Algebra and Discrete Mathematics
Number 1. **(2002).** pp. 69–78

RESEARCH ARTICLE

Radical theory in BCH-algebras

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Communicated by B.V. Novikov

Dedicated to V. V. Kirichenko on the occasion of his 60th birthday

Abstract. The notion of k-nil radical in BCH-algebras is defined, and related properties are investigated.

1. Introduction

In 1966, Y. Imai and K. Iséki [7] and K. Iséki [8] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In 1983, Q. P. Hu and X. Li [4, 5] introduced a wide class of abstract algebras: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. They have studied some properties of these algebras. In 1992, W. P. Huang [6] introduced a nil ideals in BCI-algebras. In [9], E. H. Roh and Y. B. Jun discussed the concept of nil subsets by using nilpotent elements in BCH-algebras. In this paper, we introduce the notion of k-nil radical in BCH-algebras, and study some useful properties. We prove that the k-nil radical of a subalgebra (resp. a (closed, translation, semi-) ideal) is a subalgebra (resp. a (closed, translation, semi-) ideal). Concerning the homomorphisms, we discuss related properties.

2. Preliminaries

By a *BCH-algebra* we shall mean an algebra $(X, *, 0)$ of type (2,0) satisfying the following axioms: for every $x, y, z \in X$,

2000 Mathematics Subject Classification *06F35,03G25..*
Key words and phrases: *(closed, translation) ideal, semi-ideal, k-nil radical..*

(H1) $x * x = 0$,

(H2) $x * y = 0$ and $y * x = 0$ imply $x = y$,

(H3) $(x * y) * z = (x * z) * y$.

In a BCH-algebra X, the following holds for all $x, y, z \in X$,

(p1) $x * 0 = x$,

(p2) $(x * (x * y)) * y = 0$,

(p3) $0 * (x * y) = (0 * x) * (0 * y)$,

(p4) $x * 0 = 0$ implies $x = 0$,

(p5) $0 * \big(0 * (0 * x)\big) = 0 * x$.

A nonempty subset S of a BCH-algebra X is said to be a *subalgebra* of X if $x * y \in S$ whenever $x, y \in S$. A nonempty subset A of a BCH-algebra X is called an *ideal* of X if it satisfies

(I1) $0 \in A$,

(I2) $x * y \in A$ and $y \in A$ imply $x \in A$, $\forall x, y \in X$.

A nonempty subset A of a BCH-algebra X is called a *closed ideal* of X if it satisfies (I2) and

(I3) $0 * x \in A$, $\forall x \in A$.

Note that every closed ideal of a BCH-algebra is a subalgebra, but the converse is not true (see [1]). A mapping $f : X \to Y$ of BCH-algebras is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. Note that if $f : X \to Y$ is a homomorphism of BCH-algebras, then $f(0) = 0$.

3. Main Results

Throughout this section X is a BCH-algebra and k is a positive integer. For any elements x and y of X, let us write $x * y^k$ for $(\cdots((x*y)*y)*\cdots)*y$ in which y occurs k times.

Definition 3.1. Let I be a nonempty subset of X. Then the set

$$\sqrt[k]{I} := \{x \in X \mid 0 * x^k \in I\}$$

is called the *k-nil radical* of I.

Lemma 3.2. ([9, Lemmas 3.2 and 3.3]) *For any $x, y \in X$, we have*

(1) $0 * (0 * x)^k = 0 * (0 * x^k)$,

(2) $0 * (x * y)^k = (0 * x^k) * (0 * y^k)$.

