Algebra and Discrete Mathematics
Number 3. **(2004).** pp. 126 – 134

RESEARCH ARTICLE

C^*-algebra generated by four projections with sum equal to 2

Yuri Savchuk

Communicated by L. Turowska

Abstract. We describe the C^*-algebra generated by four orthogonal projections p_1, p_2, p_3, p_4, satisfying the linear relation $p_1 + p_2 + p_3 + p_4 = 2I$. The simplest realization by 2×2-matrix-functions over the sphere S^2 is given.

Introduction

In the present paper we consider a realization of a certain C^*-algebra A with irreducible representations of dimensions equal to 1 or 2 only, as a C^*-algebra of continuous matrix-functions over S^2 with boundary conditions.

C^*-algebras with restriction on the dimensions of the irreducible representations are the object of intensive investigations, started from the works of Gelfand-Naimark, Fell, Tomiyama-Takesaki, Vasil'ev (see [4], [6], [7]).

An interesting fact is that the property for a C^*-algebra A to have irreducible representations of dimensions less or equal to n can be formulated in pure algebraic way. Let F_n denote the following polynomial of degree n in n non-commuting variables:

$$F_n(x_1, x_2, \ldots, x_n) = \sum_{\sigma \in S_n} (-1)^{p(\sigma)} x_{\sigma(1)} \ldots x_{\sigma(n)},$$

2000 Mathematics Subject Classification: *46L05, 47L50.*
Key words and phrases: *matrix-functions, projections, finitely generated C^*-algebras.*

where S_n is the symmetric group of degree n, $p(\sigma)$ is the parity of a permutation $\sigma \in S_n$. We say that an algebra A is an algebra with F_n identity if for all $x_1, \ldots, x_n \in A$, we have $F_n(x_1, \ldots x_n) = 0$. The Amitsur-Levitsky theorem says that the matrix algebra $M_n(\mathbb{C})$ is an algebra with F_{2n} identity. A C^*-algebra A has irreducible representations of dimension less or equal to n iff A satisfies the F_{2n} condition (see [5]).

One of the basic C^*-algebra classes with F_{2n} identity is the class of n-homogeneous algebras. Recall, that an algebra is called n-homogeneous iff all its irreducible representations are of dimension n. Any n-homogeneous C^*-algebra can be described in terms of algebraic bundles, see [6] or [7]. It is also convenient to realize these algebras as algebras of continuous matrix-functions. For example, it was proved in [1], that one has exactly n pairwise non-isomorphic n-homogeneous C^*-algebras having the dual space S^2 (see [1]). We will denote them by $A_{n,k}$, $k = \overline{0, n-1}$. Such algebras can be realized in the following way. Let $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ be the boundary of the unit disk D^2 in the complex plane and consider

$$V_k : S^1 \longrightarrow U(n), z \mapsto diag(z^k, 1, \ldots, 1), \; k = \overline{0, n-1}.$$

Then

$$A_{n,k} = \{f \in C(D^2 \longrightarrow M_2(\mathbb{C})) | f(z) = V_k(z)^* f(1) V_k(z), \; z \in S^1\}.$$

Evidently, the dual space is homeomorphic to $D^2/S^1 \simeq S^2$ (see [1] for more details).

An analogous realization of n-homogeneous algebra, having the two-dimensional torus as the dual space, was presented in [2]. Namely, any such algebra is isomorphic to

$$B_{V,W} = \{g \in C([0,1]^2 \longrightarrow M_n(\mathbb{C})) | g(0,s) = V^* g(1,s) V,$$
$$g(t,0) = W^* g(t,1) W, \; s, t \in [0,1]\},$$

where $V, W \in U(n)$ are some unitary matrices such that VWV^*W^* is a scalar matrix.

Note, that concrete finitely generated F_{2n}-algebras are mostly non-homogeneous. Indeed, the group C^*-algebra of any non-commutative finite group satisfies the F_{2n} condition for some n, but it is not homogeneous. The group algebra of $G = \mathbb{Z}_2 * \mathbb{Z}_2$ gives an example of F_4 algebra corresponding to infinite discrete group. One can also generate $C^*(\mathbb{Z}_2 * \mathbb{Z}_2)$ by the free pair of projections. Indeed, it is easy to see, that

$$C^*(\mathbb{Z}_2 * \mathbb{Z}_2) = C^*\langle p_1, p_2 | p_k^2 = p_k = p_k^*, \; k = 1, 2\rangle := \mathcal{P}_2.$$

A realization of $\mathcal{P}_2$ as algebra of matrix-functions was constructed in [8].

