Algebra and Discrete Mathematics
Volume 17 **(2014)**. Number 1. pp. 70 – 79 RESEARCH ARTICLE

Semiabelian and self-returning of points of n-ary groups

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Communicated by V. V. Kirichenko

Abstract. In this article new criterian Semiabelian of n-ary Groups is expressed through Self-Returning free point comparatively element specially built to sequences, consisting of mediums of all sides free polygonal figure with even number of the tops and one tops this polygonal figure in term symmetrical point and vector.

Introduction

The investigations in the theory of n-ary groups originated by W. Dörnte [1] and E. Post [2] find more and more followers (see, for example, [3–7]).

From our point of view, the interest can be explained by the fact that various applications of the theory are widely developed. Besides, the appearance of new methods such as functorial and geometrical enabled to get a number of interesting results in the field of multi-rings, polyadic multi-rings and universal algebras (see, for example, [8–11]).

Ternary groups which were studied by H. Prufer [12] and J. Certaine [13] found their application in projective geometry [14], affine geometry [15, 16] and other fields of knowledge. Having applied totally new approaches and methods of investigation S.A.Rusakov generalized the above mentioned results for the case of an n-ary group ($n \geqslant 2$) in [17]. It should

2010 MSC: 20N15.

Key words and phrases: Semiabelian n-ary group, self-returning of points, symmetric points.

be mentioned that the study of the elements of affine geometry by means of the theory of n-ary groups and the study of properties of n-ary groups connected with the properties of objects of affine geometry were realized by many authors (see, for example, [3-7, 18, 19]). We note that in such applications especially important role play so-called the semiabelian n-ary groups.

In this connection an important problem in the n-ary group theory is the problem of finding new criteria of semiabelian of n-ary groups. For more details concerning the n-ary groups and the applications of n-ary groups, the reader is referred to [18, 19].

In the note, new criteria of semiabelian of n-ary groups based on the concept of self-returning elements of n-ary groups.

1. Preliminaries

Throughout this paper, G always denotes an n-ary group.

Let m and k be integers such that $m > 0$ and $k \geqslant 0$.

1) If $k > 0$ and $m \leqslant k$, then we use x_m^k to denote the sequence $x_m x_{m+1} \dots x_k$ $(x_m, x_{m+1}, \dots, x_k \in G)$. In particular, $x_m^m = x_m$.

2) If $k > 0$, then we write $\overset{k}{x}$ to denote the sequence $xx\dots x$ of length k; if $k = 0$, then $\overset{0}{x}$ is an empty sequence, that is, the sequence which does not contain any element $(x \in G)$.

Recall that universal algebra $\langle G, (\ )\rangle$ with a single n-ary operation $(\ ) : G^n \to G$ $(n \geqslant 2)$ is called an n-ary group [20] if the following conditions are satisfied:

1) The operation $(\ )$ is associative on G, that is,

$$((a_1 \dots a_n)a_{n+1} \dots a_{2n-1}) = (a_1 \dots a_i(a_{i+1} \dots a_{i+n})a_{i+n+1} \dots a_{2n-1})$$

for all $i = 1, \dots, n$ and for all $a_1 \dots a_{2n-1} \in G$;

2) The equation

$$(a_1 \dots a_{i-1} x_i a_{i+1} \dots a_n) = b$$

has a unique solution in G for any $i = 1, \dots, n$ and $a_1, \dots, a_{i-1}$, $a_{i+1}, \dots$, a_n, $b \in G$.

Recall that G is said to be *semiabelian* if for any sequence $x_1, \dots, x_n \in G$ the equality

$$(x_1 \dots x_n) = (x_n x_2 \dots x_{n-1} x_1)$$

is true.

The succession $e_1^{k(n-1)} \in G$ where $k \geqslant 1$ is said to be a neutral $k(n-1)$-succession G if $(e_1^{k(n-1)}u) = u = (ue_1^{k(n-1)})$ for any element $u \in G$.

Any n-ary group has its neutral successions. It is conditioned, in particular, by the solvability of the equation $(ae_1^{k(n-1)-1}y) = a$ in an n-ary group. Neutral successions in an n-ary group are defined ambiguously.

The succession $b_1^j \in G$ is called a reverse succession to the succession $a_1^i \in G$ if the successions $b_1^j a_1^i$ and $a_1^i b_1^j$ are neutral.

It is clear that if b_1^j is a reverse succession to a_1^i, then a_1^i is a reverse succession to b_1^j.

It should be emphasized that a reverse succession the length of which is more than one is defined ambiguously.

An n-ary group can be an algebra with two or more operations [6]. In particular, an n-ary group can be an algebra with one associative n-ary operation and one unary operation.