Proposition 3.3. *If I and J are nonempty subsets of X, then*

$$\sqrt[k]{I \cup J} = \sqrt[k]{I} \cup \sqrt[k]{J}.$$

Proof. Note that

$$\begin{aligned} x \in \sqrt[k]{I \cup J} &\Leftrightarrow 0 * x^k \in I \cup J \\ &\Leftrightarrow 0 * x^k \in I \quad \text{or} \quad 0 * x^k \in J \\ &\Leftrightarrow x \in \sqrt[k]{I} \quad \text{or} \quad x \in \sqrt[k]{J} \\ &\Leftrightarrow x \in \sqrt[k]{I} \cup \sqrt[k]{J}. \end{aligned}$$

This completes the proof. □

Proposition 3.4. *Let $\{I_\alpha \mid \alpha \in \Lambda\}$ be a collection of nonempty subsets of X, where Λ is any index set. Then*

(i) $\sqrt[k]{\bigcap_{\alpha\in\Lambda} I_\alpha} = \bigcap_{\alpha\in\Lambda} \sqrt[k]{I_\alpha}$.

(ii) $\forall \alpha \in \Lambda,\ 0 \in I_\alpha \Rightarrow 0 \in \sqrt[k]{I_\alpha}$.

(iii) $\forall \alpha, \beta \in \Lambda,\ I_\alpha \subseteq I_\beta \Rightarrow \sqrt[k]{I_\alpha} \subseteq \sqrt[k]{I_\beta}$.

Proof. (i) Note that

$$\begin{aligned} x \in \sqrt[k]{\bigcap_{\alpha\in\Lambda} I_\alpha} &\Leftrightarrow 0 * x^k \in \bigcap_{\alpha\in\Lambda} I_\alpha \\ &\Leftrightarrow 0 * x^k \in I_\alpha \quad \text{for all} \quad \alpha \in \Lambda \\ &\Leftrightarrow x \in \sqrt[k]{I_\alpha} \quad \text{for all} \quad \alpha \in \Lambda \\ &\Leftrightarrow x \in \bigcap_{\alpha\in\Lambda} \sqrt[k]{I_\alpha}, \end{aligned}$$

and hence (i) is valid.

(ii) and (iii) are straightforward. □

Proposition 3.5. *If I is a subalgebra of X and $x \in \sqrt[k]{I}$, then $0 * x \in \sqrt[k]{I}$.*

Proof. If $x \in \sqrt[k]{I}$, then $0 * x^k \in I$. Since I is a subalgebra of X, we have $0 * (0 * x)^k = 0 * (0 * x^k) \in I$ by using Lemma 3.2(1). This shows that $0 * x \in \sqrt[k]{I}$. □

Theorem 3.6. *If I is a subalgebra of X, then so is the k-nil radical $\sqrt[k]{I}$ of I.*

Proof. Let $x, y \in \sqrt[k]{I}$. Then $0 * x^k \in I$ and $0 * y^k \in I$. Since I is a subalgebra, it follows from Lemma 3.2(2) that

$$0 * (x * y)^k = (0 * x^k) * (0 * y^k) \in I$$

so that $x * y \in \sqrt[k]{I}$. Hence $\sqrt[k]{I}$ is a subalgebra of X. □

Theorem 3.7. *If I is an ideal of X, then so is the k-nil radical $\sqrt[k]{I}$ of I.*

Proof. Assume that I is an ideal of X. Obviously $0 \in \sqrt[k]{I}$. Let $x, y \in X$ be such that $x*y \in \sqrt[k]{I}$ and $y \in \sqrt[k]{I}$. Then $0*y^k \in I$ and $(0*x^k)*(0*y^k) = 0*(x*y)^k \in I$. Since I is an ideal of X, it follows from (I2) that $0*x^k \in I$ so that $x \in \sqrt[k]{I}$. Hence $\sqrt[k]{I}$ is an ideal of X. □

Lemma 3.8. ([1, Theorem 4]) *Let I be a subalgebra of a BCH-algebra X such that $x * y \in I$ implies $y * x \in I$ for all $x, y \in X$. Then I is a closed ideal of X.*

Theorem 3.9. *For any closed ideal I of a BCH-algebra X, the k-nil radical $\sqrt[k]{I}$ of I is also a closed ideal of X.*