Namely,

$$\mathcal{P}_2 = \{f \in C([0,1] \longrightarrow M_2(\mathbb{C}))|f(0),\ f(1) \textit{ are diagonal}\}.$$

In this paper we study the C*-algebra A generated by four projections (self-adjoint idempotents) $P_1,\ P_2,\ P_3,\ P_4$ satisfying the following relation:

$$P_1 + P_2 + P_3 + P_4 = 2I.$$

The algebra A is an enveloping of the *-algebra:

$$\widetilde{A} = \mathbb{C}\big\langle P_i \ \mid \sum_{i=1}^{4} P_i = 2I,\ P_i = P_i^* = P_i^2,\ i = 1 \dots 4\big\rangle.$$

In Theorem 1, we realize A as an algebra of continuous 2×2 matrix-functions with some boundary conditions. In the theorem 2 we give the most simple of possible realizations of A.

1. Preliminaries

In this Section, for convenience of the reader, we recall some information used below.

Definition 1. Let $\mathbf{A}$ be a *-algebra, having at least one representation. Then a pair $(\mathcal{A}, \rho)$ of a C^*-algebra $\mathcal{A}$ and a homomorphism $\rho : \mathbf{A} \longrightarrow \mathcal{A}$ is called an enveloping pair for $\mathbf{A}$ if every irreducible representation $\pi{:}\mathbf{A} \longrightarrow B(H)$ factors uniquely through the $\mathcal{A}$, i.e. there is precisely one irreducible representation π_1 of algebra $\mathcal{A}$ satisfying $\pi_1 \circ \rho = \pi$. The algebra $\mathcal{A}$ is called an enveloping for $\mathbf{A}$.

The following statement is a simple corollary of the noncommutative analogue of the Stone-Weierstrass theorem for C^*-algebras (see Glimm-Stone-Weierstrass theorem in [4] or [7]).

Statement 1. *Let Y be a compact Hausdorff space. Let $\mathcal{C} \subseteq \mathcal{B}$ be subalgebras of $\mathcal{A} = C(Y \longrightarrow M_n(\mathbb{C}))$. For every pair $x_1, x_2 \in Y$ define $\mathcal{A}(x_1, x_2)$ ($\mathcal{B}(x_1, x_2)$, $\mathcal{C}(x_1, x_2)$ respectively) as:*

$$\mathcal{A}(x_1,x_2) := \{(f(x_1), f(x_2)) \in M_n(\mathbb{C}) \times M_n(\mathbb{C}) | f \in \mathcal{A} \quad (f \in \mathcal{B}, f \in \mathcal{C} \textit{ respectively})\}.$$

Then

$$\mathcal{B} = \mathcal{C} \iff \mathcal{B}(y_1, y_2) = \mathcal{C}(y_1, y_2) \quad \forall y_1, y_2 \in Y.$$

In the next section we will also need a classification of all irreducible representation of $\widetilde{A}$ (see [5] for more details). Namely, irreducible representations are either 1-dimensional or 2-dimensional. The images of generators of $\widetilde{A}$ in two-dimensional representations have the following form :

$$P_1(a,b,c) = \frac{1}{2}\begin{pmatrix} 1+a & -b-ic \\ -b+ic & 1-a \end{pmatrix},\ P_2(a,b,c) = \frac{1}{2}\begin{pmatrix} 1-a & b-ic \\ b+ic & 1+a \end{pmatrix},$$
$$P_3(a,b,c) = \frac{1}{2}\begin{pmatrix} 1-a & -b+ic \\ -b-ic & 1+a \end{pmatrix},\ P_4(a,b,c) = \frac{1}{2}\begin{pmatrix} 1+a & b+ic \\ b-ic & 1-a \end{pmatrix}.$$

where $a^2+b^2+c^2=1$ and the space of parameters (a,b,c) corresponding to irreducible pairwise non-equivalent 2-dimensional representations is (a part of the unit sphere in $\mathbb{R}^3$):

$$P = \{(a,b,c)|a>0, b>0, c\in\mathbb{R}\}\bigcup\{(a,b,c)|a=0, b>0, c>0\}\bigcup$$
$$\bigcup\{(a,b,c)|a>0, b=0, c>0\}.$$

Note that when $(a,b,c)\in\{(1,0,0),(0,1,0),(0,0,1)\}$, the formulas for P_k give reducible representations of $\widetilde{A}$, moreover, any one-dimensional representation of $\widetilde{A}$ can be obtained by decomposition of some of these reducible ones on irreducible components.

