On a common generalization of symmetric rings and quasi duo rings

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Abstract. Let $J(R)$ denote the Jacobson radical of a ring $R$. We call a ring $R$ as $J$-symmetric if for any $a, b, c \in R$, $abc = 0$ implies $bac \in J(R)$. It turns out that $J$-symmetric rings are a common generalization of left (right) quasi-duo rings and generalized weakly symmetric rings. Various properties of these rings are established and some results on exchange rings and the regularity of left SF-rings are generalized.

1. Introduction

All rings considered in this paper are associative ring with identity and $R$ denotes a ring. The symbols $J(R)$, $N(R)$, $Z(R)$, $E(R)$ respectively stand for the Jacobson radical, the set of all nilpotent elements, the set of all central elements and the set of all idempotent elements of $R$. We also denote the set $\{a \in R : a^2 = 0\}$ by $N_2(R)$, the ring of $n \times n$ upper triangular matrix over $R$ by $T_n(R)$ and the left (right) annihilator of any element $a \in R$ by $l(a)$ ($r(a)$). $R$ is abelian if all its idempotents are central. $R$ is left quasi-duo if every maximal left ideal of $R$ is an ideal. As usual, a reduced ring is a ring without non zero nilpotent elements. $R$ is semiprimitive if $J(R) = 0$. $R$ is semicommutative if $l(a)$ is an ideal of $R$ for any $a \in R$. It is well known that $R$ is semicommutative if and only if for any $a \in R$, $r(a)$ is an ideal of $R$. $R$ is symmetric if for any $a, b, c \in R$, $abc = 0$ implies $acb = 0$. $R$ is reversible if $ab = 0$ implies $ba = 0$.

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Various generalizations of symmetric rings have been done by many authors over the last several years. $R$ is weak symmetric ([5]) if for any $a, b, c \in R$, $abc \in N(R)$ implies $acb \in N(R)$. $R$ is central symmetric ([4]) if for any $a, b, c \in R$, $abc = 0$ implies $bac \in Z(R)$. $R$ is generalized weakly symmetric (GWS) ([11]) if for any $a, b, c \in R$, $abc = 0$ implies $bac \in N(R)$.

It follows that the class of GWS rings contains the class of weak symmetric rings. Again, it is known that central symmetric rings are GWS ([11]).

2. Main results

**Definition 1.** A ring $R$ is $J$-symmetric if for any $a, b, c \in R$, $abc = 0$ implies $bac \in J(R)$.

**Proposition 1.** Following conditions are equivalent for a ring $R$:
   1) $R$ is $J$-symmetric.
   2) For any $a, b, c \in R$, $abc = 0$ implies $acb \in J(R)$.

**Proof.** (1) $\Rightarrow$ (2). Let $a, b, c \in R$ such that $abc = 0$ but $acb \notin J(R)$. Then we get a maximal left ideal $M \subseteq R$ such that $acb \notin M$ so that $M + Rab = R$. Therefore $1 = x + yacb$ for some $x \in M$, $y \in R$. Now $(ya)bc = 0$. As $R$ is $J$-symmetric, $byac \in J(R)$. Thus $(1 - x)^2 = yac(byac)b \in J(R) \subseteq M$. Then using $x \in M$ we get $1 \in M$, a contradiction.

(2) $\Rightarrow$ (1). If $a, b, c \in R$ such that $abc = 0$ and $bac \notin J(R)$, then there exists a maximal left ideal $M \subseteq R$ such that $M + Rab = R$ which gives $1 = x + ybac$ for some $x \in M$, $y \in R$. Now $ab(cy) = 0$. Then by hypothesis, $acyb \in J(R)$. Therefore $(1 - x)^2 = yb(acyb)ac \in M$, whence $1 \in M$, a contradiction. Hence $R$ is $J$-symmetric.

**Proposition 2.** Let $R$ be a $J$-symmetric ring and $abc = 0$, then for each maximal left ideal $M$ of $R$, $a \in M$ or $bc \in M$.