Proof. Let I be a closed ideal of X. Then I is a subalgebra of X, and so $\sqrt[k]{I}$ is a subalgebra of X. Let $x, y \in X$ be such that $x * y \in \sqrt[k]{I}$. Then $0 * (x * y)^k \in I$. Using (H3), (p3), (p5) and Lemma 3.2(2), we have

$$\begin{aligned} 0 * (y * x)^k &= (0 * y^k) * (0 * x^k) \\ &= \big(0 * (0 * (0 * y^k))\big) * (0 * x^k) \\ &= \big(0 * (0 * x^k)\big) * \big(0 * (0 * y^k)\big) \\ &= 0 * \big((0 * x^k) * (0 * y^k)\big) \\ &= 0 * \big(0 * (x * y)^k\big) \in I, \end{aligned}$$

since I is a subalgebra. Hence $y * x \in \sqrt[k]{I}$, and so, by Lemma 3.8, $\sqrt[k]{I}$ is a closed ideal of X. □

Definition 3.10. ([1, Definition 12]) A nonempty subset I of a BCH-algebra X is called a *semi-ideal* of X if it satisfies (I1) and

(I4) $x \leq y$ and $y \in I$ imply $x \in I$

where $x \leq y$ means $x * y = 0$.

Note that every closed ideal is a semi-ideal, but the converse may not be true (see [1]).

Theorem 3.11. *If I is a semi-ideal of X, then so is $\sqrt[k]{I}$.*

Proof. Obviously $0 \in \sqrt[k]{I}$. Let $x, y \in X$ be such that $x \leq y$ and $y \in \sqrt[k]{I}$. Then $0 * y^k \in I$ and $x * y = 0$. These imply that

$$0 = 0 * (x * y)^k = (0 * x^k) * (0 * y^k), \text{ that is, } 0 * x^k \leq 0 * y^k.$$

Since I is a semi-ideal of X, it follows that $0 * x^k \in I$ or equivalently $x \in \sqrt[k]{I}$. Hence $\sqrt[k]{I}$ is a semi-ideal of X. □

Proposition 3.12. *Let $f : X \to Y$ be a homomorphism of BCH-algebras. If S is a nonempty subset of Y, then $\sqrt[k]{f^{-1}(S)} \subseteq f^{-1}(\sqrt[k]{S})$.*

Proof. Let $x \in \sqrt[k]{f^{-1}(S)}$. Then $0 * x^k \in f^{-1}(S)$, and so $0 * f(x)^k = f(0 * x^k) \in S$. Hence $f(x) \in \sqrt[k]{S}$ which implies $x \in f^{-1}(\sqrt[k]{S})$. This completes the proof. □

Theorem 3.13. *Let $f : X \to Y$ be a homomorphism of BCH-algebras. If J is a closed ideal of Y, then $f^{-1}(\sqrt[k]{J})$ is a closed ideal of X containing $\sqrt[k]{f^{-1}(J)}$.*

Proof. The inclusion $\sqrt[k]{f^{-1}(J)} \subseteq f^{-1}(\sqrt[k]{J})$ is by Proposition 3.12. Let $x, y \in f^{-1}(\sqrt[k]{J})$. Then $f(x), f(y) \in \sqrt[k]{J}$, and so $0 * f(x)^k \in J$ and $0 * f(y)^k \in J$. Since J is a subalgebra of Y, it follows from Lemma 3.2(2) that

$$\begin{aligned} f\big(0 * (x * y)^k\big) &= 0 * f(x * y)^k = 0 * \big(f(x) * f(y)\big)^k \\ &= \big(0 * f(x)^k\big) * \big(0 * f(y)^k\big) \in J \end{aligned}$$

so that $0 * (x * y)^k \in f^{-1}(J)$, that is, $x * y \in \sqrt[k]{f^{-1}(J)} \subseteq f^{-1}(\sqrt[k]{J})$. Hence $f^{-1}(\sqrt[k]{J})$ is a subalgebra of X. Now let $a, b \in X$ be such that $a * b \in f^{-1}(\sqrt[k]{J})$. Then $f(a) * f(b) = f(a * b) \in \sqrt[k]{J}$, and so $0 * \big(f(a) * f(b)\big)^k \in J$. Using Lemma 3.2(2), (p5), (H3) and (p3), we have