We will denote by $\overline{P}$ the closure of P in $\mathbb{R}^3$. Evidently

$$\overline{P} = \{(a,b,c)|a^2+b^2+c^2=1,\ a\geq 0, b\geq 0\}.$$

2. The structure of enveloping C^*-algebra

In this section we give a description of the enveloping C^*-algebra A of $\tilde{A}$. Theorem 1 realizes A by matrix-functions, and Theorem 2 gives the simplest of all descriptions for A.

Theorem 1. *Let*

$$X = \{(x,y)|(x,y)\in\mathbb{R}^2,\ |x|+|y|\leq 1\}, V = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$A_0 = \{f\in C(X\longrightarrow M_2(\mathbb{C}))|f(t,1-t) = Vf(-t,1-t)V,$$
$$f(t,t-1) = Wf(-t,t-1)W,\ t\in[0,1]\},$$

then $A\simeq A_0$.

Proof. Consider the functions $P_i = P_i(a,b,c)$, $i=\overline{1,4}$ naturally corresponding to generators P_i defined on $\overline{P}$. Let $\widehat{A}\subseteq C(\overline{P}\longrightarrow M_2(\mathbb{C}))$ be the C^*-algebra generated by P_i. It is easy to check, that $\widehat{A}$ is an enveloping C^*-algebra of $\widetilde{A}$, i.e. A. Indeed, we have homomorphism of $\widetilde{A}$ into $\widehat{A}$, which satisfies the universal property, so $\widehat{A}$ is enveloping algebra by Definition 1. We will show, that $\widehat{A}$ coincides with

$$\overline{A} = \{f \in C(\overline{P} \to M_2(\mathbb{C}))|\ Vf(s,0,t)V = f(s,0,-t),\\ Wf(0,s,t)W = f(0,s,-t), s^2+t^2=1\}.$$

To do so we apply Statement 1.

Let us check that $\widehat{A} \subseteq \overline{A}$. Indeed, it is easy to check, that P_i satisfy the boundary conditions from the definition of $\overline{A}$, so we have $P_i \in \overline{A}$.

The fact, that P is space of pairwise non-equivalent irreducible representations insures that:

$$\widehat{A}(x_1,x_2) = \overline{A}(x_1,x_2) = M_2(\mathbb{C}) \times M_2(\mathbb{C}),\ \forall x_1,\ x_2 \in P,$$

and automatically:

$$\widehat{A}(x_1,x_2) = \overline{A}(x_1,x_2) \subset M_2(\mathbb{C}) \times M_2(\mathbb{C}),\ \forall x_1,\ x_2 \in \overline{P}.$$

So, by Statement 1 we have $\widehat{A} = \overline{A}$.

Choose a homeomorphism between $\overline{P}$ and X which maps the points $(1,0,0),(0,0,\pm1),(0,1,0) \in \overline{P}$ to the points $(0,1),(\pm1,0),(0,-1) \in X$, correspondingly. This homeomorphism induces the isomorphism between $\overline{A}$ and A_0. □

Remark. It is easy to show that this theorem implies that the space of primitive ideals of algebra A is the same as for algebra of all continuous matrix-functions on the sphere S^2 having values in diagonal matrix in three fixed points. It turns out that A is isomorphic to such an algebra.

Theorem 2. *Let*

$$B = \{f \in C(S^2 \longrightarrow M_2(\mathbb{C}))|f(x_i) \in B_i \simeq M_1(\mathbb{C}) \oplus M_1(\mathbb{C}),\ i=1,2,3\},$$

where $x_1,\ x_2,\ x_3$ are fixed points of the sphere S^2, then $A \simeq B$.

Proof. We will prove this theorem in a few steps, sequently building different realizations of A.