**Proof.** If $a \notin M$, then $M + Ra = R$ which implies that $x + ya = 1$ for some $x \in M$, $y \in R$. Then using $abc = 0$ we get $(x - 1)bc = 0$. As $R$ is $J$-symmetric, $bc(x - 1) \in J(R) \subseteq M$ which leads to $bc \in M$.

**Corollary 1.** Let $R$ be a $J$-symmetric ring, then $N_2(R) \subseteq J(R)$.

**Corollary 2.** Let $R$ be a $J$ symmetric ring, then for any $a, b, c \in R$, $abc = 0$ implies $cab \in J(R)$.

The proof of the following proposition is trivial.
Proposition 3. The following conditions are equivalent for a ring $R$:

1) For any $a, b, c \in R$, $abc = 0$ implies $cab \in J(R)$.
2) For any $a, b, c \in R$, $abc = 0$ implies $cba \in J(R)$.

Proposition 4. If $R$ is a ring such that for any $a, b, c \in R$, $abc = 0$ implies $cba \in J(R)$, then $R$ is $J$-symmetric.

Proof. Let $a, b, c \in R$, $abc = 0$ but $bac \notin J(R)$. Then there exists a maximal left ideal $M \subseteq R$ such that $1 = x + ybac$ for some $x \in M$, $y \in R$. Now $ab(cy) = 0$. Then by hypothesis we get $cyba \in J(R)$. Hence $(1 - x)^2 = yba(cyba)c \in M$ leading to $1 \in M$, a contradiction. Hence $R$ is $J$-symmetric. □

Proposition 5. If $R$ is a left quasi-duo ring and $abc = 0$, then for each maximal left ideal $M$ of $R$, $a \in M$ or $b \in M$ or $c \in M$.

Proof. Let $M$ be a maximal left ideal of $R$ and $a \notin M$, then $M + Ra = R$ which implies that $x + ya = 1$ for some $x \in M$, $y \in R$ leading to $xbc = bc$. As $R$ is left quasi-duo and $x \in M$, we get $bc \in M$. If $b \notin M$, then $M + Rb = R$ yielding $u + vb = 1$ for some $u \in M$, $v \in R$, whence $1 - vb \in M$ and so $(1 - vb)c \in M$. Therefore using $bc \in M$ we obtain $c \in M$. □

Proposition 6. A left quasi-duo ring is $J$-symmetric.

Proof. Let $R$ be a left quasi duo ring and $abc = 0$ and $M$ be a maximal left ideal of $R$. It follows from Proposition 5 that $a \in M$ or $b \in M$ or $c \in M$. As $R$ is left quasi-duo, we get $bac \in M$. Therefore $bac \in J(R)$ which proves that $R$ is $J$-symmetric. □

Proposition 7. Central symmetric rings are $J$-symmetric.

Proof. Let $R$ be a central symmetric ring which is not $J$-symmetric. Then there exists $a, b, c \in R$ such that $abc = 0$ but $bac \notin J(R)$ so that there exists a maximal left ideal $M \subseteq R$ such that $1 = x + ybac$ for some $x \in M$, $y \in R$. Now for any $r_1, r_2 \in R$, $(ab)(cr_1)1 = 0$ and $(r_2a)bc = 0$. Hence $cr_1ab, br_2ac \in Z(R)$. Therefore

$$
(1 - x)^4 = (ybac)^4 = ybacyba(cybac)ybac = ybacyba(baccy)ybac
= ybacybabacc(yybac) = ybacybabacc(baccy)
= y(b(acybab)ac)cacy = ycba(b(acybab)ac)cyy
= ycba(c(yb)ab)cacy = ycba(c(yb)ab)aacy
= ycba(abc)yabaacccyy = 0.
$$

This leads to $1 \in M$, a contradiction. Hence $R$ is $J$-symmetric. □
Proposition 8. Generalized weakly symmetric rings are $J$-symmetric.