$$\begin{aligned} 0 * f(b * a)^k &= 0 * \big(f(b) * f(a)\big)^k \\ &= \big(0 * f(b)^k\big) * \big(0 * f(a)^k\big) \\ &= \big(0 * (0 * (0 * f(b)^k))\big) * \big(0 * f(a)^k\big) \\ &= \big(0 * (0 * f(a)^k)\big) * \big(0 * (0 * f(b)^k)\big) \\ &= 0 * \big((0 * f(a)^k) * (0 * f(b)^k)\big) \\ &= 0 * \big(0 * (f(a) * f(b))^k\big) \in J, \end{aligned}$$

because J is a subalgebra. Hence $f(b * a) \in \sqrt[k]{J}$, and so $b * a \in f^{-1}(\sqrt[k]{J})$. Using Lemma 3.8, we know that $f^{-1}(\sqrt[k]{J})$ is a closed ideal of X. □

Theorem 3.14. *Let $f : X \to Y$ be a homomorphism of BCH-algebras. If U is a semi-ideal of Y, then $f^{-1}(\sqrt[k]{U})$ is a semi-ideal of X containing $\sqrt[k]{f^{-1}(U)}$.*

Proof. Obviously $0 \in f^{-1}(\sqrt[k]{U})$. Let $x, y \in X$ be such that $x \leq y$ and $y \in f^{-1}(\sqrt[k]{U})$. Then $x * y = 0$ and $f(y) \in \sqrt[k]{U}$, that is, $0 * f(y)^k \in U$. Using Lemma 3.2(2), we have

$$\big(0*f(x)^k\big)*\big(0*f(y)^k\big) = 0*\big(f(x)*f(y)\big)^k = 0*f(x*y)^k = 0*f(0)^k = 0,$$

and so $0 * f(x)^k \leq 0 * f(y)^k$. Since U is a semi-ideal, it follows that

$$f(0 * x^k) = f(0) * f(x)^k = 0 * f(x)^k \in U$$

so that $0 * x^k \in f^{-1}(U)$, i.e., $x \in \sqrt[k]{f^{-1}(U)} \subseteq f^{-1}(\sqrt[k]{U})$. Therefore $f^{-1}(\sqrt[k]{U})$ is a semi-ideal of X. □

Theorem 3.15. *Let $f : X \to Y$ be a homomorphism of BCH-algebras. Then $f(\sqrt[k]{I}) \subseteq \sqrt[k]{f(I)}$ for every nonempty subset I of X. Moreover, the equality is valid when f is one-to-one.*

Proof. Let $y \in f(\sqrt[k]{I})$. Then there exists $x \in \sqrt[k]{I}$ such that $f(x) = y$. Hence $0 * x^k \in I$ and

$$0 * y^k = f(0) * f(x)^k = f(0 * x^k) \in f(I),$$

and so $y \in \sqrt[k]{f(I)}$. Thus $f(\sqrt[k]{I}) \subseteq \sqrt[k]{f(I)}$. Assume that f is one-to-one and let $y \in \sqrt[k]{f(I)}$. Then $y = f(x)$ for some $x \in X$, and

$$f(0 * x^k) = 0 * f(x)^k = 0 * y^k \in f(I).$$

Since f is one-to-one, it follows that $0 * x^k \in f^{-1}\big(f(I)\big) = I$ so that $x \in \sqrt[k]{I}$. Therefore $y = f(x) \in f(\sqrt[k]{I})$. This completes the proof. □

Definition 3.16. [10] A *translation ideal* of X is defined to be an ideal U of X satisfying an additional condition:

$$\forall x, y, z \in X,\ x*y \in U,\ y*x \in U \Rightarrow (x*z)*(y*z) \in U,\ (z*x)*(z*y) \in U.$$