The algebra $G = \langle X, (\ ),^{[-2]} \rangle$ of the type, where $n \geqslant 2$ is called an n-ary group [6] if

1) An n-ary operation ( ) on the set X is associative;

2) For any elements x and y from X the equations $(x^{[-2]} \overset{n-2}{x} (\overset{n-1}{x} y)) = y = ((y \overset{n-1}{x}) \overset{n-2}{x} x^{[-2]})$ are true.

The symbol $x^{[-2]}$ which can be seen in the given equation is the solution of the equation $(y \overset{2(n-1)}{x}) = x$, i.e. $(x^{[-2]} x \overset{2(n-1)}{x}) = x$, where $x, y \in G$.

It follows from the last equation that $x^{[-2]} \overset{2n-4}{x}$ will be a reverse succession to any $x \in G$. Indeed, $(x^{[-2]} x \overset{2(n-1)}{x}) = (xx^{[-2]} \overset{2n-4}{x} x) = x$. Apparently, $xx^{[-2]} \overset{2n-4}{x} = x^{[-2]} \overset{2n-4}{x} x$ are neutral successions.

Hereinafter, the elements of an n-ary group G will be called points. According to [17] the set of two points a and b is called a segment and designated either $[ab]$ or $[ba]$. The succession of k arbitrary points $\langle a_1, a_2, \ldots, a_k \rangle$ is called a k-angle G, where $a_1, a_2, \ldots, a_k \in G$.

It is proved in [17] that for any points $a, b, c \in G$ the equations

$$(ab^{[-2]} \overset{2n-4}{b} c) = b, \tag{1}$$

$$(cb^{[-2]} \overset{2n-4}{b} a) = b \tag{2}$$

are equivalent.

If equation (1) or (2) is true b is called the middle of the segment $[ac]$. If equation (1) (equation (2)) is true the point c (point a) is called a point symmetric to the point a (point c) with respect to the point b and designated by $S_b(a)$ (by $S_b(c)$), i.e. $c = S_b(a)$ ($a = S_b(c)$).

It follows from (1) or (2) that

$$S_b(a) = (ba^{[-2]} \overset{2n-4}{a} b), \tag{3}$$

$$S_b(c) = (bc^{[-2]} \overset{2n-4}{c} b) \tag{4}$$

It is established in [2] that an n-ary group G will be semiabelian if for any $x, y, z \in G$ the equation $(xy^{[-2]} \overset{2n-4}{y} z) = (zy^{[-2]} \overset{2n-4}{y} x)$ is true.

Hereinafter, the reverse succession to any $x \in G$ will be designated by x^{-1}, i.e. $x^{-1} = x^{[-2]} \overset{2n-4}{x}$. Then equations (3) and (4) can be written in the forms $S_b(a) = (ba^{-1}b), S_b(c) = (bc^{-1}b)$, accordingly. And an n-ary group G will be semiabelian if for any $x, y, z \in G$ the equation $(xy^{-1}z) = (zy^{-1}x)$ is true.

We remind from [21] that the point $S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)$ is called a round of the elements of the succession $\langle a_1, \ldots, a_k\rangle$ by a point p, where $a_1, \ldots a_k, p \in G$. If $S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots) = p$, they say that p self-returning with respect to the elements of the succession $\langle a_1, \ldots, a_k\rangle$.

2. Semiabelian and self-returning of points of n-ary groups

Theorem 1. *Let $b_1, \ldots, b_k$ be arbitrary points of an n-ary group G ($k \in R$, $k \geqslant 4$, k is even), and $a_1, \ldots, a_k \in G$ so that*

$$b_2 = S_{a_1}(b_1),\ b_3 = S_{a_2}(b_2), \ldots,\ b_k = S_{a_{k-1}}(b_{k-1}),\ b_1 = S_{a_k}(b_k). \tag{5}$$

An n-ary group G will be semiabelian when an arbitrary point $p \in G$ self-returning with respect to the elements of the succession $\langle a_1, \ldots, a_k\rangle$, i.e. when the equation

$$S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots) = p \tag{6}$$

is true.

Proof. Let G be a semiabelian n-ary group. We shall find out the validity of equality (6).