I. It is easy to see, that the algebra A_0 is isomorphic to the algebra A_1, where

$$A_1 = \{(f_1,f_2)|f_1,f_2 \in C(X_1 \longrightarrow M_2(\mathbb{C})),\ f_1(s,0) = f_2(s,0),\ s \in [-1,1],\\ Vf_1(t,1-t)V = f_1(-t,1-t),\ Wf_2(t,1-t)W = f_2(-t,1-t),\ t \in [0,1]\},\\ X_1 = \{(x,y)|(x,y) \in \mathbb{R}^2,\ |x|+y \le 1,\ y \ge 0\}$$

(the norm on the algebra A_1 is natural: $\| (f_1, f_2) \| = max(\| f_1 \|, \| f_2 \|)$).

The boundary conditions for A_1 imply that:

$$f_1(0,1) \in \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \right\}_{a,b \in \mathbb{C}}, f_2(0,1) \in \left\{ \begin{pmatrix} c & d \\ d & c \end{pmatrix} \right\}_{c,d \in \mathbb{C}},$$

$$f_1(1,0) = f_2(1,0) = V f_1(-1,0) V = W f_2(-1,0) W \in \left\{ \begin{pmatrix} e & f \\ -f & e \end{pmatrix} \right\}_{e,f \in \mathbb{C}}.$$

Let

$$R_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \ R_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}.$$

One can check, that $R_1^* \begin{pmatrix} c & d \\ d & c \end{pmatrix} R_1$, $R_2^* \begin{pmatrix} e & f \\ -f & e \end{pmatrix} R_2$ are diagonal matrices for any $c, d, e, f \in \mathbb{C}$. So, one has natural isomorphism, which will be used in considerations below.

$$\left\{ \begin{pmatrix} c & d \\ d & c \end{pmatrix} \right\}_{c,d \in \mathbb{C}} \simeq M_1(\mathbb{C}) \oplus M_1(\mathbb{C}), \left\{ \begin{pmatrix} e & f \\ -f & e \end{pmatrix} \right\}_{e,f \in \mathbb{C}} \simeq M_1(\mathbb{C}) \oplus M_1(\mathbb{C}).$$

II. Let

$$\lambda_1 : [0,\ 1] \longrightarrow U(2),\ t \mapsto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi t} \end{pmatrix},$$

$$\lambda_2 : [0,\ 1] \longrightarrow U(2),\ t \mapsto e^{i\frac{\pi t}{2}} \begin{pmatrix} cos\frac{\pi t}{2} & -isin\frac{\pi t}{2} \\ -isin\frac{\pi t}{2} & cos\frac{\pi t}{2} \end{pmatrix}$$

be homotopies joining the unit matrix E with V and W respectively.

Construct maps $\mu_i : X_1 \longrightarrow U(2)$ by the rule:

$$(x,y) \mapsto \lambda_i \left(\frac{x+1-y}{2(1-y)} \right),\ (x,y) \neq (0,1),$$
$$(0,1) \mapsto E.$$

Neither μ_1 nor μ_2 is continuous, nevertheless it is easy to check, that $\forall (f_1, f_2) \in A_1$, $(\mu_1^* f_1 \mu_1,\ \mu_2^* f_2 \mu_2)$ is a pair of continuous matrix-functions (here $\mu_i^*(x)$, $x \in X_1$, means the adjoint of the matrix $\mu_i(x)$). The correspondence:

$$A_1 \ni (f_1, f_2) \mapsto (\mu_1^* f_1 \mu_1,\ \mu_2^* f_2 \mu_2),$$

induces an isomorphism:

$$A_1 \simeq A_2 = \{(\mu_1^* f_1 \mu_1,\ \mu_2^* f_2 \mu_2) | (f_1, f_2) \in A_1\} =$$
$$= \{(g_1, g_2) |\ g_1, g_2 \in C(X_1 \longrightarrow M_2(\mathbb{C})), g_i(0,1) \in A_2^{(i)} \simeq M_1(\mathbb{C}) \oplus M_1(\mathbb{C}),$$
$$g_1(t, 1-t) = g_1(-t, 1-t), g_2(t, 1-t) = g_2(-t, 1-t), t \in [0,1],$$
$$\lambda_1((s+1)/2) g_1(s,0) \lambda_1^*((s+1)/2) =$$
$$= \lambda_2((s+1)/2) g_2(s,0) \lambda_2^*((s+1)/2), s \in [-1,1], \}.$$