Theorem 3.17. *If U is a translation ideal of X, then so is $\sqrt[k]{U}$.*

Proof. If U is a translation ideal of X, then U is an ideal of X and so $\sqrt[k]{U}$ is an ideal of X (see Theorem 3.7). Let $x, y, z \in X$ be such that $x * y \in \sqrt[k]{U}$ and $y * x \in \sqrt[k]{U}$. Then

$$(0 * x^k) * (0 * y^k) = 0 * (x * y)^k \in U$$

and

$$(0 * y^k) * (0 * x^k) = 0 * (y * x)^k \in U.$$

Since U is a translation ideal, it follows from Lemma 3.2(2) that

$$0 * \big((x * z) * (y * z)\big)^k = \big((0 * x^k) * (0 * z^k)\big) * \big((0 * y^k) * (0 * z^k)\big) \in U$$

and

$$0 * \big((z * x) * (z * y)\big)^k = \big((0 * z^k) * (0 * x^k)\big) * \big((0 * z^k) * (0 * y^k)\big) \in U,$$

and so $(x * z) * (y * z) \in \sqrt[k]{U}$ and $(z * x) * (z * y) \in \sqrt[k]{U}$. Therefore $\sqrt[k]{U}$ is a translation ideal of X. □

Theorem 3.18. *Let $f : X \to Y$ be a homomorphism of BCH-algebras. If U is a translation ideal of Y, then $f^{-1}(\sqrt[k]{U})$ is a translation ideal of X containing $\sqrt[k]{f^{-1}(U)}$.*

Proof. Let $x, y, z \in X$ be such that $x*y \in f^{-1}(\sqrt[k]{U})$ and $y*x \in f^{-1}(\sqrt[k]{U})$. Then $f(x) * f(y) = f(x * y) \in \sqrt[k]{U}$ and $f(y) * f(x) = f(y * x) \in \sqrt[k]{U}$. Hence

$$\big(0 * f(x)^k\big) * \big(0 * f(y)^k\big) = 0 * \big(f(x) * f(y)\big)^k \in U$$

and

$$\big(0 * f(y)^k\big) * \big(0 * f(x)^k\big) = 0 * \big(f(y) * f(x)\big)^k \in U.$$

Since U is a translation ideal of Y, it follows that

$$\begin{aligned} & 0 * f\big((x * z) * (y * z)\big)^k \\ = \ & 0 * \big(f(x * z) * f(y * z)\big)^k \\ = \ & \big(0 * f(x * z)^k\big) * \big(0 * f(y * z)^k\big) \\ = \ & \big(0 * (f(x) * f(z))^k\big) * \big(0 * (f(y) * f(z))^k\big) \\ = \ & \big((0 * f(x)^k) * (0 * f(z)^k)\big) * \big((0 * f(y)^k) * (0 * f(z)^k)\big) \in U \end{aligned}$$

and

$$\begin{aligned} & 0 * f\big((z * x) * (z * y)\big)^k \\ = \ & 0 * \big(f(z * x) * f(z * y)\big)^k \\ = \ & \big(0 * f(z * x)^k\big) * \big(0 * f(z * y)^k\big) \\ = \ & \big(0 * (f(z) * f(x))^k\big) * \big(0 * (f(z) * f(y))^k\big) \\ = \ & \big((0 * f(z)^k) * (0 * f(x)^k)\big) * \big((0 * f(z)^k) * (0 * f(y)^k)\big) \in U \end{aligned}$$

so that $f\big((x * z) * (y * z)\big) \in \sqrt[k]{U}$ and $f\big((z * x) * (z * y)\big) \in \sqrt[k]{U}$. Hence $(x * z) * (y * z) \in f^{-1}(\sqrt[k]{U})$ and $(z * x) * (z * y) \in f^{-1}(\sqrt[k]{U})$, completing the proof. □