Let us consider the left part of equality (6) with account of the fact that $S_a(b) = (ab^{-1}a)$ and the neutrality of successions xx^{-1} and $x^{-1}x$ for any $x \in G$. We have

$$S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots) = S_{a_k}(\ldots(S_{a_2}(a_1p^{-1}a_1))\ldots) =$$

$$= S_{a_k}(\dots(S_{a_3}(a_2a_1^{-1}pa_1^{-1}a_2)\dots).$$

Making the similar transformations and taking into account that k is an even natural number we obtain

$$S_{a_k}(\dots(S_{a_2}(S_{a_1}(p)))\dots) = (a_ka_{k-1}^{-1}\dots a_2a_1^{-1}pa_1^{-1}a_2\dots a_{k-1}^{-1}a_k). \qquad (7)$$

As the element $p \in G$ is arbitrary we consider $p = b_1$. In this case we re-write equality (7) in the form

$$S_{a_k}(\dots(S_{a_2}(S_{a_1}(b_1)))\dots) = (a_ka_{k-1}^{-1}\dots a_2a_1^{-1}b_1a_1^{-1}a_2\dots a_{k-1}^{-1}a_k). \qquad (8)$$

On the other hand, with account of equalities (5) the expression $S_{a_k}(\dots(S_{a_2}(S_{a_1}(b_1)))\dots)$ is equal to b_1, i.e.

$$S_{a_k}(\dots(S_{a_2}(S_{a_1}(b_1)))\dots) = b_1. \qquad (9)$$

Then with account (8) and (9)

$$b_1 = (a_ka_{k-1}^{-1}\dots a_2a_1^{-1}b_1a_1^{-1}a_2\dots a_{k-1}^{-1}a_k). \qquad (10)$$

We multiply both the parts of equality (10) on the right by the expression $a_k^{-1}a_{k-1}\dots a_2^{-1}a_1b_1^{-1}p$, we obtain

$$\begin{aligned}&(b_1a_k^{-1}a_{k-1}\dots a_2^{-1}a_1b_1^{-1}p) = \\ &= (a_ka_{k-1}^{-1}\dots a_2a_1^{-1}b_1a_1^{-1}a_2\dots a_{k-1}^{-1}a_ka_k^{-1}a_{k-1}\dots a_2^{-1}a_1b_1^{-1}p). \qquad (11)\end{aligned}$$

With account of the neutrality of successions xx^{-1} and $x^{-1}x$ for any $x \in G$ we re-write equality (11) in the form

$$(b_1a_k^{-1}a_{k-1}\dots a_2^{-1}a_1b_1^{-1}p) = (a_ka_{k-1}^{-1}\dots a_2a_1^{-1}p). \qquad (12)$$

With account of (12) equality (7) can be re-written in the form

$$\begin{aligned}S_{a_k}(\dots(S_{a_2}(S_{a_1}(p)))\dots) &= (b_1a_k^{-1}a_{k-1}\dots a_2^{-1}a_1b_1^{-1}pa_1^{-1}a_2\dots a_{k-1}^{-1}a_k) = \\ &= ((b_1a_k^{-1}a_{k-1}\dots a_2^{-1}a_1b_1^{-1}p)a_1^{-1}a_2\dots a_{k-1}^{-1}a_k) = \\ &= (pa_k^{-1}a_{k-1}\dots a_2^{-1}a_1b_1^{-1}b_1a_1^{-1}a_2\dots a_{k-1}^{-1}a_k) = p. \qquad (13)\end{aligned}$$

Based equality (13) we conclude that

$$S_{a_k}(\dots(S_{a_2}(S_{a_1}(p)))\dots) = p.$$

2. Let equality (6) be true. We shall prove that G is a semiabelian n-ary group.

Since we did not use the property of semi-commutativity of the group G in the transformations from point 1 to equality (13) inclusive we consider without repeating the reasoning that

$$S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots) = (b_1 a_k^{-1} a_{k-1} \ldots a_2^{-1} a_1 b_1^{-1} p a_1^{-1} a_2 \ldots a_{k-1}^{-1} a_k).$$

Then with account of equality (6) and (7) we have

$$(b_1 a_k^{-1} a_{k-1} \ldots a_2^{-1} a_1 b_1^{-1} p a_1^{-1} a_2 \ldots a_{k-1}^{-1} a_k) = p. \tag{14}$$

Let x_1^n be arbitrary points from G. We consider $p = (x_1^n)$, $a_1 = x_n$, $a_2 = a_3 = \ldots = a_k = b_1 = x_1$.