III. Further, the boundary conditions

$$g_1(t,1-t)=g_1(-t,1-t), g_2(t,1-t)=g_2(-t,1-t), \quad t\in[0,1]$$

for algebra A_2 allow us to replace X_1 by $X_1/\sim$ where the equivalence relation $\sim$ is defined as follows:

$$(t,1-t)\sim(-t,1-t), t\in[0,1],$$

and we can consider the algebra A_2 as an algebra of pairs of functions on the quotient space $X_1/\sim$. Evidently $X_1/\sim$ is homeomorphic to the closed unit disk D^2 in $\mathbb{R}^2$. We denote this disk by X_2. In the following, it will be convenient for us to consider X_2 as the unit disk with center $(0,1)$. In the polar coordinates one has:

$$X_2=\{(rcos\phi, rsin\phi)\in\mathbb{R}^2|\ r\leq 2sin\phi,\ 0\leq\phi\leq\pi\}.$$

Below, for any $x\in X$, by $[x]$ we denote its class in $X_1/\sim$. We can suppose that the homeomorphism $\psi: X_1/\sim\longrightarrow X_2$ maps $[(0,1)]$ to the center of disk and the image of $[-1,1]\times\{0\}$ is the boundary of D^2

$$\partial X_2=\{(rcos\phi, rsin\phi)\in\mathbb{R}^2|r=2sin\phi,\ 0\leq\phi\leq\pi\}.$$

To be more precise, one can choose ψ such that:

$$[(0,1)]\mapsto(0,1)\in D^2,$$
$$[(s,0)]\mapsto(2\sin(\pi(s+1)/2),\pi(s+1)/2)\in\partial D^2, s\in[-1,1].$$

The explanations given above show that one can consider the elements of A_2 as the functions on the quotient space. So one has the isomorphism:

$$A_2\simeq A_3=\{(h_1,h_2)=(h_1(r,\phi),h_2(r,\phi))|h_i\in C(X_2\longrightarrow M_2(\mathbb{C})),$$
$$h_i(1,\pi/2)\in A_3^{(i)}\simeq M_1(\mathbb{C})\oplus M_1(\mathbb{C}),$$
$$h_1(2sin\phi,\phi)=\lambda_1^*\left(\frac{\phi}{\pi}\right)\lambda_2\left(\frac{\phi}{\pi}\right)h_2(2sin\phi,\phi)\lambda_2^*\left(\frac{\phi}{\pi}\right)\lambda_1\left(\frac{\phi}{\pi}\right),\phi\in[0,\pi]\}.$$

The boundary conditions in the point $(0,0)$ imply that

$$h_1(0,0)=h_2(0,0)\in\left\{\begin{pmatrix} e & f\\ -f & e\end{pmatrix}\right\}_{e,f\in\mathbb{C}}.$$

IV. To prove an isomorphism $A\simeq B$ we construct a map (non-continuous!):

$$\nu=\nu(r,\phi): X_2\longrightarrow M_2(\mathbb{C}),$$
$$(r,\phi)\mapsto\begin{pmatrix} e^{i\frac{r\phi}{4sin\phi}}cos\frac{\phi}{2} & e^{i\frac{r(\phi-\pi)}{4sin\phi}}sin\frac{\phi}{2}\\ -e^{-i\frac{r(\phi-\pi)}{4sin\phi}}sin\frac{\phi}{2} & e^{-i\frac{r\phi}{4sin\phi}}cos\frac{\phi}{2}\end{pmatrix}, r\neq 0, (0,0)\mapsto\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Note, that the restriction of ν on the set $\{(2sin\phi,\phi)|\phi\in[0,\pi]\}=\partial X_2$ coincides with

$$\lambda_1^*\left(\frac{\phi}{\pi}\right)\lambda_2\left(\frac{\phi}{\pi}\right)=\begin{pmatrix} e^{i\frac{\phi}{2}}cos\frac{\phi}{2} & -ie^{i\frac{\phi}{2}}sin\frac{\phi}{2}\\ -ie^{-i\frac{\phi}{2}}sin\frac{\phi}{2} & e^{-i\frac{\phi}{2}}cos\frac{\phi}{2}\end{pmatrix}= \\ =\begin{pmatrix} e^{i\frac{\phi}{2}}cos\frac{\phi}{2} & e^{i\frac{\phi-\pi}{2}}sin\frac{\phi}{2}\\ -e^{-i\frac{\phi-\pi}{2}}sin\frac{\phi}{2} & e^{-i\frac{\phi}{2}}cos\frac{\phi}{2}\end{pmatrix}.$$

