S_p^{\uparrow}$ is a poset of the form (I);

(d) if S is of the form (IV) and p denotes the minimal element of the semichain of length $k \geqslant 2$, then $S_p^{\uparrow}$ is a poset of the form (II);

(e) if S is of the form (V) and p denotes the minimal element of the second chain, then $S_p^{\uparrow}$ is a poset of the form (IV).

So, by Corollary 3, it is sufficient to consider only the case of posets of the form (I) with $k = 0$, i.e. the case of semichains of 2-length 2. By formulas (3) and (18) of [2], the quadratic Tits form $q_P(z)$ of a semichain $P = \{P_1 < P_2 < \cdots < P_s\}$ of 2-length 2 with two-element sets $P_i = \{u_1, u_2\}$, $P_j = \{v_1, v_2\}$ $(i \neq j)$ and one-element sets $P_k = \{p_k\}$ $(p \neq i, j)$ satisfies the following equality:

$$2q_P(z) = z_0^2 + (z_0 - \sum_{k\neq i,j} z_{p_k} - z_{u_1} - z_{u_2} - z_{v_1} - z_{v_2})^2 + \sum_{k\neq i,j} z_{p_k}^2 + (z_{u_1} - z_{u_2})^2 + (z_{v_1} - z_{v_2})^2.$$

From here it follows that $q_P(z)$ is principal, and so P is principal.

Thus the sufficiency of the Theorem 3 is proved.

Since all subposets of posets of the forms (I)–(V) also have such forms (and all posets $S(m)$ in the definition of serial principal posets are also the same ones), for the proof of the necessity of Theorem 3 and Theorem 4 it suffices to show that the next statement holds.

Proposition 2. *Any principal poset of order $n > 8$ is one of the form (I)–(V).*

We prove first the following lemma.

Lemma 1. *Let $A = \{a\} \coprod B$ be a principal poset of order $n > 8$ with B to be a positive poset. Then $B = \{b\} \coprod C$, where C is an almost chain (consequently, A is of the form* (II) *with $k = 1$).*

Proof. For proof, we need the following facts: The posets

$T_1 = \{1, 2, 3, 4, 5, 6, 7, 8 \,|\, 2 \prec 3 \prec 4, 5 \prec 6 \prec 7 \prec 8\}$,

$T_2 = \{1, 2, 3, 4, 5, 6, 7, 8, 9 \,|\, 2 \prec 3, 4 \prec 5 \prec 6 \prec 7 \prec 8 \prec 9\}$,

$T_3 = \{1, 2, 3, 4, 5, 6, 7, 8, 9 \,|\, 2 \prec 9, 3 \prec 4 \prec 5 \prec 6 \prec 7 \prec 8 \prec 9\}$,

$T_4 = \{1, 2, 3, 4, 5, 6, 7, 8, 9 \,|\, 2 \prec 3, 2 \prec 9, 4 \prec 5 \prec 6 \prec 7 \prec 8 \prec 9\}$,

$T_5 = \{1, 2, 3, 4, 5 \,|\, 4 \prec 5\}$.

are not non-negative[1]. They are follows from the equalities

$q_{T_1}(8, 4, 2, 2, 2, 2, 2, 1, 1) = q_{T_2}(12, 6, 4, 4, 2, 2, 2, 2, 1, 1) =$
$= q_{T_3}(8, 4, 6, 2, 2, 2, 2, 1, 1, -4) = q_{T_4}(8, 4, 4, 2, 2, 2, 2, 1, 1, -2) =$
$= q_{T_5}(4, 2, 2, 2, 1, 1) = -1.$

Since the poset B is positive of order $n' = n - 1 > 7$, it is of the form (1) or (2) or (3) (see Theorems 1 and 2).

In case (1) $\{a\} \coprod B$ is positive if $k \leqslant 1$ (by (1) and (3) of Theorem 1), and is not non-negative if $k > 1$ (because it contains a subposet isomorphic to T_2 when $k = 2$, and to T_1 when $k > 2$). So A can not be principal.