With account of equality (14) and neutral successions xx^{-1} and $x^{-1}x$ for any $x \in G$ we have

$$\begin{aligned}
&(x_1^n) = ((x_1^n)x_n^{-1}x_n \underbrace{x_1^{-1}x_1 \ldots x_1^{-1}x_1}_{k}) = \\
&= (\underbrace{x_1 x_1^{-1} \ldots x_1 x_1^{-1}}_{k/2} x_n x_1^{-1}(x_1^n)x_n^{-1}x_1 \underbrace{x_1^{-1}x_1 \ldots x_1^{-1}x_1}_{k/2-1}) = \\
&= (x_n x_1^{-1}(x_1^n)x_n^{-1}x_1) = ((x_n x_1^{-1} x_1 x_2^{n-1}(x_n x_n^{-1} x_1)) = \\
&= (x_n x_2^{n-1} x_1).
\end{aligned}$$

G is a semiabelian n-ary group. □

Theorem 2. *Let b_1^k be a succession of points of an n-ary group G ($k \in N$, $k \geqslant 4$, k is even) and $a_1^k \in G$ so that*

$$b_2 = S_{a_1}(b_1), b_3 = S_{a_2}(b_2), \ldots, b_k = S_{a_{k-1}}(b_{k-1}), b_1 = S_{a_k}(b_k). \tag{15}$$

An n-ary group will be semi-commutative only when the equality

$$\begin{gathered}
\overrightarrow{pS_{a_1}(p)} + \overrightarrow{S_{a_1}(p)S_{a_2}(S_{a_1}(p))} + \ldots + \\
+\overrightarrow{S_{a_{k_1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)S_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots))} = \overrightarrow{0}.
\end{gathered} \tag{16}$$

is valid.

Proof. **1.** Let G be a semi-commutative n-ary group. We shall establish the validity of equality (16).

We shall substitute the expression in equality (15) for the expression from the last but one and obtain

$$b_1 = S_{a_k}(S_{a_{k-1}}(b_{k-1})). \tag{17}$$

As a result of the similar procedure we obtain

$$b_1 = S_{a_k}(S_{a_{k-1}}(\dots(S_{a_2}(S_{a_1}(b_1)))\dots)). \tag{18}$$

We shall consider the right part of equality (18) with account of determining symmetric points. We have

$$S_{a_1}(b_1) = (a_1 b_1^{-1} a_1),$$

$$S_{a_2}(S_{a_1}(b_1)) = S_{a_2}(a_1 b_1^{-1} a_1) = (a_2(a_1 b_1^{-1} a_1)^{-1} a_2) = (a_2 a_1^{-1} b_1 a_1^{-1} a_2).$$

Keeping the same procedure and taking into account that k is even we have

$$S_{a_k}(S_{a_{k-1}}(\dots(S_{a_2}(S_{a_1}(b_1)))\dots)) =$$

$$= (a_k a_{k-1}^{-1} \dots a_2 a_1^{-1} b_1 a_1^{-1} a_2 \dots a_{k-1}^{-1} a_k). \tag{19}$$

With account of (18) and (19) we obtain that

$$b_1 = (a_k a_{k-1}^{-1} \dots a_2 a_1^{-1} b_1 a_1^{-1} a_2 \dots a_{k-1}^{-1} a_k). \tag{20}$$

We multiply both the parts (20) on the right by the expression $a_k^{-1} a_{k-1} \dots a_2^{-1} a_1 b_1^{-1} p$. We obtain

$$(b_1 a_k^{-1} a_{k-1} \dots a_2^{-1} a_1 b_1^{-1} p) =$$

$$= (a_k a_{k-1}^{-1} \dots a_2 a_1^{-1} b_1 a_1^{-1} a_2 \dots a_{k-1}^{-1} a_k a_k^{-1} a_{k-1} \dots a_2^{-1} a_1 b_1^{-1} p).$$

From which with account of neutrality of the successions we have

$$(b_1 a_k^{-1} a_{k-1} \dots a_2^{-1} a_1 b_1^{-1} p) = (a_k a_{k-1}^{-1} \dots a_2 a_1^{-1} p). \tag{21}$$

We transform the left part of equality (16) with account of definition 5 from [17]. We have

$$\overrightarrow{pS_{a_1}(p)} + \overrightarrow{S_{a_1}(p) S_{a_2}(S_{a_1}(p))} + \dots +$$

$$+\overrightarrow{S_{a_{k_1}}(\dots(S_{a_2}(S_{a_1}(p)))\dots) S_{a_k}(S_{a_{k-1}}(\dots(S_{a_2}(S_{a_1}(p)))\dots))} =$$

$$= \overrightarrow{pS_{a_k}(S_{a_{k-1}}(\dots(S_{a_2}(S_{a_1}(p)))\dots))}. \tag{22}$$

We transform the expression $S_{a_k}(S_{a_{k-1}}(\dots(S_{a_2}(S_{a_1}(p)))\dots))$ with account of determining symmetric points. We have

$$S_{a_1}(p) = (a_1 p^{-1} a_1),$$

$$S_{a_2}(S_{a_1}(p)) = (a_2(a_1p^{-1}a_1)^{-1}a_2) = (a_2a_1^{-1}pa_1^{-1}a_2).$$