One can check that for every pair $(h_1,h_2)\in A_3$, $(h_1,\nu h_2\nu^*)$ is also a pair of continuous matrix function, so the correspondence:

$$A_3\ni(h_1,h_2)\mapsto(h_1,\ \nu h_2\nu^*).$$

induces an isomorphism:

$$A_3\simeq A_4=\{(h_1,\ \nu h_2\nu^*)|(h_1,h_2)\in A_3\}= \\ =\{(k_1,k_2)|\ k_1,k_2\in C(X_2\longrightarrow M_2(\mathbb{C}));k_1(0,0)= \\ =k_2(0,0)\in\left\{\begin{pmatrix} e & f\\ -f & e\end{pmatrix}\right\}_{e,f\in\mathbb{C}},k_i(1,\pi/2)\in A_4^{(i)}\simeq M_1(\mathbb{C})\oplus M_1(\mathbb{C}); \\ k_1(2sin\phi,\phi)=k_2(2sin\phi,\phi),\ \phi\in[0,\pi]\}.$$

V. The last two conditions for the algebra A_4 allow us to unite pairs of functions in one. Namely, let $S^2=S^2_+\cup S^2_-$, where S^2 is a unit sphere in $\mathbb{R}^3$, $S^2_+,\ S^2_-$ are the upper and the lower closed half-spheres and $\chi_\pm: X_2\longrightarrow S^2_\pm$ be homeomorphisms such that

$$\chi_+(2sin\phi,\phi)=\chi_-(2sin\phi,\phi),\ \phi\in[0,\pi).$$

Denote by $x_1,x_2,x_3\in S^2$ the points $\chi_+(1,\pi/2),\chi_-(1,\pi/2),\chi_+(0,0)=\chi_-(0,0)$. The pair of homeomorphisms $\chi_\pm$ defines an isomorphism:

$$A_4\simeq A_5=\{l\in C(S^2\longrightarrow M_2(\mathbb{C}))| \\ l(x_i)\in A_5^{(i)}\simeq M_1(\mathbb{C})\oplus M_1(\mathbb{C}),i=1,2,3\},$$

given by the rule:

$$A_4\ni(k_1(x),k_2(x))\mapsto l(x)=\begin{cases} k_1(\chi_+^{-1}(x)), & x\in S^2_+,\\ k_2(\chi_-^{-1}(x)), & x\in S^2_-.\end{cases}$$

Evidently $A_5\simeq B$. The proof is completed. □

Acknowledgements.
Author expresses his gratitude to Prof. Yuriĭ S. Samoĭlenko for the problem stating and permanently attention. I am also indebted to Dr. Daniil P. Proskurin for his critical remarks and helpful discussions.

References

[1] Antonevich A., Krupnik N.: On trivial and non-trivial n-homogeneous C^*-algebras. *Integr.equ.oper.theory* **38**(2000), 172–189.

[2] Disney S., Raeburn I.:Homogeneous C^*-algebras whose spectra are tori. *J.Austral.Math.Soc.* (Series A)**38**(1985), 9–39.

[3] Dixmier J.: Les C^*-algebres et leurs representations.*Paris, Gauthier-Villars Editeur* (1969).

[4] Fell J.M.G.:The structure of algebras of operator fields, *Acta Math.*, **106**, n. 3-4, (1961), 233–280.

[5] Ostrovskyĭ, V. and Samoĭlenko, Yu.: Introduction to the Theory of Representations of Finitely Presented *-Algebras. I. Representations by bounded operators. *The Gordon and Breach Publishing group, London* (1999).

[6] Tomiyama J., Takesaki M.: Application of fibre bundles to certain class of C^*-algbras. *Tohoku Math. Journ.***13** n.3, (1961), 498–522.

[7] Vasil'ev N.: C^*-algebras with finite dimensional irreducible representations. *Uspehi mat. nauk.* **XXI.**, n.1, (1966), 136–154.

[8] Vasilevski N., Spitkovski I.: On algebra generated by two projections, *Dokl. Akad. Nauk Ukr. SSR*, Ser. A **8**,(1981),10–13, (Russian).

Contact information

Y. Savchuk
Department of Computer Science,
Kyiv Taras Shevchenko University,
64 Volodymyrska st.,
01033 Kyiv Ukraine
E-Mail: yurius@univ.kiev.ua

Received by the editors: 20.04.2004
and final form in 01.10.2004.