In case (2) $\{a\} \coprod B$ is not non-negative, because it contains a subposet isomorphic to T_3 when $k = 1$, to T_4 when $k = 2$, to T_1 when $k = 3, 4, 5$, to T_4^{op} when $k = 6$, and to T_3^{op} when $k \geqslant 7$. So A can not be principal.

In case (3) $\{a\} \coprod B$ is positive if $s = 0$ (by (3) of Theorem 1), and is not non-negative if $s > 1$ (because it contains a subposet isomorphic to T_5). So $s = 1$ and B has the form indicated in the formulation of our lemma (then A is principal as a poset of the form (II)). □

Let now S be a principal poset of order $n > 8$ and let $t' \neq 0$ be such that $q_S(t') = 0$. Fix $d \in S$ such that $t'_d \neq 0$. Then by the definition of principal poset $S_0 = S \setminus d$ is a positive poset. Put $A := \{x \in S \,|\, x < d\}$ and $B := \{x \in S \,|\, x > d\}$. Then $S_d := S_{AB}^{\uparrow\downarrow} = \{d\} \coprod T$ for some subposet T of S. By Corollaries 2 and 3 the poset T is positive and the poset S_d is principal. Since $|T| > 7$, it follows from Theorems 1, 2 and Lemma 1 that T is of the form (3) with $s = 1$.

We will mention the posets of the form (I), (II), …, (V) up to replacement left sums by right sums.

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[1] T_1–T_5 are minimal posets, that are not non-negative (i.e. with the quadratic Tits form, which are not non-negative). All such posets are classificated in [7]. This classification provides a criterion for a poset S to be non-negative.

Let $T=\{x_0\}\coprod C$ with an almost chain C. Since $S^{\uparrow\downarrow}_{AB}=\{d\}\coprod T$, implies $S=(\{d\}\coprod T)^{\uparrow\downarrow}_{AB}$, or equivalently $S=\{d\}\coprod T^{\uparrow\downarrow}_{AB}$ (because $d\notin A,B$), and the set of all posets of the form (I)–(V) is closed with respect to duality, then by Lemma 1 (written as $S^{\downarrow}_{A}=[(S^{\mathrm{op}})^{\uparrow}_{A^{\mathrm{op}}}]^{\mathrm{op}}$), to complete the proof it suffices to show that $\{d\}\coprod T^{\uparrow}_{A}$ is one of the form (I)–(V) for any lower subposet of T.

Put $C:=\{x_1<x_2<\cdots x_p<\{u,v\}<y_1<y_2<\cdots<y_q\}$ and write out all types of lower subposets of $T=\{x_0\}\coprod C$:

$A_0=\varnothing$; $A_1=\{x_1,\dots,x_i\}$, $1\leqslant i\leqslant p$; $A_2=\{x_1,\dots,x_p,u\}$;

$A_3=A_2\cup v$; $A_4=A_3\cup\{y_1,\dots,y_j\}$, $1\leqslant j\leqslant q$; $A_5=x_0$;

$A_6=x_0\cup A_1$; $A_7=x_0\cup A_2$; $A_8=x_0\cup A_3$; $A_9=x_0\cup A_4$.

By the definition of $S^{\uparrow}_{A}$ we have that $\{d\}\coprod T^{\uparrow}_{A}=\{d\}\coprod(\{x_0\}\coprod C)^{\uparrow}_{A}$ is of the form (II) for $A=A_0,A_1$, of the form (III) for $A=A_2$, of the form (I) for $A=A_3,A_4$, of the form (III) for $A=A_5,A_6$, of the form (V) for $A=A_7$, of the form (IV) for $A=A_8,A_9$.

Proposition 2 is proved.

The fact that all posets of the form (I)–(V) (see Theorem 3) are pairwise non-isomorphic is obvious.

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