Proof. Let $R$ be a generalized weakly symmetric ring and $abc = 0$. If $R$ is not $J$-symmetric, then there exists a maximal left ideal $M$ of $R$ such that $1 = x + ybac$ for some $x \in M$, $y \in R$. As $R$ is generalized weakly symmetric and $abcy = 0$, $bacy \in N(R)$ so that $(bacy)^k = 0$ for some positive integer $k$. Therefore

$$(1-x)^{k+1} = (ybac)^{k+1} = y(bacy)^k bac = 0 \in M.$$ 

This together with $x \in M$ implies that $1 \in M$, a contradiction. Hence $R$ is $J$-symmetric. \hfill \square

Corollary 3. Weak symmetric rings are $J$-symmetric.

Remark 1. For a field $\mathbb{F}$ and $n > 1, R = T_n(\mathbb{F})$ is weak symmetric ([5], Proposition 2.3) and hence GWS and $J$-symmetric. As $R$ is not abelian, $R$ is neither central symmetric nor semicommutative. Also, it is worth mentioning here that an abelian ring need not be $J$-symmetric.

Take

$$R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}, a - d \equiv b \equiv c \equiv 0 \pmod{2} \right\}$$

Then $E(R) = \{0, I\}$ where $I$ is the identity matrix over $\mathbb{Z}$. Therefore $R$ is abelian. Consider $A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$. Then $A^2 = 0$ but $A \notin J(R)$ as for

$$K = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, I - KA$$ is not a unit in $R$. Therefore $N_2(R) \not\subseteq J(R)$, hence $R$ is not $J$-symmetric.

A ring $R$ is directly finite if for any $a, b \in R$, $ab = 1$ implies $ba = 1$.

Proposition 9. Every $J$-symmetric ring is directly finite.

Proof. Let $a, b \in R$ such that $ab = 1$. Take $e = ba$, then $e^2 = e$. If $c = b(1 - e)$, then $c^2 = 0$ so that by Corollary 1, $c \in J(R)$ which implies that $ac \in J(R)$ and hence $1 - ac = 1 - ab(1 - e) = e$ is invertible which leads to $e = ba = 1$. \hfill \square

Recall that a ring $R$ is left min-abel if $(1-e)Re = 0$ for any $e \in E(R)$ satisfying $Re$ is a minimal left ideal of $R$.

Lemma 1. For any $e \in E(R)$, $J(eRe) = eJ(R)e$
Theorem 1. Let $R$ be a $J$-symmetric ring. Then

(1) If $e \in E(R)$ such that $ReR = R$, then $e = 1$.
(2) If $e \in E(R)$ and $M$ be a maximal left ideal of $R$, then either $e \in M$ or $(1 - e) \in M$.
(3) $Ra + R(ac - 1) = R$ for any $a \in R$ and $e \in E(R)$.
(4) $R$ is left min-abel.
(5) For any $e \in E(R)$, $eRe$ is $J$-symmetric.

Proof. (1) Since $R$ is $J$-symmetric and $Re(1 - e) = 0$, $eR(1 - e) \subseteq J(R)$. By hypothesis, $ReR = R$ which implies that $R(1 - e) = ReR(1 - e) \subseteq J(R)$, whence $1 - e \in J(R)$ so that $e = 1$.

(2) Follows from Proposition 2 as $e(1 - e) = 0$.

(3) Assume $Ra + R(ac - 1) \neq R$ for some $a \in R$ and $e \in E(R)$, then there exists a maximal left ideal $M$ of $R$ such that $Ra + R(ac - 1) \subseteq M$. If $e \in M$, then $ae \in M$, hence $1 = -(ae - 1) + ae \in M$, a contradiction. If $e \notin M$, then $1 - e \in M$ implying $a - ae = a(1 - e) \in M$. As $ae - 1 \in M$, this leads to $1 \in M$, a contradiction. Hence $Ra + R(ac - 1) = R$ for each $a \in R$ and $e \in E(R)$.

(4) Let $e \in E(R)$ and $Re$ be a minimal left ideal and $(1 - e)Re \neq 0$. Then $R(1 - e)Re = Re$. Now $e \in eRe = eR(1 - e)Re \subseteq J(R)$ which is a contradiction. Therefore $(1 - e)Re = 0$ and $R$ is left min-abel.