Since k is an even number, then keeping the similar reasoning we obtain

$$S_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)) =$$
$$= (a_ka_{k-1}^{-1}\ldots a_2a_1^{-1}pa_1^{-1}a_2\ldots a_{k-1}^{-1}a_k). \tag{23}$$

We substitute the expression $a_ka_{k-1}^{-1}\ldots a_2a_1^{-1}p$ in (23) for the corresponding expression from (21). We have

$$S_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)) =$$
$$= (b_1a_k^{-1}a_{k-1}\ldots a_2^{-1}a_1b_1^{-1}pa_1^{-1}a_2\ldots a_{k-1}^{-1}a_k). \tag{24}$$

We transform the right part of equality (24) with account of the property of semi-commutativity of the group G and the neutrality of successions. We have

$$S_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)) =$$
$$= ((b_1a_k^{-1}a_{k-1}\ldots a_2^{-1}a_1b_1^{-1}p)a_1^{-1}a_2\ldots a_{k-1}^{-1}a_k) =$$
$$= (pa_k^{-1}a_{k-1}\ldots a_2^{-1}a_1b_1^{-1}b_1a_1^{-1}a_2\ldots a_{k-1}^{-1}a_k) = p. \tag{25}$$

Taking into account (22) and (25) equality (16) can be re-written in the form

$$\overrightarrow{pS_{a_1}(p)} + \overrightarrow{S_{a_1}(p)S_{a_2}(S_{a_1}(p))} + \ldots +$$
$$+\overrightarrow{S_{a_{k_1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)S_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots))} =$$
$$= \overrightarrow{pS_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots))} = \overrightarrow{pp} = \overrightarrow{0}.$$

Thus, the first part of the theorem is proved. Let equality (16) be true. We shall prove that G is semi-commutative

2. As in the first part of our proof we used the property of semi-commutativity of an n-ary group only in equality (25) we shall consider all the preceding reasoning true.

From equality (22) and the condition of the theorem it follows that

$$\overrightarrow{pS_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots))} = \overrightarrow{0}. \tag{26}$$

Since according to definition 6 from [17] $\overrightarrow{pp} = \overrightarrow{0}$, equality (26) can be re-written in the form

$$\overrightarrow{pS_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots))} = \overrightarrow{pp}. \tag{27}$$

From equality (27) with account of definitions 2 and 4 from [17] it follows that

$$S_{a_k}(S_{a_{k-1}}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)) = p. \tag{28}$$

With account of equality (23) equality (28) can be re-written in the form

$$(a_k a_{k-1}^{-1} \ldots a_2 a_1^{-1} p a_1^{-1} a_2 \ldots a_{k-1}^{-1} a_k) = p. \tag{29}$$

We shall substitute the expression $a_k a_{k-1}^{-1} \ldots a_2 a_1^{-1} p$ in (29) for the corresponding one with account equality (21). We obtain

$$((b_1 a_k^{-1} a_{k-1} \ldots a_2^{-1} a_1 b_1^{-1} p) a_1^{-1} a_2 \ldots a_{k-1}^{-1} a_k) = p. \tag{30}$$

We shall suppose in equality (30) that

$$p = (x_1^n), b_1 = x_1, a_k = x_1, a_1 = a_2 = \ldots = a_{k-1} = x_n.$$

We obtain

$$(x_1 x_1^{-1} x_n \underbrace{x_n^{-1} x_n \ldots x_n^{-1} x_n}_{k-2} x_1^{-1} (x_1^n) x_n^{-1} \underbrace{x_n x_n^{-1} \ldots x_n x_n^{-1}}_{k-2} x_1) = (x_1^n). \tag{31}$$

With account of the neutrality of successions and the fact that that k is an even number equality (31) can be re-written in the form

$$(x_n x_1^{-1} (x_1 x_2^{n-1} x_n) x_n^{-1} x_1) = (x_1^n). \tag{32}$$

Since an n-ary operation ( ) is associative equality (32) has the form

$$((x_n x^{-1} x_1) x_2^{n-1} (x_n x_n^{-1} x_1)) = (x_1^n). \tag{33}$$

From which

$$(x_n x_2^{-1} x_1) = (x_1^n). \tag{34}$$

On the basis of equality (34) and the definition of semi-commutativity of an n-ary group we conclude that G is semi-commutative. □

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Received by the editors: 01.03.2014
and in final form 01.03.2014.