Let U be a translation ideal of X and define a relation "$\sim$" on X by $x \sim y$ if and only if $x * y \in U$ and $y * x \in U$ for every $x, y \in X$. Then "$\sim$" is a congruence relation on X. By $[x]$ we denote the equivalence class containing x, and by X/U we denote the set of all equivalence classes, that is, $X/U := \{[x] \mid x \in X\}$. Then $(X/U; \odot, [0])$ is a BCH-algebra, where $[x] \odot [y] = [x * y]$ for every $x, y \in X$ (see [10]). If U is a translation ideal of X, then so is $\sqrt[k]{U}$ (see Theorem 3.17). Hence $(X/\sqrt[k]{U}; \odot, [0])$ is a BCH-algebra and $[0] = \sqrt[k]{U}$. For any two BCH-algebras X and Y, the *product BCH-algebra* is defined to be a BCH-algebra $(X \times Y; *, 0)$, where $X \times Y = \{(x, y) \mid x \in X,\ y \in Y\}$, $(x_1, y_1) * (x_2, y_2) = (x_1 * x_2, y_1 * y_2)$ for all $(x_1, y_1), (x_2, y_2) \in X \times Y$, and $0 = (0, 0)$ (see [4, 5]).

Lemma 3.19. *Let X and Y be BCH-algberas. For any $(x, y) \in X \times Y$, we have $(0, 0) * (x, y)^k = (0 * x^k, 0 * y^k)$.*

Proof. It is straightforward. □

Theorem 3.20. *Let A and B be nonempty subsets of BCH-algebras X and Y, respectively. Then*

(i) $\sqrt[k]{A} \times \sqrt[k]{B} = \sqrt[k]{A \times B}$,

(ii) if A and B are translation ideals of X and Y respectively, then $\sqrt[k]{A \times B}$ is a translation ideal of $X \times Y$ and

$$\frac{X \times Y}{\sqrt[k]{A \times B}} \cong X/\sqrt[k]{A} \times Y/\sqrt[k]{B}.$$

Proof. (1) We have that

$$\begin{aligned}
\sqrt[k]{A \times B} &= \{(a, b) \in X \times Y \mid (0, 0) * (a, b)^k \in A \times B\} \\
&= \{(a, b) \in X \times Y \mid (0 * a^k,\ 0 * b^k) \in A \times B\} \\
&= \{(a, b) \in X \times Y \mid 0 * a^k \in A,\ 0 * b^k \in B\} \\
&= \{(a, b) \in X \times Y \mid a \in \sqrt[k]{A},\ b \in \sqrt[k]{B}\} \\
&= \{a \in X \mid a \in \sqrt[k]{A}\} \times \{b \in X \mid b \in \sqrt[k]{B}\} \\
&= \sqrt[k]{A} \times \sqrt[k]{B}
\end{aligned}$$

(ii) Obviously $\sqrt[k]{A \times B}$ is a translation ideal of $X \times Y$. Consider natural homomorphisms

$$\pi_X : X \to X/\sqrt[k]{A},\ x \mapsto [x] \text{ and } \pi_Y : Y \to Y/\sqrt[k]{B},\ y \mapsto [y].$$

Define a mapping $\Phi : X \times Y \to X/\sqrt[k]{A} \times Y/\sqrt[k]{B}$ by $\Phi(x, y) = ([x], [y])$ for all $(x, y) \in X \times Y$. Then clearly Φ is a well-defined onto homomorphism.

Moreover,

$$\begin{aligned} \mathrm{Ker}\Phi &= \{(x,y) \in X \times Y \mid \Phi(x,y) = ([0],[0])\} \\ &= \{(x,y) \in X \times Y \mid ([x],[y]) = ([0],[0])\} \\ &= \{(x,y) \in X \times Y \mid [x] = [0],\ [y] = [0]\} \\ &= \{(x,y) \in X \times Y \mid x \in \sqrt[k]{A},\ y \in \sqrt[k]{B}\} \\ &= \sqrt[k]{A} \times \sqrt[k]{B} = \sqrt[k]{A \times B}. \end{aligned}$$

By the homomorphism theorem (see [10, Theorem 3.7]), we have

$$\frac{X \times Y}{\sqrt[k]{A \times B}} = \frac{X \times Y}{\mathrm{Ker}\Phi} \cong X/\sqrt[k]{A} \times Y/\sqrt[k]{B}.$$

□

Acknowledgements. One of the authors (Young Bae Jun) would like to express his thanks to the KOSEF and PAS for providing the necessary funds for his trip to the Technical University of Wroclaw, Poland exploring the issues of this paper.

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Received by the editors: 05.10.2002.