(5) Let $e \in E(R)$ and $eae, ebe, ece \in eRe$ with $(eae)(ebe)(ece) = 0$. By hypothesis, $(ebe)(eae)(ece) \in J(R)$ and so $e(ebe)(eae)(ece)e = (ebe)(eae)(ece) \in eJ(R)e = J(eRe)$ by Lemma 1. \hfill $\square$

Converse of (5) of Theorem 1 need not be true. The following example shows this fact.

Example 1. Take $R = M_2(\mathbb{F})$, where $\mathbb{F}$ is a field and consider the idempotent $e = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. It is easy to check that $eRe = \begin{pmatrix} \mathbb{F} & 0 \\ 0 & 0 \end{pmatrix}$ is $J$-symmetric but $R$ is not.

Proposition 10. If $R$ is a $J$-symmetric ring and idempotents can be lifted modulo $J(R)$, then $R/J(R)$ is abelian.

Proof. Let $\overline{R} = R/J(R)$ and $\overline{e} \in E(\overline{R})$. As idempotents can be lifted modulo $J(R)$, there exists $e \in E(R)$ such that $\overline{e} = \overline{e}$. For any $\overline{e} \in \overline{R}$, write $h = xe - exe$. Then $h^2 = 0$ and hence by Corollary 1, $h \in J(R)$. Therefore $\overline{eh} = \overline{e} \overline{x} \overline{e}$, that is $\overline{eh} = \overline{exe}$. Similarly $\overline{eh} = \overline{x} \overline{e}$. Hence $\overline{R}$ is abelian. \hfill $\square$

Proposition 11. If $R/J(R)$ is symmetric, then $R$ is $J$-symmetric.
Proof. Let \( a, b, c \in R \) such that \( abc = 0 \). Then \( \overline{abc} = 0 \). As \( R/J(R) \) is symmetric, \( \overline{bac} = 0 \) which yields \( bac \in J(R) \). Therefore \( R \) is \( J \)-symmetric. 

**Proposition 12.** Direct product of arbitrary family of \( J \)-symmetric rings is \( J \)-symmetric.

**Proof.** For any arbitrary family of rings \( \{R_i : i \in I \} \), we know that \( J(\prod_{i \in I} R_i) = \prod_{i \in I}(J(R_i)) \). Hence the result easily follows.

**Corollary 4.** A ring \( R \) is \( J \)-symmetric if \( eR \) and \( (1-e)R \) are \( J \)-symmetric for any central idempotent \( e \).

**Example 2.** A homomorphic image of a \( J \)-symmetric ring need not be \( J \)-symmetric

Consider \( \mathbb{Z}_2(y) \), the rational functions field of polynomial ring \( \mathbb{Z}_2[y] \) and \( R = \mathbb{Z}_2(y)[x] \) be the ring of polynomials in \( x \) over \( \mathbb{Z}_2(y) \) subject to the relation \( xy + yx = 1 \). Now by ([4], Example 2.11), \( R \) is central symmetric and therefore \( J \)-symmetric. Let \( L = x^2R \), which is a maximal ideal of \( R \). Consider \( \overline{R} = R/L \). Now \( (\overline{x})^2 = 0 \). So \( 0 \neq \overline{1} \in N_2(\overline{R}) \). But \( \overline{R} \) being a simple ring, we have \( J(\overline{R}) = 0 \). Thus we have \( N_2(\overline{R}) \nsubseteq J(\overline{R}) \), hence \( \overline{R} \), a homomorphic image of \( R \) is not \( J \)-symmetric.

The next two propositions gives the condition on an ideal of a ring which forces the ring to be \( J \)-symmetric.

**Proposition 13.** Let \( I \) be a nil ideal of a ring \( R \) such that \( R/I \) is \( J \)-symmetric. Then \( R \) is \( J \)-symmetric.

**Proof.** Let \( a, b, c \in R \) such that \( abc = 0 \). Then \( \overline{abc} = 0 \) in \( R/I \). Since \( R/I \) is \( J \)-symmetric, \( \overline{bac} \in J(R/I) \). Then for any \( r \in R \), there exists \( t \in R \) such that \( 1 - t(1 - rbac) \in I \subseteq J(R) \) since \( I \) is nil. It follows that \( (1 - rbac) \) is left invertible and hence \( bac \in J(R) \).

**Proposition 14.** Let \( I \) be an ideal of a \( J \)-symmetric ring \( S \) and let \( R \) be a subring of \( S \) containing \( I \). Then \( R/I \) is \( J \)-symmetric implies \( R \) is \( J \)-symmetric.

**Proof.** Let \( a, b, c \in R \) such that \( abc = 0 \) in \( R \subseteq S \). Since \( S \) is \( J \)-symmetric, \( bac \in J(S) \). Then for any \( r \in R \subseteq S \), there exists \( s \in S \) such that \( s(1 - rbac) = 1 \). Now \( \overline{abc} = 0 \) in \( R/I \). Since \( R/I \) is \( J \)-symmetric, \( \overline{bac} \in J(R/I) \). Therefore there exists \( t \in R \) such that \( (1 - (1 - rbac)t) \in I \). This yields \( s - s(1 - rbac)t \in I \) and so \( s - t \in I \subseteq R \). This implies \( s \in R \) and hence \( (1 - rbac) \) is left invertible in \( R \) so that \( bac \in J(R) \).
Theorem 2. The following conditions are equivalent for a ring $R$:

1. $R$ is $J$-symmetric.
2. $T_n(R)$ is $J$-symmetric for any $n \geq 2$.
3. $R[x]/(x^n)$ is $J$-symmetric for any $n \geq 2$.
4. $S_n(R) = \left\{ \begin{pmatrix} a & a_{12} & \ldots & a_{1n} \\ 0 & a & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & a \end{pmatrix} : a, a_{ij} \in R, i < j \leq n \right\}$ is $J$-symmetric for any $n \geq 2$.

Proof. Let

\[ I = \left\{ \begin{pmatrix} 0 & a_{12} & a_{13} & \ldots & a_{1n} \\ 0 & 0 & a_{23} & \ldots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \ldots & 0 \end{pmatrix} : a_{ij} \in R, i < j \leq n \right\}. \]

Then $I$ is a nil ideal of $T_n(R)$ as well as $S_n(R)$.

(2) $\Rightarrow$ (1), (3) $\Rightarrow$ (1), (4) $\Rightarrow$ (1) are trivial.

(1) $\Rightarrow$ (2). $T_n(R)/I$ is isomorphic to direct product of $n$-copies of $R$. Hence by Proposition 12 and Proposition 13, $T_n(R)$ is $J$-symmetric.

(1) $\Rightarrow$ (3). Since $S_n(R)/I \simeq R$, it follows that $S_n(R)$ is also $J$-symmetric.

(1) $\Rightarrow$ (4). $R[x]/(x^n) \simeq V_n(R)$ where

\[ V_n(R) = \left\{ \begin{pmatrix} a_0 & a_1 & a_2 & \ldots & a_{n-1} & a_n \\ 0 & a_0 & a_1 & \ldots & a_{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & a_0 \end{pmatrix} : a_i \in R, i = 0, 1, 2, \cdots n \right\}. \]
As $K = I \cap V_n(R)$ is a nil ideal of $V_n(R)$ and $V_n(R)/K \simeq R$, $V_n(R)$ is $J$-symmetric.

If $R$ is $J$-symmetric then $M_n(R)$ need not be $J$-symmetric. The following example shows this fact:

**Example 3.** Let $\mathbb{F}$ be a field and consider $R = M_2(\mathbb{F})$. Now $J(M_2(\mathbb{F})) = M_2(J(\mathbb{F})) = 0$. If $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, then $ABC = 0$, but $BAC \neq 0$.

$R$ is *(von Neumann)* regular if for any $a \in R$, there exists some $b \in R$ such that $a = aba$. $R$ is strongly regular if for any $a \in R$, there exists some $b \in R$ such that $a = a^2b$. It is known that $R$ is strongly regular if and only if $R$ is reduced regular. $R$ is *left SF-ring* if its simple left modules are flat. In 1975, Ramamurthy initiated the study of SF-rings in [10]. It is known that regular rings are left SF-rings. However, till date, it is unknown whether left SF-rings are regular. The regularity of left SF-rings satisfying certain additional conditions have been proved by various authors over the last four decades (see, [6], [9], [10], [11], [14]). The strong regularity of left (right) quasi-duo left SF-rings, central symmetric left SF rings are proved respectively in [6], [11]. These results are generalized as follows:

**Theorem 3.** A $J$-symmetric left SF-ring is strongly regular.

*Proof.~* $R/J(R)$ is a left SF-ring by ([6], Proposition 3.2). Let $b^2 \in J(R)$ such that $b \notin J(R)$. We claim that $Rr(b) + J(R) \neq R$. If this is not true, then $1 = c + \sum r_it_i$, where $c \in J(R)$, $r_i \in R$, $t_i \in r(b)$. This yields $b = cb + \sum r_it_ib$. Now for each $i$, $(t_ib)^2 = t_i(bt_i)b = 0$ and hence by Corollary 1, $t_ib \in J(R)$. Therefore $\sum r_it_ib \in J(R)$ yielding $b \in J(R)$, a contradiction to $b \notin J(R)$. Therefore $Rr(b) + J(R) \neq R$ and so there exists a maximal left ideal $M$ of $R$ containing $Rr(b) + J(R)$. Since $R$ is a left SF-ring and $b^2 \in J(R) \subseteq M$, by ([6], Lemma 3.14), there exists some $d \in M$ such that $b^2 = b^2d$, that is $b - bd \in r(b) \subseteq M$, whence $b \in M$. Hence, again there exists some $e \in M$ such that $b = be$. Then $1 - e \in r(b) \subseteq M$, so that $1 \in M$, contradicting $M \neq R$. Therefore $R/J(R)$ is reduced. Hence by ([6], Remark 3.13), $R/J(R)$ is strongly regular. This implies that $R$ is left quasi-duo. Therefore by ([6], Theorem 4.10), $R$ is strongly regular. □
$R$ is clean if every element of $R$ can be written as a sum of an idempotent and a unit. $R$ is exchange if for any $a \in R$, there exists $e \in E(R)$ such that $e \in Ra$ and $(1 - e) \in R(1 - a)$. In [7], Nicholson proved that every clean ring is exchange. Exchange rings need not be clean but under certain additional conditions exchange rings turns out to be clean (see [1], [2], [3], [7], [11], [12]). It is known that left (right) quasi-duo exchange rings are clean ([12]). Also GWS exchange rings are clean ([11]). These results are extended to $J$-symmetric rings as follows:

**Theorem 4.** Let $R$ be a $J$-symmetric exchange ring. Then $R$ is clean.

**Proof.** Let $x \in R$. By hypothesis, there exists $e \in E(R)$ such that $e \in Rx$ and $(1 - e) \in R(1 - x)$. It is easy to see that $e = yx$ and $1 - e = z(1 - x)$ for some $y, z \in R$ such that $y = ey$ and $z = (1 - e)z$. Therefore $(ze)^2 = 0 = [y(1 - e)]^2$ and so by Corollary 1, $ze, y(1 - e) \in J(R)$. Now $1 - ze - y(1 - e) = (e - zx + z) - ze - y(1 - e) = yx - zx + z - ze - y + ye = (y - z)(x - (1 - e))$. As $ze, y(1 - e) \in J(R)$, $1 - ze - y(1 - e)$ is a unit so that that $x - (1 - e)$ is left invertible. Since a $J$-symmetric ring is directly finite, it follows that $x - (1 - e)$ is a unit and hence $x$ is clean which implies that $R$ is clean.

$R$ has stable range one if for any $a, b \in R$ satisfying $aR + bR = R$, there exists $y \in R$ such that $a + by$ is a unit. It is known that left (right) quasi-duo exchange rings have stable range one. In [11], Wei proved that GWS exchange rings have stable range one. Observing that a $J$-symmetric ring $R$ satisfies $eR(1 - e) \subseteq J(R)$ for any $e \in E(R)$ and using ([8], Theorem 5.4(1)), we get the following theorem which is a generalization of these existing results.

**Theorem 5.** A $J$-symmetric exchange ring have stable range one.

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